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INVESTIGATION OF THE SOURCES OF MEASUREMENT UNCERTAINTY IN THE CALIBRATION OF MEASURING INSTRUMENTS

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ABSTRACT

Measurement uncertainty in calibration depends not only on the calculation method but also on the completeness of the measurement model and the relative contribution of individual sources across the measuring range. This study investigates uncertainty sources in the calibration of a capacitive relative humidity hygrometer by comparison with a reference chilled-mirror hygrometer in a climatic chamber at 25 °C. An eight-input measurement model was evaluated at 20, 50, and 80 %RH using the GUM uncertainty framework, with results validated by the Monte Carlo method. The uncertainty budget changed substantially with humidity. At 20 %RH, calibration and drift of the reference instrument accounted for 72% of the combined variance. At 80 %RH, hysteresis of the instrument under calibration became the largest contribution at 45%, followed by reference calibration and chamber non-uniformity. Expanded uncertainty increased from 0.70 to 1.37 %RH across the range. Monte Carlo validation confirmed the GUM interval at 20 %RH, was borderline at 50 %RH, and failed at 80 %RH because the dominant rectangular hysteresis component produced a non-Gaussian output distribution. Targeted reduction of hysteresis and chamber non-uniformity decreased expanded uncertainty by up to 30%. The findings support point-specific uncertainty budgets and Monte Carlo validation when non-Gaussian components dominate.

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1. Introduction

Calibration establishes the relationship between the indications of a measuring instrument and the values realized by measurement standards, and the uncertainty stated in a calibration certificate is transferred to every subsequent measurement made with the instrument. ISO/IEC 17025:2017 (clause 7.6) requires calibration laboratories to identify all significant contributions to measurement uncertainty. Current JCGM guidance emphasizes that the quality of an uncertainty evaluation is determined primarily by the completeness and correctness of the measurement model rather than by the calculation technique (Joint Committee for Guides in Metrology [JCGM], 2023; Possolo et al., 2024; van der Veen et al., 2022).

In practice, uncertainty budgets are often compiled once for a whole measuring range using a fixed list of components. This approach overlooks the fact that the relative significance of sources may change along the range, and that the GUM uncertainty framework (GUF) based on the law of propagation of uncertainty may give an inaccurate coverage interval when a non-Gaussian component dominates (Castro, 2024; García-Berrocal et al., 2024). Humidity calibration is a representative example: capacitive sensors

exhibit humidity-dependent hysteresis and temperature sensitivity, and chamber conditions contribute effects comparable to the uncertainty of the reference (Kapić et al., 2023).

The aim of this work is to identify and quantify the sources of uncertainty in the calibration of a relative humidity hygrometer, to investigate how their contributions change along the measuring range, to validate the GUF results by the Monte Carlo method (MCM), and to substantiate measures for reducing the calibration uncertainty.

2. Materials and methods

The object of the study is the calibration of a capacitive thermohygrometer (instrument under calibration, UUC) with a resolution of 0.1 %RH by comparison with a reference chilled-mirror hygrometer in a climatic chamber at 25 °C. Calibration is performed at 20, 50 and 80 %RH with five readings at each point; the points are passed in ascending and descending order to evaluate hysteresis. The input estimates correspond to values typical of this class of equipment (calibration certificate of the reference, instrument specifications and chamber characterization), so the study is a model case study.

The measurand is the error of the UUC indication E, described by the measurement model:

$$E = H_x + \delta H_{x,res} + \delta H_{hys} - H_s - \delta H_{s,cal} - \delta H_{s,drift} - \delta H_{s,res} - \delta H_{unif} - \delta H_{stab} \qquad (1)$$

where H_x and H_s are the mean indications of the UUC and the reference; $\delta H_{x,res}$ and $\delta H_{s,res}$ are corrections for resolution; δH_{hys} is the correction for hysteresis of the UUC; $\delta H_{s,cal}$ and $\delta H_{s,drift}$ are corrections for the calibration and drift of the reference; δH_{unif} and δH_{stab} are corrections for spatial non-uniformity and temporal instability of the chamber. All corrections have zero estimates, and all sensitivity coefficients are equal to ±1, so the combined standard uncertainty is:

$$u_c(E) = \sqrt{\Sigma_{i=1}^{8} u_i^2} \qquad (2)$$

The non-uniformity of relative humidity is caused mainly by temperature gradients: at a constant dew point, $|\partial H/\partial T| \approx 0.06H$ K^{-1} at 25 °C, therefore for a maximum temperature difference ΔT_{max} between the sensors:

$$u_{\text{unif}} = \frac{0.06H\Delta T_{max}}{\sqrt{3}} \qquad (3)$$

The hysteresis component is evaluated from the maximum difference h between the readings in ascending and descending order, assuming a rectangular distribution: $u_{\text{hys}} = \frac{h}{2\sqrt{3}}$. The significance of each source is characterized by its share in the combined variance $r_i = \frac{u_i^2}{u_c^2}$.

The GUF results were validated by the MCM in accordance with GUM Supplement 1 (JCGM 101) using $M = 2 \cdot 10^6$ trials. The GUF is considered validated if the endpoints of the 95 % coverage intervals obtained by both methods differ by no more than the numerical tolerance $\delta = 0.005$ %RH, corresponding to two significant digits of u_c:

$$d_{max} = \max\left(|-1.96u_c - y_{low}|, |1.96u_c - y_{high}|\right) \leq \delta \qquad (4)$$

3. Results

The uncertainty budgets at the three calibration points are given in Table 1, and the structure of the combined variance is shown in Figure 1a.

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Table 1

Uncertainty Budget of the Hygrometer Calibration (Standard Uncertainties, %RH)

Source of uncertainty	Type, distribution	20 %RH	50 %RH	80 %RH
Calibration of the reference	B, normal	0.240	0.300	0.360
Drift of the reference	B, rectangular	0.173	0.173	0.173
Resolution of the reference	B, rectangular	0.003	0.003	0.003
Resolution of the UUC	B, rectangular	0.029	0.029	0.029
Repeatability of the UUC (n = 5)	A, normal	0.045	0.054	0.067
Spatial non-uniformity of the chamber	B, rectangular	0.070	0.175	0.279
Temporal instability of the chamber	B, rectangular	0.115	0.115	0.115
Hysteresis of the UUC	B, rectangular	0.115	0.231	0.462
Combined standard uncertainty u_c		0.349	0.470	0.685
Expanded uncertainty U, $k = 2$		0.70	0.94	1.37

At 20 %RH, the budget is dominated by the reference instrument: its calibration and drift together account for 72 % of the combined variance. At 50 %RH, the contributions are distributed more evenly (reference calibration 41 %, hysteresis 24 %, chamber non-uniformity 14 %, reference drift 14 %). At 80 %RH, the hysteresis of the UUC becomes the main source (45 %), followed by the reference calibration (28 %) and chamber non-uniformity (17 %). Repeatability and resolution do not exceed 2 % at any point. The expanded uncertainty increases almost twofold from 0.70 to 1.37 %RH across the range.

The results of the GUF validation are given in Table 2 and Figure 1b.

Table 2

Validation of the GUM Uncertainty Framework by the Monte Carlo Method (95% Coverage Intervals, %RH)

Point, %RH	u_c	GUF interval	MCM interval	d_{max}	GUF validated
20	0.349	±0.685	[−0.680; +0.681]	0.004	yes
50	0.470	±0.921	[−0.916; +0.915]	0.006	borderline
80	0.685	±1.343	[−1.319; +1.320]	0.024	no

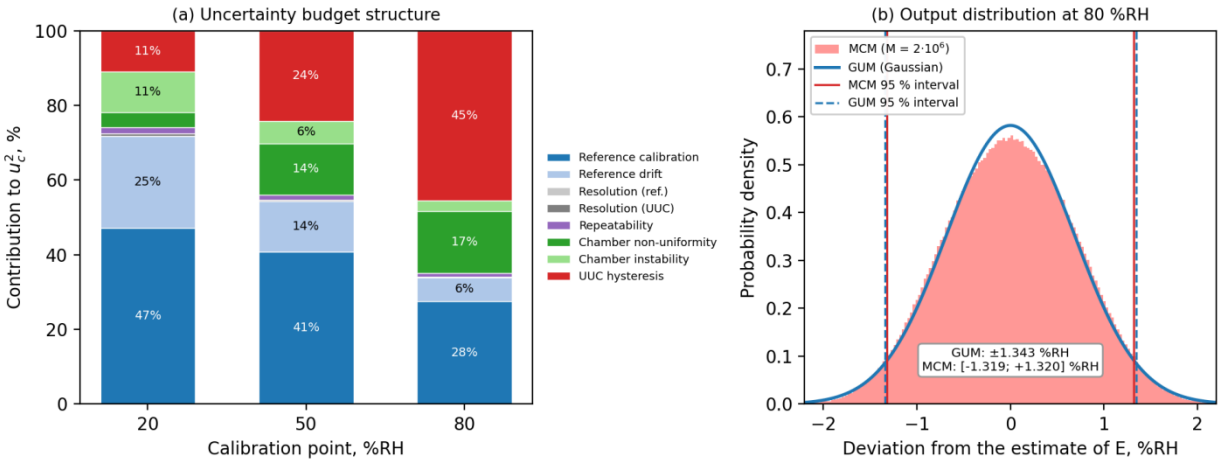


Figure 1. (a) Contributions of uncertainty sources to the combined variance at three calibration points; (b) probability distribution of the calibration error at 80 %RH obtained by the Monte Carlo method in comparison with the Gaussian distribution assumed by the GUF

At 20 %RH, the GUF is validated. At 50 %RH, the difference is at the level of the numerical tolerance. At 80 %RH, the GUF interval is wider than the MCM interval by 0.024 %RH, which exceeds the tolerance almost fivefold; the output distribution is flatter than the Gaussian (excess kurtosis −0.29), as a result of the dominant rectangular hysteresis component.

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4. Discussion

The results show that the list of significant sources and their ranking depend on the calibration point. A single budget for the whole range either overestimates the uncertainty at low humidity or underestimates it at high humidity, where the humidity-dependent hysteresis and the temperature-induced non-uniformity grow. This agrees with the data of Kapić et al. (2023), who showed a strong dependence of the uncertainty components of capacitive sensors on the conditions of use. The uncertainty of the reference should also be treated as time-dependent: the drift component (14–25 % at low and medium humidity) is directly linked to the recalibration interval of the reference and can be reduced by uncertainty-based scheduling of its recalibration (Schaude & Hausotte, 2024).

The failure of GUF validation at 80 %RH is explained not by the non-linearity of the model, which is linear, but by the dominance of a rectangular component, which violates the conditions of the central limit theorem. The same effect was reported for the calibration of volume meters and tank provers (Castro, 2024; García-Berrocal et al., 2024). In this case, the GUF gives a conservative interval; the MCM makes it possible to report a smaller but still reliable uncertainty, which is important for instruments operating close to the tolerance limits.

The investigation of sources makes it possible to direct improvement measures to the dominant components. For the 80 %RH point, the calculation according to model (1) shows that reporting separate corrections for increasing and decreasing humidity, which reduces the residual hysteresis from 1.6 to 0.6 %RH, decreases the expanded uncertainty from 1.37 to 1.07 %RH (by 22 %). Improving air mixing in the chamber so that the temperature difference between the sensors does not exceed ±0.05 K reduces it to 1.28 %RH (by 6 %), and both measures together reduce it to 0.95 %RH (by 30 %). At the same time, improving the resolution of the UUC or increasing the number of readings would have a negligible effect, since these sources contribute less than 2 %.

5. Conclusion

An investigation of the sources of measurement uncertainty in the calibration of a relative humidity hygrometer was carried out on the basis of an explicit measurement model with eight input quantities. It was established that the structure of the uncertainty budget changes significantly along the measuring range: the reference instrument dominates at 20 %RH, whereas the hysteresis of the instrument under calibration dominates at 80 %RH. The GUM uncertainty framework is validated by the Monte Carlo method at low humidity but not at high humidity, where a dominant rectangular component makes the output distribution non-Gaussian. The results justify point-specific uncertainty budgets, validation of the GUF by the MCM when non-Gaussian components dominate, and targeted improvement measures that reduce the expanded uncertainty by up to 30 %. Further research will focus on experimental confirmation of the results with data from an accredited calibration laboratory.

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