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URBAN BUS ROUTE OPTIMIZATION USING DYNAMIC PROGRAMMING AND PARETO-EFFICIENT SELECTION

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ABSTRACT

The paper proposes a route-optimization framework that combines dynamic programming with Pareto-efficient selection for urban bus networks operating under variable demand and service conditions. The network is represented as a finite set of states connected by admissible transitions, so route formation can be treated as a sequence of linked decisions rather than as a single shortest-path calculation. The procedure evaluates candidate trajectories using passenger- and operator-oriented measures, including travel time, waiting time, transfer convenience, service reliability, and resource requirements. At each stage, Bellman recursion is used to propagate cumulative performance through the state graph, while a nondominance test removes alternatives that are inferior across the selected criteria. The retained labels form an efficient set that can be updated when GPS/GNSS observations, fare transactions, dispatching records, or passenger-flow estimates change. The analytical results show that the combined procedure preserves meaningful trade-offs between user convenience and operating efficiency and avoids premature reduction of the problem to a single weighted objective. For Nukus, the framework can serve as a decision-support mechanism for adaptive route planning and subsequent coordination with timetables, fleet allocation, and digital transport services.

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Introduction

Designing an urban bus network requires decisions about route alignment, demand coverage, service frequency, transfers, fleet use, and reliability. These objectives are interdependent and often conflict. A path that reduces passenger travel time may require additional buses, whereas broader territorial coverage can increase running time and make transfers more complex. Consequently, route design is better viewed as a multistage decision problem than as a conventional shortest-path task.

Daily operating conditions add another layer of uncertainty. Passenger volumes change by time of day, congestion alters link travel times, dwell times are not constant, and service intervals may become unstable. Because the attractiveness of a route can change during the operating day, a static representation based on fixed costs is insufficient. A more suitable approach is to describe the network through states and feasible transitions whose performance values can be refreshed as new operational information becomes available.

Recent studies in Uzbekistan point in the same direction. Matrasulov and Yo‘ldoshev (2025) used several expert indicators to assess the condition of urban public transport infrastructure. Mirzabekov and Kurbonova (2025) highlighted the role of real-time monitoring, sensor information, and integrated data in intelligent transport management. Jurayev and Ikromov (2025) examined mathematical approaches to improving bus regularity, while Abdullayev (2025) considered passenger-flow forecasting as an input for service and fleet planning. Together, these works show that route decisions increasingly depend on combined demand, operational, and digital information.

International research provides complementary evidence. Dou and Li (2023) addressed timetable coordination when travel time is uncertain. Risso et al. (2023) formulated a bi-objective problem that balances transfer synchronization against operating cost. Lin et al. (2023) used Pareto analysis to represent heterogeneous comfort preferences, while Hu et al. (2023) jointly optimized timetables and vehicle schedules with attention to fleet cost and passenger waiting. Real-time Pareto-based control has also been investigated by Pasquale et al. (2024). More recent studies have linked adaptive dispatching to changing demand (Barrera Hernandez et al., 2025) and used Pareto frontiers to balance passenger comfort with operator efficiency (Cheng & He, 2024). Reviews of bus-system planning further confirm the importance of treating network design, operation, and control as connected decisions (Ibarra-Rojas et al., 2015), while Mandl’s (1980) classical work provides an early foundation for network-level route optimization. The aim of this study is therefore to formulate a reproducible route-search procedure in which dynamic programming structures the sequential decision process and Pareto screening preserves efficient alternatives under conflicting criteria.

Materials and methods. The route network is modeled as a directed state graph. A state can represent a stop, transfer point, route segment, or another information condition relevant to passenger or vehicle movement. From each state, only admissible decisions are allowed, and every decision leads to a successor state. A complete route is therefore constructed as a chain of local transitions from an origin to a terminal state. The method is designed so that another researcher can reproduce the procedure once the state set, feasible transitions, criterion values, and operating constraints are specified.

Table 1

Core Elements of the Dynamic Route-Optimization Model

Model component	Symbol	Role in computation
Admissible states	D	Set of movement and information states included in the route-search space.
Terminal set	F	Destination states at which the recursive search stops.
Available actions	$V(x)$	Feasible decisions that may be taken from state x.
State transition	$f(x, v)$	Rule that maps a current state and selected action to a successor state.
Local measure	$s(x, v)$	Criterion value assigned to a permitted transition.
Route trajectory	T	Ordered state sequence linking an origin with a terminal state.
Local aggregate	D_i	Optional weighted indicator constructed from criterion-specific partial evaluations.

The state-space description used in the model is:

$$\Omega = \{D, F, V(x), f(x,v), s(x,v)\} \quad (1)$$

A feasible route is described by a finite sequence $T = \{x^0, x^1, x^2, \ldots, x^n\}$. Consecutive states must satisfy the admissible transition rule:

$$x^t = f(x^{t-1}, v^t), \quad v^t \in V(x^{t-1}), \quad t = 1,2,\ldots,n \quad (2)$$

For a single cost criterion, the continuation value is computed recursively according to Bellman’s principle of optimality (Bellman, 1957):

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$$S(x) = min_{v} \in V(x) \{ s(x,v) + S(f(x,v)) \}, \quad x \in D \backslash F \quad (3)$$

Terminal states are assigned a continuation value of zero. The calculation then proceeds backward from the destination toward the origin. For each predecessor state, the admissible successor producing the best continuation value is stored. Because downstream values are reused, common route segments do not need to be evaluated repeatedly. This makes the procedure suitable for layered networks in which several alternatives merge at the same transfer point.

To represent several objectives at the local level, a weighted performance indicator may be written as:

$$D_i = \Sigma_{j=1}^{m} b_{ij} c_j \rightarrow max, \qquad \Sigma_{j=1}^{m} c_j = 1 \quad (4)$$

In this expression, b_{ij} denotes the performance of a partial decision under criterion j, and c_j is the relative importance assigned to that criterion. For Equation (4), larger b_{ij} values represent better performance, so the weighted indicator is maximized. The scalar form is useful when stable preferences are available, but it may conceal meaningful trade-offs. For that reason, the main selection procedure works with criterion vectors and applies Pareto dominance before the next dynamic-programming stage.

The dominance rule is implemented in minimization form. Candidate a dominates candidate b when a is no worse than b on every criterion and is strictly better on at least one criterion. Criteria expressed as benefits, such as reliability or service quality, are transformed so that all components have the same optimization direction. Only nondominated labels are retained at each state. The computational sequence is repeated whenever updated demand, travel-time, or fleet information changes the local criterion values.

Table 2

Reproducible Optimization Procedure

Step	Required information	Computation	Result
1. Build the network	Stops, transfer points, route segments	Encode admissible states and directed transitions.	State graph.
2. Attach data	Demand, travel times, monitoring and dispatch records	Assign criterion values to each permitted transition.	Transition-value database.
3. Initialize destination	Terminal state set F	Set the continuation value at terminal states to zero.	Boundary values.
4. Run backward recursion	Predecessor states and admissible actions	Evaluate predecessor labels using Bellman recursion.	Candidate cumulative labels.
5. Screen by dominance	Criterion vectors for candidate labels	Discard labels dominated on the selected criteria.	Efficient labels at each state.
6. Check operations	Fleet availability and service constraints	Remove routes that violate practical operating limits.	Feasible efficient routes.
7. Refresh when data change	Updated GPS/GNSS, fare, demand, or dispatch information	Revise affected transition values and repeat Steps 4–6.	Updated efficient set.

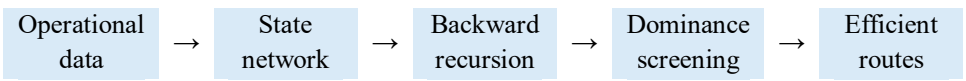


Figure 1. Computational sequence used to generate efficient route alternatives.

The procedure is intended for repeated updating rather than one-time optimization. When observed demand, route travel time, transfer conditions, or fleet availability change, only the affected local values are revised before backward recursion and nondominance screening are performed again. GPS/GNSS

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observations, fare transactions, dispatching logs, and passenger-flow estimates can therefore be incorporated without rebuilding the entire state structure.

Results. The resulting architecture divides route optimization into two coordinated operations. Dynamic programming propagates the consequences of local choices through the network, whereas Pareto screening limits the candidate set to criterion combinations that remain efficient. This structure prevents the solution from being driven by a single arbitrary cost measure and avoids carrying every possible trajectory to the final stage.

The first result is a finite-state representation of the route-stop-transfer system. The graph contains an origin, intermediate decision states, transfer states, and a destination connected by admissible arcs. Each arc stores local performance information, and a complete route is assembled from the corresponding transition sequence. This formulation makes route formation directly compatible with algorithmic search and allows the same network structure to support different demand scenarios.

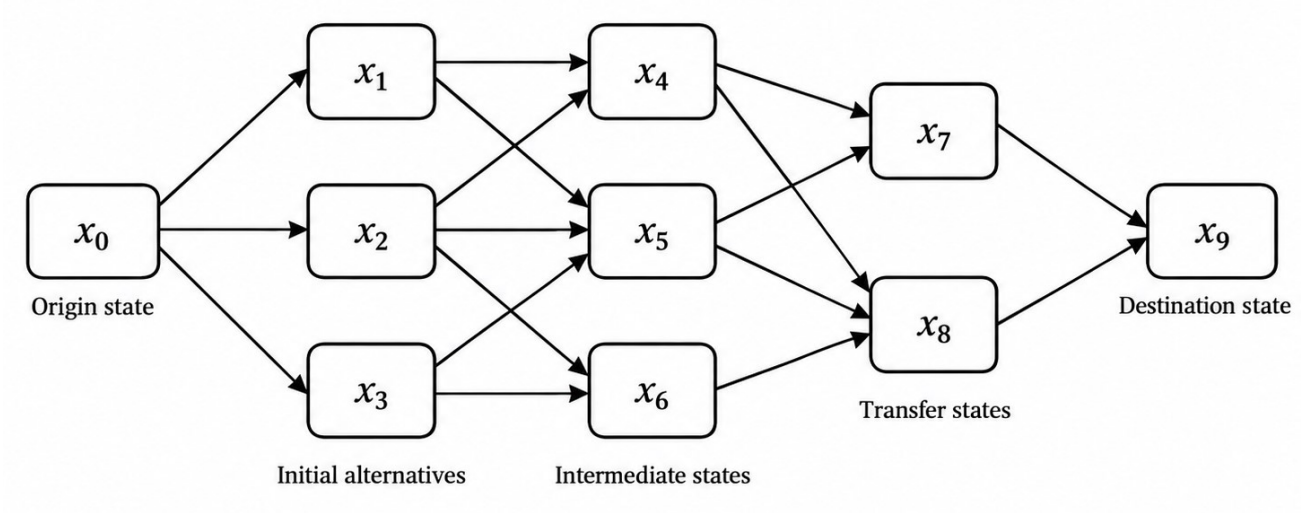


Figure 2. Layered state-transition representation used in the route-search procedure.

The second result is a recursive search mechanism. Any feasible trajectory can be separated into an immediate decision and a remaining subproblem. Equation (3) exploits this structure by storing the continuation value associated with each downstream state. When several alternatives share the same transfer point, their common continuation is evaluated once and reused, thereby reducing unnecessary repetition in the search process.

The third result is the preservation of trade-offs among several objectives. Equation (4) can be applied when criterion weights are known and sufficiently stable. If the weights are uncertain, or if the decision-maker needs to see competing performance profiles explicitly, Pareto screening is preferable. It can retain, for example, a route with a slightly higher operating requirement when that route substantially reduces passenger time or transfers, while also retaining a lower-cost alternative when its service penalty remains acceptable.

The fourth result is the ability to react to changing operating conditions without altering the model structure. A congestion event may increase the value attached to one transition, a new passenger-flow pattern may change the importance of a transfer point, and fare or dispatching records may reveal a shift in demand. After such changes, only the affected labels are recalculated and the efficient set is refreshed. This feature is particularly relevant to an urban system such as Nukus, where both demand and service conditions vary over time.

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Table 3

Main Analytical Outcomes and Their Operational Meaning

Analytical outcome	Methodological role	Use in planning
Finite-state representation	Encodes route construction as admissible state transitions.	Enables algorithmic comparison of route alternatives.
Recursive evaluation	Reuses downstream continuation values across multistage decisions.	Reduces repeated evaluation of shared route segments.
Pareto screening	Keeps only alternatives that remain nondominated across conflicting criteria.	Makes user–operator trade-offs explicit.
Data-driven refresh	Allows transition values to change when new observations arrive.	Supports re-optimization after demand or traffic changes.
Digital-system compatibility	Connects route, stop, transfer, vehicle, passenger, and payment information.	Facilitates linkage with monitoring, dispatching, and mobility services.

The study is analytical and does not report a universal percentage improvement across all routes. Its contribution is the decision structure itself: a finite-state formulation, explicit transition rules, backward recursion, multicriteria local evaluation, and state-by-state elimination of dominated alternatives. Quantitative gains should be reported only after the procedure has been tested using measured route data against a defined baseline and evaluation protocol.

Discussion and conclusion. The proposed method is consistent with current public-transport optimization research, but it organizes the decision problem at the route-state level. Dou and Li (2023) concentrate on timetable coordination under uncertain travel time, whereas the present framework starts from the sequence of route states through which a passenger or vehicle can move. Risso et al. (2023) obtain Pareto-efficient timetable solutions, and Lin et al. (2023) preserve trade-offs among passenger, operator, and policy objectives. These studies support the decision to keep efficient alternatives visible instead of compressing all objectives into a single fixed score too early.

Hu et al. (2023) show that timetable and vehicle scheduling can be coordinated while balancing fleet cost and passenger waiting. Pasquale et al. (2024) demonstrate that Pareto-efficient decisions can also be selected in real time. In the framework proposed here, the same idea is embedded within a dynamic route search: nondominated alternatives are preserved at intermediate states, not only after all complete routes have been enumerated. This gives the decision-maker access to efficient trade-offs throughout the search process.

The adaptive character of the method is also consistent with the work of Barrera Hernandez et al. (2025), who combine demand forecasting with responsive dispatching. Cheng and He (2024) similarly show the value of balancing passenger comfort with operating efficiency. In the present model, new digital observations revise the local state values, after which only the affected recursive calculations need to be updated. This creates a practical link between strategic route design and continuously changing operational information.

Several limitations remain. The quality of the efficient set depends on the accuracy of passenger-flow, travel-time, and fleet data. The number of nondominated labels may also grow rapidly when many criteria are introduced, so large networks may require additional pruning or preference rules. Finally, a weighted scalar indicator is sensitive to expert judgment when the coefficients are not well established. For planning applications, Pareto representation is therefore useful because the final preference is postponed until clearly inefficient alternatives have been removed.

For Nukus, the method is best interpreted as a decision-support layer within a wider digital transport environment. Information about routes, stops, transfer points, buses, passengers, payments, and monitoring

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can be connected to the same state representation. The efficient route set can then be tested against fleet availability, timetable constraints, service coverage, and transfer coordination before implementation.

Conclusion

Dynamic programming and Pareto-efficient selection complement one another in urban bus route optimization. The recursive component divides a complex network problem into manageable subproblems, while the dominance test preserves only alternatives that remain competitive across the selected criteria. The combined procedure makes passenger-operator trade-offs transparent, avoids dependence on a single objective, and can be updated as digital observations change. Its main scientific value lies in connecting state-based route representation, recursive optimization, and multicriteria efficient selection within one reproducible framework. Applied to Nukus, it offers a methodological basis for adaptive route planning and for subsequent coordination with timetables, dispatching, electronic fare collection, and other digital transport services.

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