ISSN: 2181-3337 | SCIENTISTS.UZ VOLUME 5 ISSUE 7 JULY 2026

# DETERMINATION OF THE BALANCING MOMENT OF A CRANK-SLOTTED MECHANISM USING THE N.E. ZHUKOVSKY LEVER METHOD

**Siddikov Shuhrat**[1], **Abdazimov Anvar**[1*]

[1]Tashkent State Technical University named after Islam Karimov,
2 University St., 100174 Tashkent, Uzbekistan
* Corresponding author: anvarabdazimov95996@gmail.com

**DOI: 10.67620/sai.57a0014**

**ABSTRACT**

*This study presents the determination of the balancing force and balancing moment of a multiloop crank–slotted cotton-bush guiding mechanism used in the spindle drum of a cotton-harvesting machine. The analysis is performed using the N.E. Zhukovsky lever method, which enables the required balancing parameters to be obtained without directly determining reaction forces in the kinematic pairs or carrying out a complete force analysis of Assur groups. The investigated mechanism consists of a crank, sliding block, and rotating slotted link. Calculations are performed for a crank rotation angle of φ1=30∘, using previously determined kinematic parameters together with the inertia forces of the mechanism links and the experimentally established technological resistance force. The velocity diagram is rotated by 90∘ and treated as the Zhukovsky lever, after which the acting forces and inertia moment are transferred to the corresponding points. The balancing force is obtained from the moment-equilibrium equation of the transformed velocity diagram. For the considered mechanism position, the calculated balancing force is approximately 59.74 N, corresponding to a balancing moment of approximately 8.72 N·m. The proposed procedure provides a relatively simple and effective approach for evaluating the dynamic loading of the mechanism and can be applied to other crank positions to determine the variation of balancing force and moment throughout the operating cycle.*

**ACCEPTED**

**10.08.26**

***Keywords:*** *crank-slotted mechanism; cotton-bush guiding mechanism; cotton picker; N.E. Zhukovsky lever; force analysis; balancing force; balancing moment.*

CC BY

## Introduction

Slotted-link mechanisms are classical mechanisms in the theory of mechanisms and machines and are widely used in mechanical engineering. They are employed when rotary or oscillatory motion must be transformed into reciprocating linear motion and can provide high linear speeds of working members. Typical applications include valve-actuation systems in internal-combustion engines, shaping and slotting machines, reciprocating pumps, presses, and packaging equipment. Table 1 summarizes several common configurations.

**Table 1.**

**Classification of slotted-link mechanisms**

| Mechanism type | Motion of the slotted link | Parameter relationship | Typical applications |
|---|---|---|---|
| Crank-slotted mechanism with an oscillating slotted link | Reciprocating angular motion | l > r (l: center distance; r: crank radius) | Shaping machines; reciprocating pumps |
| Crank-slotted mechanism with a rotating slotted link | Continuous rotation | l < r | Slotting machines; packaging equipment |
| Slotted-link slider mechanism | Oscillatory | Variable | Automatic machines; presses |
| Double-slotted-link mechanism | Motion transmission | i = 1 (constant) | Compensating couplings |

The operating principle of a slotted-link mechanism is governed by the fundamental laws of applied mechanics, kinematics, and statics [1,2]. The links are idealized as rigid bodies of finite dimensions and mass. Dynamic analysis therefore requires information about mass distribution, displacements, velocities, accelerations, external loads, inertia forces, and inertia-force moments. The balancing force and balancing moment are then determined with allowance for the working resistance force.

The trajectory of each point of a crank-slotted mechanism is determined by the link geometry and the working radii of its elements. A rotating or oscillating link acts on a translationally moving member at their kinematic connection. The guided member moves with one degree of freedom until it reaches an extreme position, after which its direction of motion reverses as the driving link continues its cycle.

A balancing force (or balancing moment) is the force (or moment) supplied by the drive that balances all forces acting on the mechanism, including inertia forces and inertia-force couples. Determination of the balancing force P′bal and/or balancing moment M′bal is therefore a necessary part of mechanism force analysis. Their point of application and direction are prescribed by the drive and transmission arrangement.

One approach for determining these quantities is the N.E. Zhukovsky lever method. The Zhukovsky lever is obtained by rotating the previously constructed velocity diagram by 90° and treating the rotated diagram as a rigid body fixed at the velocity pole Pv. External forces are transferred, parallel to themselves, to the corresponding points of this rotated diagram. The moment equilibrium of the resulting lever gives the balancing force. The method avoids the explicit solution of reaction forces in the kinematic pairs and the force analysis of Assur groups, although the required kinematic quantities, including accelerations used to obtain inertia forces, must already be known [1,2].

The objective of this work is to determine the balancing force and balancing moment of the multiloop crank-slotted cotton-bush guiding mechanism of the spindle drum of a vertical-spindle cotton picker [3] using the N.E. Zhukovsky lever method. The object of the study is the mechanism with a rotating slotted link at the crank position $\varphi_1 = 30°$.

**Materials and Methods**

The balancing force P′bal is first obtained from the moment-equilibrium condition of the Zhukovsky lever. It is assumed to act at point $B_1$ perpendicular to crank $OB_1$. The balancing moment is then calculated from $M'bal = P'bal \cdot OB_1$. The analysis uses the mechanism diagram at $\varphi_1 = 30°$ (Fig. 1), the previously determined velocity diagram (Fig. 2), and the previously determined acceleration quantities for this mechanism [4,5,7].



ISSN: 2181-3337 | SCIENTISTS.UZ

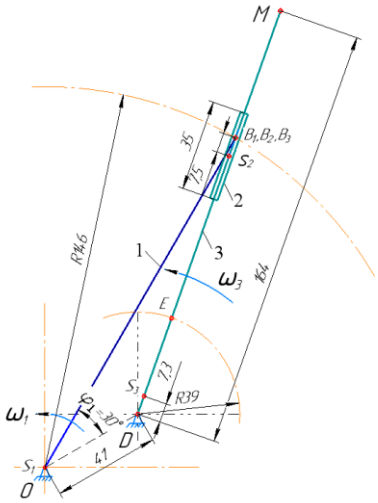



**Fig. 1. Mechanism diagram at a crank rotation angle $\varphi_1 = 30°$: 1 - crank; 2 - sliding block; 3 - rotating slotted link; O and D - fixed supports; $S_1$, $S_2$, and $S_3$ - link centers of mass.**

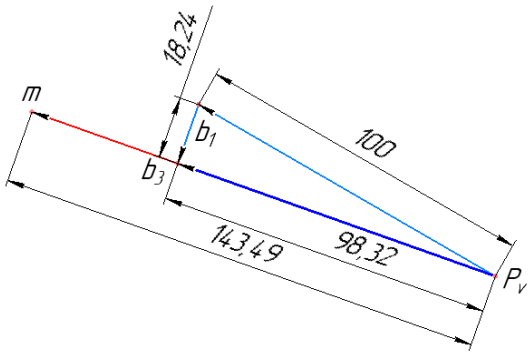



**Fig. 2. Velocity diagram of the mechanism at $\varphi_1 = 30°$ ($\mu_v = 0.001$ (m/s)/mm).**

**Initial data and inertia quantities**

The geometric and inertial data used for the considered position are as follows:

1) OD = 0.041 m - distance between the fixed supports O and D.
2) $OB_1 = 0.146$ m - crank length (link 1); crank mass $m_1 = 32$ kg. The crank center of mass $S_1$ coincides with the rotation axis at O, so $OS_1 = 0$. A later parameter determination for the same mechanism gives $\omega_1 = 13.2$ rad/s [7]; because $OS_1 = 0$, the centripetal acceleration of the crank mass center is zero.
3) $B_2$ - sliding block (link 2), length $\ell_2 = 0.035$ m and mass $m_2 = 0.036$ kg.
4) DM - rotating slotted link (link 3), length DM = 0.164 m and mass $m_3 = 1.349$ kg [7].
5) Qres = 36.61 N - working resistance (technological load) acting on the rotating slotted link at $\varphi_1 = 30°$, determined previously by electrical strain-gauge measurement [6].

$$aS_1 = \omega_1^2 \cdot OS_1 = (13.2)^2 \cdot 0 = 0 \text{ m/s}^2$$

For a link i, the inertia force and the inertia-force moment are written as [1,2]:

$$F_i = -m_i \cdot aS_i \quad (1)$$

where $m_i$ is the link mass, kg, and $aS_i$ is the total acceleration vector of the link center of mass, m/s². The minus sign indicates that the inertia force acts opposite to the center-of-mass acceleration.

$$M_i = -JS_i \cdot \varepsilon_i \quad (2)$$

where $JS_i$ is the mass moment of inertia of the link about its center-of-mass axis, kg·m², and $\varepsilon_i$ is the angular acceleration, rad/s². The minus sign indicates that the inertia moment acts opposite to the angular acceleration.

Because $aS_1 = 0$, the crank inertia force is $F_{i1} = m_1 \cdot aS_1 = 32 \cdot 0 = 0$ N.

For the sliding block at $\varphi_1 = 30°$, the previously determined center-of-mass acceleration is $aS_2 = 28.57$ m/s² [5]. Hence $F_{i2} = m_2 \cdot aS_2 = 0.036 \cdot 28.57 = 1.029$ N.

For the rotating slotted link, $aS_3 = 2.14$ m/s² [5]. Using the consistent link mass $m_3 = 1.349$ kg gives $F_{i3} = m_3 \cdot aS_3 = 1.349 \cdot 2.14 = 2.887$ N.


ISSN: 2181-3337 | SCIENTISTS.UZ

For the same mechanism and crank position, the later inertia study by the authors reports a link-3 mass moment of inertia $J_3 = 0.001566$ kg·m² and an inertia moment $M_{i3} \approx 0.099$ N·m [7]. This value is used in the equilibrium calculation below.

**Construction of the Zhukovsky lever**

The velocity diagram is rotated by 90° clockwise. At the corresponding points of the rotated velocity diagram, the force vectors from the loading diagram are transferred parallel to themselves: the working resistance Qres, the inertia forces $F_{i2}$ and $F_{i3}$, and the balancing force P′bal. Forces tending to rotate the Zhukovsky lever clockwise are assigned a negative sign, and forces tending to rotate it counterclockwise are assigned a positive sign.

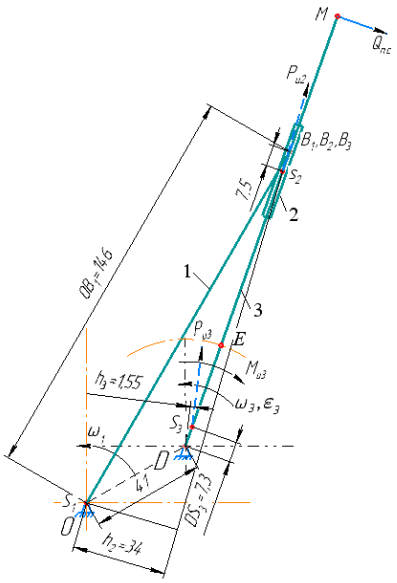



**Fig. 3. Loading diagram of the mechanism at $\varphi_1 = 30°$ ($\mu_l = 0.001$ m/mm).**

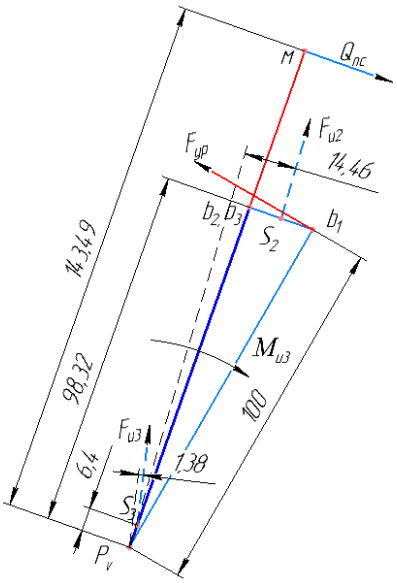



**Fig. 4. Force diagram on the velocity diagram rotated by 90° for the mechanism at $\varphi_1 = 30°$.**

**Results and Discussion**

For the considered mechanism position, the moment-equilibrium equation about the velocity pole Pv is:

$$F_{i3}\cdot hF_{i3} + F_{i2}\cdot hF_{i2} + P'bal\cdot hbal - Qres\cdot hQres - M_{i3}/\mu_l = 0 \qquad (3)$$

The scale factor is $\mu_l = 0.001$ m/mm. From the rotated diagram, the corresponding moment arms are $hF_{i3} = 1.38$ mm, $hF_{i2} = 14.46$ mm, hbal = 100 mm, and hQres = 143.49 mm.

Solving Eq. (3) for the balancing force gives:

$$P'bal = (-F_{i3}\cdot hF_{i3} - F_{i2}\cdot hF_{i2} + Qres\cdot hQres + M_{i3}/\mu_l) / hbal$$

$$= [-2.887\cdot 1.38 - 1.029\cdot 14.46 + 36.61\cdot 143.49 + 0.099/0.001] / 100 = 53.333 \text{ N}$$

The balancing moment referred to the crank is therefore:



ISSN: 2181-3337 | SCIENTISTS.UZ

$$M'bal = P'bal \cdot OB_1 = 53.333 \cdot 0.146 = 7.787 \text{ N·m} \quad (4)$$

Thus, at $\varphi_1 = 30°$, the corrected balancing force is approximately 53.33 N and the corrected balancing moment is approximately 7.79 N·m. The largest positive term in Eq. (3) is the moment of the working resistance Qres, while the translational inertia-force terms are comparatively small at this particular crank position.

The Zhukovsky-lever construction is advantageous because the balancing action can be obtained from a moment equilibrium on the rotated velocity diagram without explicitly solving the reaction forces in the kinematic pairs or carrying out the full force analysis of Assur groups. Nevertheless, it is not independent of kinematic analysis: acceleration data are required to determine the inertia forces and inertia moments. The present calculation is for one representative, highly loaded crank position; a complete operating-cycle assessment would require repetition over the full crank-angle range.

**Conclusions**

1. The N.E. Zhukovsky lever method was applied to the multiloop crank-slotted cotton-bush guiding mechanism of a vertical-spindle cotton picker at $\varphi_1 = 30°$.

2. Using consistent link data and the supported inertia moment for the rotating slotted link, the balancing force is P′bal = 53.333 N and the balancing moment is M′bal = 7.787 N·m.

3. The method avoids explicit determination of kinematic-pair reaction forces and the force analysis of Assur groups, but it uses previously obtained kinematic quantities, including accelerations.

4. The same procedure can be applied at additional crank positions to obtain balancing-force and balancing-moment curves over the mechanism cycle; this is recommended for a full dynamic assessment.

**REFERENCES**


1. I. I. Artobolevsky, Theory of Mechanisms and Machines. Moscow: KnoRus, 2021, 639 p. ISBN 978-5-4365-5793-9.
2. K. V. Frolov, S. A. Popov, A. K. Musatov et al., Theory of Mechanisms and Machines, K. V. Frolov, Ed. Moscow: Vysshaya Shkola, 1987, 496 p.
3. K. Turanov, A. Abdazimov, M. Shaumarova, and S. Siddikov, “Type Analysis of a Multiloop Coulisse Mechanism of a Cotton Harvester,” in International Scientific Conference Energy Management of Municipal Facilities and Sustainable Energy Technologies (EMMFT 2019), Advances in Intelligent Systems and Computing, vol. 1258, Springer, 2021, pp. 290–305. DOI: 10.1007/978-3-030-57450-5_27
4. S. Siddikov, A. Sadriddinov, A. Abdazimov, and M. Khajiev, “Graphic and analytical study of the kinematics of the multiloop coulisse mechanism of the cotton harvester,” IOP Conference Series: Earth and Environmental Science, vol. 1231, 2023, art. 012014. DOI: 10.1088/1755-1315/1231/1/012014
5. S. Siddikov, A. Abdazimov, M. M. Atadjanova, and G. A. Abdikayimova, “Developing motion profiles for the rotating coulisse within a complex mechanism used for harvesting cotton in a vertical spindle machine,” IOP Conference Series: Earth and Environmental Science, vol. 1390, 2024, art. 012007. DOI: 10.1088/1755-1315/1390/1/012007
6. Sh. Sh. Siddikov, “Determination of the loading of a spindle drum with a cotton-bush guiding mechanism by electrical strain-gauge measurement,” Proceedings of the 2nd International Scientific Conference “Development and Production of Competitive Agricultural Machinery and Machines for Processing Agricultural Products,” Tashkent, 1997, pp. 36–37.
7. Sh. Siddikov and A. Abdazimov, “Determination of the Center of Gravity and Moment of Inertia of Moving Links of a Multi-Contour Crank-Slotted Mechanism,” Science and Innovation, 2026. DOI: 10.5281/zenodo.18149871




ISSN: 2181-3337 | SCIENTISTS.UZ