# GEOMETRIC VISUALIZATION AS A FACTOR IN THINKING DEVELOPMENT

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***Abstract***. *This article analyzes the role of geometric visualization in the formation and development of students' thinking from a scientific and methodological perspective. Geometric visualization not only facilitates the visual perception of shape, space, size, relationships, and position, but also activates intellectual processes such as logical reasoning, abstract thinking, modeling, analysis, and generalization. The article highlights the interrelationship between geometric visualization and mathematical thinking, the didactic conditions necessary for its development, methods of enhancing spatial imagination in school mathematics education, and the possibilities of using modern digital tools for this purpose. Geometric visualization is considered an important factor in the development of students' logical, visual, algorithmic, and creative thinking. Furthermore, the purposeful use of visual aids, dynamic models, drawings and diagrams, practical tasks, and digital visualization tools in teaching geometric concepts contributes significantly to improving the effectiveness of education.*

***Keywords:*** *geometric visualization, thinking, mathematical thinking, spatial perception, visualization, geometric model, logical reasoning, abstraction, mathematics education, didactic approach.*

## Introduction

In the modern educational system, it is important not only for students to acquire ready-made knowledge but also to develop the ability to think independently, analyze information, draw conclusions, and find rational solutions in problematic situations. In particular, the development of thinking occupies a central place in mathematics education. The content of mathematics contributes to the formation of logical consistency, a culture of reasoning and proof, abstraction, generalization, modeling, and analytical thinking among students.

Geometry is one of the most important branches of mathematics that exerts a strong influence on students' cognitive development. In geometry, students work not only with formulas and theorems but also perceive and interpret relationships involving shape, space, size, movement, symmetry, angles, distance, similarity, and spatial arrangement. In this process, geometric visualization plays a particularly significant role.

Furthermore, geometric visualization can be understood as a system of mental images representing geometric objects, concepts, laws, principles, and facts that may not have been previously encountered by an individual. In students' minds, fundamental geometric concepts such as point, line, plane, belonging, inclusion, position, and spatial relations are gradually accumulated and organized. As a result of this cognitive accumulation, new and more complex mental images and representations emerge. Consequently, students' geometric visualization develops, expands, and becomes enriched, enabling them to create higher levels of abstraction and engage in more sophisticated mathematical reasoning.

The relevance of this topic lies in the fact that, in many general secondary schools, the teaching of geometry often leads students to focus primarily on memorizing theorems, applying formulas, and working with ready-made diagrams. However, they frequently encounter difficulties when required to visualize the essence of a geometric problem, understand relationships among geometric figures, construct diagrams independently, analyze spatial situations, and identify alternative solution strategies. This situation highlights the necessity of purposefully developing students' geometric visualization skills.

The aim of this article is to provide a scientific and methodological justification of the role of geometric visualization in the development of students' thinking and to analyze the didactic approaches that contribute to its formation and enhancement.

The main objectives of this article are: to reveal the scientific and methodological essence of the concept of geometric visualization; to analyze the relationship between geometric visualization and thinking; to identify the pedagogical conditions necessary for the development of geometric visualization; to substantiate methods for fostering geometric visualization in school mathematics education; and to demonstrate the potential of visual models and digital technologies in enhancing students' geometric visualization skills.

**Literature Review**

The problem of geometric visualization and spatial thinking has been studied across various fields, including mathematics education methodology, psychology, pedagogy, and cognitive sciences. This issue is closely connected with students' cognitive activity, visual perception, abstract thinking, and the process of mathematical modeling.

In psychological research, imagination is interpreted as the ability of the human mind to reconstruct, transform, and create new images of objects and phenomena. Geometric visualization represents a mathematically enriched form of this general psychological process. In this context, students perceive geometric figures not merely as objects to be seen or drawn, but as mental objects that can be analyzed, transformed, and modeled.

In the methodology of mathematics education, the importance of geometric visualization has been emphasized by many scholars. In particular, it is noted that visualization, drawing, modeling, construction, proof, and problem-solving processes in geometry contribute significantly to the development of students' thinking. The effective acquisition of geometric knowledge is directly related to the learner's ability to mentally visualize figures, their elements, and the relationships between them.

Jean Piaget emphasizes that the development of children's thinking occurs in stages, and the understanding of spatial relations is considered one of the key indicators of cognitive development. L.S. Vygotsky, in turn, substantiates the relationship between learning and development, showing that tasks given within the learner's zone of proximal development expand their cognitive capabilities. From this perspective, geometric tasks serve as a mediator between students' existing knowledge and new intellectual operations.

The theory of geometric thinking levels developed by the Dutch scientist P. van Hiele plays an important role in the methodology of teaching geometry. According to this theory, students first perceive geometric objects at a visual level, then identify their properties, understand relationships between these properties, and subsequently progress toward deductive reasoning and axiomatic thinking. This theory explains the transition of geometric visualization from simple visual perception to higher-level logical thinking.

In Uzbekistan, research in the methodology of mathematics teaching has also focused on developing students' mathematical thinking, spatial visualization, independent reasoning, and problem-solving skills. The analysis of existing literature shows that although the development of geometric visualization has been widely studied, its comprehensive treatment as an independent factor in the formation of thinking remains an important scientific and methodological issue in contemporary education.

**Methodology**

In this article, a set of research methods was used to determine the role of geometric visualization in the development of students' thinking, including analysis and synthesis, comparative analysis, generalization, modeling, and pedagogical observation.

The method of analysis and synthesis was applied to study the scientific and methodological essence of the concepts of geometric visualization, spatial perception, mathematical thinking, and visual modeling. Through comparative analysis, different aspects of geometric visualization were examined in relation to visual, logical, abstract, and creative thinking.

The method of generalization was used to identify the key pedagogical conditions that contribute to the development of students' thinking in the process of teaching geometry. The modeling method was employed to analyze the possibilities of explaining geometric concepts through drawings, diagrams, dynamic models, and spatial representations.

**Results**

The research findings indicate that geometric visualization develops several key components of students' thinking. Through geometric visualization, students are able to mentally represent figures, identify their properties, analyze relationships, predict transformations, and draw conclusions. In this process, different forms of thinking become actively engaged.

In geometry, every concept is to a certain extent based on a mental image. For example, concepts such as triangle, circle, parallel lines, prism, or cone are formed in the student's mind as specific visual representations. If students are unable to correctly visualize these images, they also cannot deeply understand their properties.

Visual thinking enables the holistic perception of a geometric object and the ability to mentally represent its shape and structure. For instance, the ability to visualize a cube unfolding into its net and then reconstructing it in three-dimensional space is an important manifestation of spatial thinking.

Geometric visualization is not limited to perceptual imagery; it is also closely connected with logical reasoning. In geometry, proving theorems, justifying properties, determining necessary and sufficient conditions, and identifying relationships between given and unknown quantities all require logical thinking.

For example, the conclusion "if two lines are perpendicular to the same line, then they are parallel to each other" can be visually represented through a diagram, but its true meaning is understood through logical justification. Thus, the diagram initiates thinking, while logic provides its justification.

Geometric figures in mathematics may resemble real-world objects; however, they are treated as mathematical abstractions. For instance, a point has no dimensions, a straight line extends infinitely, and a plane is considered an ideal geometric object. In the process of understanding these concepts, students move from real-world perception to mathematical modeling.

Geometric problems often have multiple solution approaches. A single problem can be solved using synthetic, analytic, algebraic, coordinate, or vector methods. As students explore different solution strategies, their creative thinking becomes significantly developed.

**The Relationship Between Geometric Visualization and Types of Thinking**

| Geometric Visualization Activity | Type of Thinking Developed | Methodological Significance |
|---|---|---|
| Visualizing a geometric figure | Visual (imagery-based) thinking | Helps perceive geometric objects as a whole |
| Analyzing relationships in a diagram | Logical thinking | Supports identifying cause–effect relationships |
| Transition from real object to mathematical model | Abstract thinking | Ensures generalization of mathematical concepts |
| Mentally rotating or transforming a figure | Spatial thinking | Strengthens the ability to visualize spatial situations |
| Searching for different solution methods | Creative thinking | Develops independent and non-standard thinking |
| Proving a theorem | Deductive thinking | Forms a culture of logical justification |

**Discussion**

Viewing geometric visualization as a factor of thinking requires a new approach to the content and methodology of teaching geometry. In traditional education, geometry is often taught as a system of theorems, definitions, formulas, and proofs. Although this approach is necessary, it is not sufficient. If students do not properly visualize geometric content, they cannot fully understand the logical structure of proofs.

Several methodological conditions are essential for developing geometric visualization. First, visualization and logical reasoning must be integrated in the teaching of geometry. Relying solely on drawings may strengthen students' perceptual understanding, but it does not fully develop logical thinking. Therefore, every diagram, scheme, or model should be connected with questions, analysis, and conclusions.

Second, students should be taught not only to use ready-made drawings but also to construct geometric figures independently. During the process of constructing a diagram, students analyze the conditions of the problem, identify given and unknown elements, and determine relationships between geometric objects. This significantly activates their thinking processes.

Third, it is important to present geometric objects in different positions and representations. For example, a triangle should not always be shown in a standard orientation; it should also be presented in various rotations, positions, and scales. Otherwise, students may memorize only a specific visual appearance of the figure rather than understanding the geometric concept itself.

Fourth, the use of models, mock-ups, nets, and digital visualization tools is highly effective in studying spatial figures. In particular, software such as GeoGebra, Desmos 3D, and Cabri 3D allows students to observe figures dynamically, rotate them, create cross-sections, and modify parameters interactively. These tools significantly enhance spatial understanding and engagement.

Fifth, comparing multiple methods of solving geometric problems contributes to the development of students' critical and creative thinking. For instance, after solving a problem using

the synthetic method, analyzing it again through coordinate or vector approaches helps students understand the multidimensional nature of mathematical relationships.

**Methodological Model for Developing Geometric Visualization**

The process of developing geometric visualization can be organized through the following stages: visual perception, analytical understanding, modeling, logical justification, and creative application.

At the stage of visual perception, students observe a geometric object, identify its external features, and name its shape and elements. At the stage of analytical understanding, the components, properties, and relationships of the figure are examined in detail. For example, elements such as sides, angles, diagonals, heights, and medians are identified and analyzed.

During the modeling stage, real objects or problem conditions are represented through geometric models. Diagrams, schemes, or spatial models are constructed to represent the given situation. At the stage of logical justification, the properties of the figure are used to prove theorems, rules, or solution paths.

Finally, at the stage of creative application, students apply geometric knowledge in new situations, explore different solution strategies, and draw independent conclusions. This stage is particularly important for developing flexible and innovative thinking skills.

**Problems**

For instance, the well-known property that the medians of a triangle intersect at a single point and divide each other in the ratio 2:1 (measured from the vertex) can be generalized through the development of geometric thinking (Figure 1). As a result of such generalization, it becomes possible to imagine a situation in which segments drawn from the vertices of a triangle intersect at a single interior point and intersect the opposite sides at specific points. In this case, a more generalized geometric representation can be mentally constructed and visualized, extending beyond the standard case of medians.

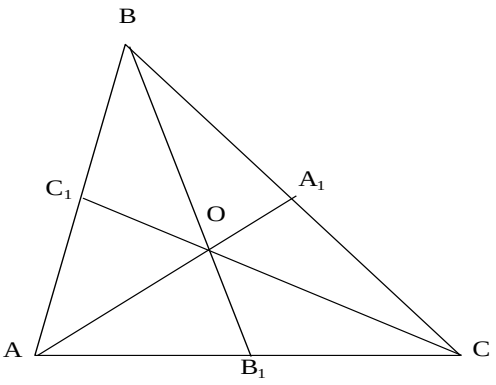



***Figure 1***

This form of visualization teaches students to construct generalized geometric patterns that arise from a logical sequence of reasoning, often leading to previously unconsidered generalizations. Therefore, the closest and most frequently accompanying cognitive process of imagination is logical thinking, which not only ensures a complete and coherent mental representation of geometric objects but also plays a crucial role in the formation of such representations.

In geometry, visualization is of fundamental importance, since only when a student can fully imagine a theorem, a geometric problem, and its content together with its diagram in their mind, are they able to prove the theorem or solve the problem without difficulty. This, in turn, contributes to the development of students' mathematical culture and modern forms of thinking.

For example, a median of a triangle divides the opposite side into two equal parts. If a student perceives the median not merely as a line segment but as an element that divides the area of the triangle into two equal parts, their geometric understanding becomes significantly deeper.

In this case, visualization leads to a logical conclusion: if two triangles have equal heights and equal bases, then their areas are also equal.

A diagram is an important tool when explaining the relationship between a central angle in a circle and its corresponding arc. However, the diagram should not be used only for illustration; it must also serve to develop understanding of the relationship itself. If a student is able to mentally visualize that when the measure of an angle changes, the length of the corresponding arc also changes, they will gain a deeper understanding of the meaning of the formula.

Section formation: predicting the shape of a cross-section formed when a prism is intersected by a plane requires spatial thinking. In such problems, students analyze the relationships between points, edges, and planes in space. This type of reasoning develops their ability to mentally manipulate three-dimensional geometric structures.

**Novelty of the Research**

The scientific novelty of this article lies in the fact that geometric visualization is interpreted not merely as an auxiliary tool for developing students' thinking, but as an independent didactic factor that significantly influences the formation of mathematical thinking. In addition, the interrelated aspects of geometric visualization with visual, logical, abstract, spatial, and creative thinking are systematically substantiated.

The following components necessary for the formation and development of students' geometric visualization skills have been identified:

- the mathematical content required for forming and developing students' geometric visualization skills, along with the hierarchical structure of its development;
- the internal and external relationships of geometric laws, rules, and formulas, as well as teaching methodologies that integrate them with other branches of mathematics;
- the content, structure, and implementation mechanisms of learning material that ensure the dynamic development of geometric visualization and its components;
- the levels and hierarchical structure of students' geometric visualization development within the system of continuous education.

**Conclusion**

Based on the conducted analysis, the following conclusions can be drawn: geometric visualization is an important didactic factor in the development of students' mathematical thinking; it integrates visual perception, logical analysis, abstraction, modeling, and creative thinking into a unified cognitive process; and the effective teaching of geometry requires the purposeful use of diagrams, models, schemes, and digital visualization tools.

Students should be taught not only to work with ready-made drawings but also to construct geometric figures independently, transform them, and mentally visualize spatial situations. Solving geometric problems using different methods enhances students' creative and critical thinking abilities.

If the process of developing geometric visualization is organized in a step-by-step manner, students' skills in theorem proving, problem solving, and mathematical modeling will be significantly strengthened. In modern mathematics education, the development of geometric visualization is not only essential for mastering geometry but also a key condition for improving overall intellectual capacity.

**Recommendations**

In geometry lessons, it is advisable that each new concept be explained through diagrams, practical examples, and logical analysis. Students should be given tasks that involve representing geometric figures in different positions, rotating, transforming, and reconstructing them.

When teaching spatial figures, it is necessary to use models, nets, and 3D representations. The use of digital tools such as GeoGebra, Desmos, and Cabri 3D enables the creation of dynamic geometric models and significantly improves the effectiveness of instruction.

When solving geometric problems, it is recommended not to rely on a single method but to compare synthetic, analytic, coordinate, and vector approaches. Such comparative analysis helps students develop a deeper understanding of mathematical relationships and enhances their problem-solving flexibility.

The approaches presented in this article can be used in the teaching of geometry in general secondary schools, in the development of methodological recommendations for mathematics teachers, and in the selection of geometric problems and organization of lesson processes. Furthermore, the materials of this article may serve as a theoretical and practical resource in higher education institutions for teaching the course “Methods of Teaching Mathematics.”

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