PROPAGATION OF WAVES IN INFINITELY LONG LAYERED VISCOELASTIC CYLINDRICAL MECHANICAL BODIES

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https://doi.org/10.5281/zenodo.18997320

Abstract. *The paper considers the propagation of natural waves in a thin cylindrical shell and constructs The dispersion dependence of a flexural wave in the walls of a viscoelastic cylindrical shell. The wall thickness of the cylindrical shell was assumed to be so small that all effects associated solely with its changes over time were neglected. In this approximation, the waveguide, which represents such a shell, is single-mode across the entire frequency range, with the exception of a small range in which attenuation-free propagation is impossible. For a more detailed examination of wave processes in an extended cylindrical shell, both within this frequency range and beyond, it is necessary to consider an approximation in order of smallness. From here on, it is appropriate to refer to the wave of change in the radius of the centerline of the cross-section, as a flexural wave.*

Keywords: *cylindrical shell, dispersion dependence, wave propagation, shell wall, natural wave.*

1.Introduction

The dispersion relation of a circular cylindrical shell has been a subject of great interest for several decades [1-4]. In some studies, numerical procedures were used exclusively for calculating dispersion relations. In [5], an analytical expression for the dispersion relations from Flügge shell theory was obtained using the Mathematica symbolic algebra package. Karchub's main concern was to verify the correspondence between his analytical results and those obtained previously using numerical methods, and was limited to harmonic orders. $n \geq 1$. Axisymmetric modes (n=0) are important for the transmission of longitudinal waves, and are particularly important in acoustics because of their high radiation efficiency. In order to have complete analytical solutions for the shell dispersion relations, solutions for the n=0 cases are included and discussed in this paper. Solutions to the modal models for each of the propagating and evanescent modes are also important and are not discussed in [6].

These solutions are crucial for determining the vibration of a finite shell under various allowable boundary conditions and arbitrary external forces. The dispersion relations and their associated eigenvectors also provide a means by which transmission matrices for vibroacoustic transmission in cylindrical shell structures or pipe-and-hose systems can be constructed [7–9]. A traditional and standard method for finding the eigenvectors has been described in detail in [10];

an eigenvector was normalized such that its radial displacement component was equal to unity, while the longitudinal and circumferential components were expressed as ratios of the displacement with respect to the radial displacement. In some cases these ratios could become extremely large and the eigenvector could thus deviate significantly from the eigenvector commonly used in mathematical physics [11,12]. To improve this problem of finding normalized eigenvectors with norms equal to unity, an alternative method is proposed using the built-in eigenvalue problem in any numerical package [13,14].

Continuously changing mode models for all kinds of wave types in the desired frequency range will be quickly obtained, and the physical meanings of wave propagation can also be shown more clearly and straightforwardly IN [15]. Apparently, the term "Lamb wave" is commonly used for this mode. This likely stems from the analogy of this wave with the conventional antisymmetric Lamb wave defined for a flat layer. However, in the case of waves in the wall of a cylindrical shell, this can lead to confusion, since at low frequencies, when the wavelength is commensurate with the radius of curvature, the wave in the pipe wall and the wave in the flat layer behave differently. Furthermore, the literature on pipes does not always specify the type of plane Lamb wave with which the analogy is drawn—antisymmetric or symmetric—despite the fact that these phenomena already have significant qualitative differences from their "flat" prototypes..

Problem statement and solution methods

Here we consider the problem of wave propagation in dissipatively homogeneous and inhomogeneous three-layer cylindrical bodies (without fluid). The equations of motion of the surrounding medium and the cylindrical body are as follows [16]:

$$\tilde{\mu}_n\nabla^2\vec{u} + (\tilde{\lambda}_n + \tilde{\mu}_n \quad) graddi\vartheta \quad \vec{u} = \rho_n \frac{\partial^2\vec{u}}{\partial t^2}$$

, (n =1, 2, 3…..N.) (1)

Here $\vec{u}(u_x, u_y, u_z)$ -vector of displacement of points of the environment; ρ_j - density of the material; u – displacement components; ν - Poisson's ratio;

$$.\tilde{\lambda}_n\phi(t) = \frac{v_n\tilde{E}_n\phi(t)}{(1+v_n)(1-2v_n)}\ ; \qquad \tilde{\mu}_n\phi(t) = \frac{v_n\tilde{E}_n\phi(t)}{2(1+v_n)}$$

In the case where the Koltunov-Rzhanitsen relaxation kernel is used in place of (1), then the elastic modulus is replaced by operators, i.e. $\tilde{E}_n$ – the operator modulus of elasticity has the form [17]:

$$(2)\tilde{E}_n\phi(t) = E_{0n}\left[\phi(t) - \int_0^t R_{En}(t-\tau)\phi(t)d\tau\right]$$

$\varphi(t)$ – an arbitrary function of time; $R_{En}(t-\tau)$ – relaxation core; E_{01} –instantaneous modulus of elasticity.

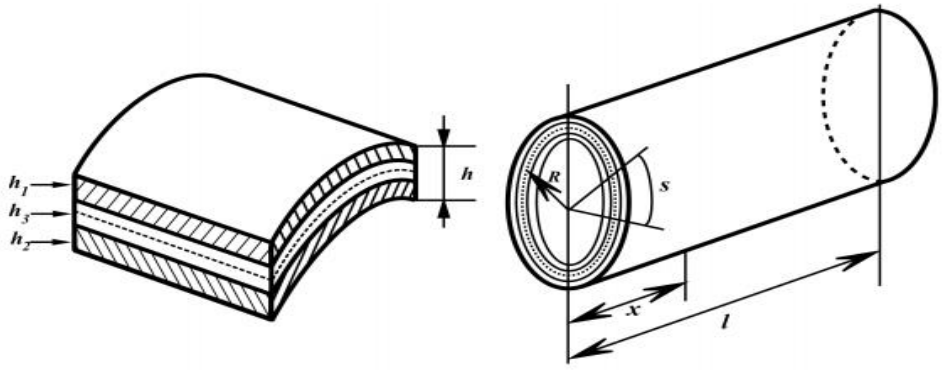


Fig. 1. Calculation scheme. Three-layer structure.(h1- first layer, h2- second layer, 3- filler).

Then the boundary conditions (internal and external boundaries are free from loads, the condition of rigid fixation at the boundary of two layers) have the following form:

$$\begin{aligned}
&r = r_1, \qquad \sigma_{rr1} = 0, \quad \sigma_{r\theta 1} = 0, \quad \sigma_{rz1} = 0,\\
&r = r_2, \qquad \sigma_{rr1} = \sigma_{rr2}, \quad \sigma_{r\theta 1} = \sigma_{r\theta 2}, \quad \sigma_{rz1} = \sigma_{rz2},\\
&\qquad\qquad u_{r1} = u_{r2}, \quad u_{\theta 1} = u_{\theta 2}, \quad u_{z1} = u_{z2},\\
&r = r_3, \qquad \sigma_{rr2} = \sigma_{rr3}, \quad \sigma_{r\theta 2} = \sigma_{r\theta 3}, \quad \sigma_{rz2} = \sigma_{rz3},\\
&\qquad\qquad u_{r2} = u_{r3}, \qquad u_{\theta 2} = u_{\theta 3}, \qquad u_{z2} = u_{z3},\\
&r = r_4, \qquad \sigma_{rr3} = 0, \qquad \sigma_{r\theta 3} = 0, \qquad \sigma_{rz3} = 0.
\end{aligned} \tag{3}$$

The displacements of the elements of a three-layer cylinder are expressed through the Bessel and Neumann functions:

$$\begin{aligned}
u_{r\kappa} &= \sum_{n=0}^{\infty} \Big\{ \gamma_\kappa \left[A_{1\kappa} J'_\kappa(\gamma_{1\kappa} r) + A_{2\kappa} Y'_\kappa(\gamma_{1\kappa} r) \right] + \frac{n}{r} \left[A_{3\kappa} J_\kappa(\gamma_{2\kappa} r) + A_{4\kappa} Y_\kappa(\gamma_{2\kappa} r) \right] - \\
&- \frac{\alpha \gamma_{2\kappa}}{\mu_{2\kappa}} \left[A_{5\kappa} J'_\kappa(\gamma_{2\kappa} r) + A_{6\kappa} Y'_\kappa(\gamma_{2j} r) \right] \Big\} \cos n\varphi e^{i(-\omega t + \gamma z)},\\
u_{\varphi\kappa} &= -\sum_{n=0}^{\infty} \frac{n}{r} \Big\{ \gamma_\kappa \left[A_{1\kappa} J_\kappa(\gamma_{1\kappa} r) + A_{2\kappa} Y_\kappa(\gamma_{1\kappa} r) \right] + \frac{r\gamma_{2\kappa}}{n} \left[A_{3r} J'_k(\gamma_{2k} r) + A_{4k} Y'_k(\gamma_{2k} r) \right] - \\
&- \frac{\alpha}{\mu_{2r}} \left[A_{5k} J_k(\gamma_{2k} r) + A_{6k} Y_k(\gamma_{2k} r) \right] \Big\} \sin n\varphi e^{i(-\omega t + \gamma z)},\\
u_{zk} &= \sum_{n=0}^{\infty} \Big\{ \gamma_k \left[A_{1k} J_k(\gamma_{1k} r) + A_{2k} \gamma_k(\gamma_{1k} r) \right] + \\
&+ \frac{\gamma_{2k}^2}{n} \left[A_{5k} J_k(\gamma_{2k} r) + A_{6k} Y_k(\gamma_{2k} r) \right] \Big\} \cos n\varphi e^{i(\omega t + \gamma z)}.
\end{aligned} \tag{4}$$

Here

$$\gamma_{1k}^2 = \bar{\mu}_{1k}^2 - \gamma^2, \quad \bar{\mu}_{1k} = \frac{\omega}{a_{1k}\Gamma_k}, \quad \gamma_{2k}^2 = \bar{\mu}_{2k}^2 - \gamma^2,$$

$$\bar{\mu}_{2k} = \omega / a_{2k}\Gamma_{2k}, \quad a_{2k}^2 = \frac{\bar{\mu}_k}{\rho_k}, \quad \gamma = m\pi / l, \ (m = 1,2,\ldots) \quad (k = 1,2,3).$$

If the displacements (4) are known, then the components of the deformations and stresses can be found. Substituting into (4) the components of the displacements and the stresses found through them, then to determine arbitrary constants $A_{1k}, A_{2k}, A_{3k}, A_{4k}, A_{5k}, A_{6k}$ we obtain a system of 18 homogeneous algebraic equations (with complex coefficients) with 18 unknowns

$$(3)[\mathrm{C}]\{q\} = \{0\}$$

In order for a homogeneous system of algebraic equations to have non-trivial solutions, the main determinant of the system of equations must be equal to zero: ($[C]$=0), the elements of the last equations are expressed through the Bessel functions of the first and second kind of the nth

order. The order of the determinant given above is 18. When expanding this determinant, we obtain a transcendental equation with respect to the complex amplitude . This equation is solved using a specially developed algorithm (based on the Müller and Gauss methods, as well as methods for calculating the determinant of a matrix) and a program.$C_{ij}(i = 1,2,\dots,18; \quad j = 1,2,\dots,18)\omega$

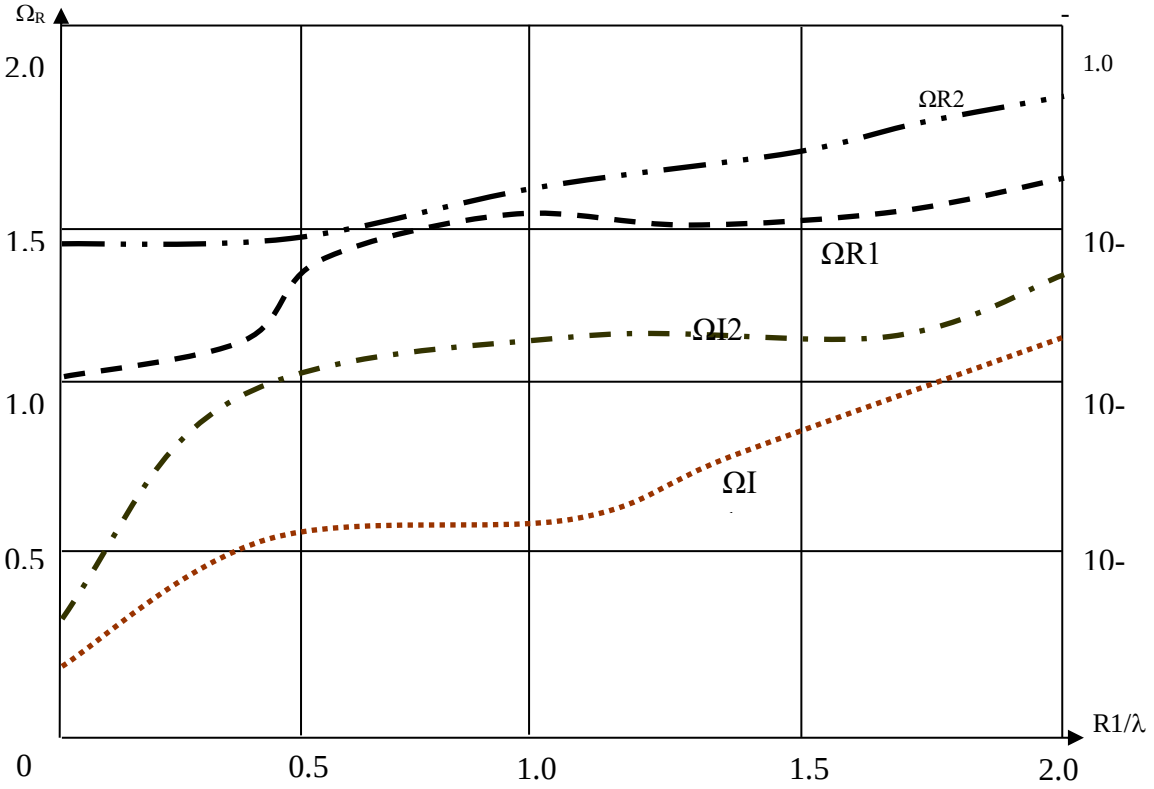


Fig. 1. Change in actual$\Omega_R$and imaginary$\Omega_I$parts of the complex frequency as a function of the wave number (dissipative homogeneous system)

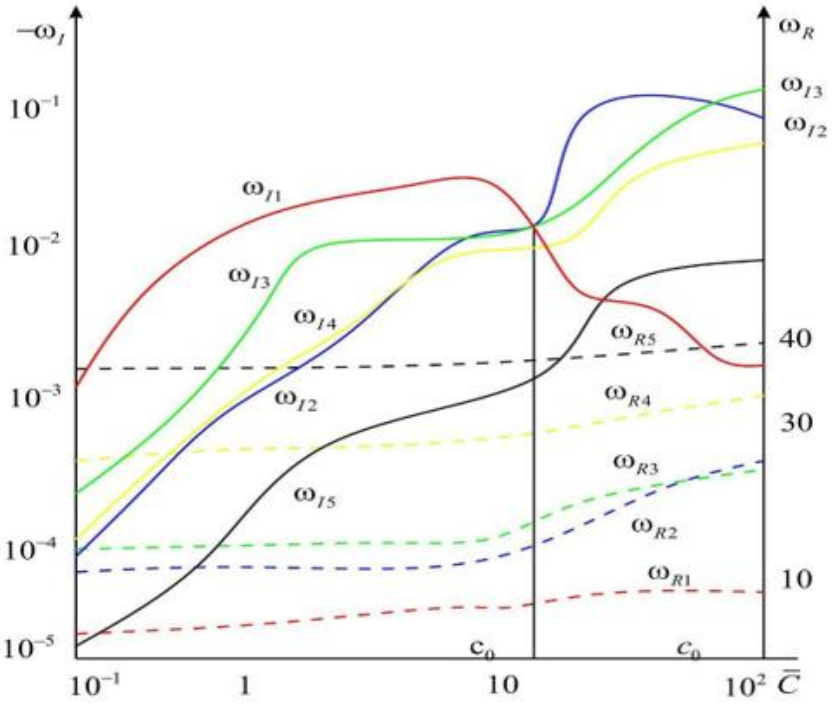


Fig. 2. Change in actualΩ_R and imaginary$\Omega_I$parts of the complex frequency as a function of the wave number (dissipative-inhomogeneous system)

In all the cases considered, the following were adopted: Poisson's ratio of the midsurface is 0.25; the density ratio is:ρ_{With}/ρ=0.35 (the ratio of the density of the middle surface to the density of the outer layer); Gc/μ- the ratio of the bulk compression modulus to the shear modulus of the outer layer is 0.20; μ_{With}/μ- the ratio of the shear modulus of the middle surface to the shear modulus of the outer layer is equal to 0.11(1+Γκ), $\Gamma_{\kappa} = 1 - \Gamma^{C}(\omega_R) - i\Gamma^{S}(\omega)$. The calculation results (changes in the real and imaginary parts of the complex frequency as a function of the wave number) are shown in Fig. 2 and Fig. 3 (h/b=0.1) for dissipative-homogeneous and inhomogeneous systems. The effect of expressing the change in the attenuation coefficient as a function of the wave number through monotonic functions (at values of maximum convergence of the real parts of the frequencies or phase velocity) is confirmed. New branches of this effect are revealed. The imaginary parts of the first, second, and third frequencies act as the global attenuation coefficient for dissipative-inhomogeneous systems. This opens up new possibilities for optimizing the dissipative properties of a mechanical system.

Conclusions

1. To characterize the dissipative properties of a mechanical system, the parameter δ (the global damping coefficient (GDC)) was introduced. For dissipatively homogeneous systems, the absolute value of δ is determined by the imaginary part of the first complex frequency. For dissipatively inhomogeneous mechanical systems, the role of δ is played by the imaginary parts of the first, second, third, etc. frequencies.

2. It has been established that the group velocity in a non-dispersible medium will be 16% greater than the group velocity of a dispersible medium.

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