RESEARCH ON MODELING APPLICATION OF ORDINARY DIFFERENTIAL EQUATIONS TO PRACTICAL PROBLEMS AND CASE-ORIENTED TEACHING

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Abstract. *Ordinary differential equation is an important component of higher mathematics, and its modeling method is widely applied to solving various practical problems. Traditional teaching of ordinary differential equation lays particular emphasis on theoretical deduction, which makes it difficult for students to perceive the application value of mathematical knowledge. In this paper, the case-based teaching method is adopted, and three typical practical problems including solution dilution, identification of painting and calligraphy counterfeits and computer virus propagation are selected. The corresponding ordinary differential equation models are established and solved respectively, with the internal laws and mathematical principles of each problem analyzed. Through the modeling and solution of multi-field cases, the advantages of ordinary differential equations in depicting natural phenomena and practical problems are demonstrated, and the teaching concept of integrating theory with practice is reflected. Practice has proved that case-based teaching can effectively reduce the learning difficulty of ordinary differential equations, stimulate students' learning interest, help students grasp the idea of mathematical modeling and improve their ability of knowledge application, which plays a positive role in improving the teaching effect of the ordinary differential equation course.*

Keywords: *ordinary differential equation; mathematical modeling; case-based teaching; practical application; model solution.*

1 Introduction

As one of the core branches of higher mathematics, ordinary differential equations have distinctive characteristics in the field of mathematical modeling, and serve as an important bridge connecting mathematical theories with practical problems. Case-based teaching is a highly targeted teaching mode in the course of ordinary differential equations, which can vividly interpret the teaching concept of integrating theory with practice [1]. The traditional teaching mode mostly follows a fixed process of "theorem elaboration - proof and deduction - example explanation". Tedious theoretical deductions often occupy the core time of classroom teaching, leading to students' distraction and a significant decline in the efficiency of knowledge acquisition in the subsequent example explanation session. Therefore, how to effectively attract students' attention and continuously stimulate their learning interest has become a key issue to improve the teaching quality of the ordinary differential equation course.

Case-based teaching can exactly make up for this deficiency. This teaching mode abandons tedious pure theoretical indoctrination, evokes students' resonance through concrete practical problems, and is more conducive to focusing students' attention [2]. Among all branches of higher mathematics, ordinary differential equations are particularly suitable for case-based teaching due to their distinctive characteristics of wide application scenarios and close integration of theory with

practice. Carrying out teaching with specific cases can not only help students achieve an in-depth integration of theory and practice, but also make the abstract basic theories of ordinary differential equations intuitive and perceptible, thereby improving students' ability to analyze and solve problems and strengthening their abstract thinking literacy [3-6]. The following part will combine specific cases to explore the teaching approaches of theoretical knowledge of ordinary differential equations under the case-based teaching mode.

2 The Modeling and Application Examples of Ordinary Differential Equations in Practical Problems

2.1 Modeling and Solution of Ordinary Differential Equation for the Solution Dilution Problem

Case 1: Consider a salt solution with a salt concentration of 0.3 kg/L. This salt solution is poured into a container holding 10 L of pure water at a constant flow rate of 2 L/min. After the solution in the container is fully stirred and diluted, it flows out at a rate of 2 L/min. The question is: what is the mass of salt remaining in the container after 5 minutes?

Analysis: The core of this problem is to embody the idea of mathematical modeling, that is, to extract key data from specific practical problems, construct a suitable mathematical model and solve it. Among the model systems of various branches of mathematics, ordinary differential equation models are widely applied, which can accurately depict the variation laws of numerous natural phenomena in real life. Therefore, for this problem, we can adopt the thinking of ordinary differential equation modeling, and finally obtain the answer to the problem by establishing the corresponding ordinary differential equation model and solving its particular solution.

Solution: Let time t be the independent variable, and let the mass of salt in the container at time t minutes after injection be the unknown function $y(t)$ to be solved. Take an arbitrary time interval $[t, t+\Delta t]$, then the variation of the mass of salt in the container over this time interval is

$$y(t+\Delta t)-y(t)\approx 0.3\cdot 2\Delta t-\frac{y(t)}{10}\cdot 2\Delta t.$$

Divide both sides by Δt, and let $\Delta t\to 0$, then we obtain the initial value problem

$$\begin{cases}\dfrac{dy}{dt}=0.6-0.2y(t)\\ y(0)=0\end{cases} \qquad (1)$$

The solution of equation (1) is

$$y(t)=3-3e^{-0.2t}.$$

Therefore, when t=5, the mass of salt remaining in the container is

$$y(5)=3-3e^{-0.2\times 5}\approx 1.9 \text{ kg}.$$

This example fully embodies the core idea of mathematical modeling. Taking the salt solution dilution problem as a carrier, a time variable is introduced to construct an ordinary differential equation model. In the process of explaining this example, students can not only truly understand the ideas and methods of mathematical modeling and experience the pleasure of constructing mathematical models to solve practical problems, but also fully demonstrate the inherent beauty and application value of mathematics.

2.2 Construction and Application of Radioactive Decay Model for Identification of Painting and Calligraphy Counterfeits

Case 2: In the process of human cultural development, the works of renowned painters and calligraphers are often targeted for forgery due to their dual artistic and collection values. Therefore, the authenticity identification of paintings and calligraphy has always been an important subject in the fields of cultural inheritance and art collection. At present, the identification of such works mostly relies on manual judgment by experts with years of experience, and this method has obvious limitations and risks — some works initially identified as authentic by experts are subsequently proven to be counterfeits. Based on this, we cannot help but ponder: can we start from a mathematical perspective and apply rational thinking and quantitative analysis methods to provide a brand-new technical approach and support for the identification of ancient paintings and calligraphy?

Analysis: Before constructing the mathematical model, it is necessary to clarify the characteristics of raw material components for the creation of paintings and calligraphy. Artists often adopt mineral substances as pigments in their creation, and some minerals such as radium and lead are radioactive. According to the definition in physics, radioactivity refers to a phenomenon in which the atomic nuclei of certain elements undergo fission and release microscopic particles, leading to a decrease in the number of protons in the atoms and their eventual transformation into other elements, among which lead is a typical radioactive element. Based on this characteristic, if we can deduce the variation law of lead content $M(t)$ with time t during the decay process, and then obtain the actual lead content in the painting and calligraphy works through detection, we can calculate the actual age of the works, and further judge whether they are modern imitations.

Solution: According to the principles of physics, the decay rate of lead is proportional to the mass M of undecayed lead atoms at that time. Set the initial condition: the mass of lead is $M_{t_0} = M_0$ when time $t = t_0$. The decay rate of lead can be expressed as the derivative $\frac{dM}{dt}$ of mass M with respect to time t. Since the mass decreases with time during the decay process and the decay rate is a negative value, a mathematical model C

$$\begin{cases} \dfrac{\mathrm{d}M}{\mathrm{d}t} = -kM \\ M(t_0) = M_0 \end{cases} \qquad (2)$$

can be established accordingly, where k (k>0) is a constant, referred to as the decay coefficient.

Solving equation (2), we obtain

$M = M_0 \mathrm{e}^{-k(t-t_0)}$.

This law not only depicts the decay characteristics of lead, but also represents the common decay law shared by most radioactive elements. It can be inferred that the content of lead shows an exponential decreasing trend with the passage of time t. Based on this, we can calculate the survival age of a painting and calligraphy work by detecting its actual lead content. It should be clarified that if the age calculated by this method is inconsistent with the claimed creation age of the work, the work can be judged as a counterfeit. Even if the calculated age is consistent with the claimed age, it cannot directly prove that the work is authentic — because this model can only quantify the survival time of the work, and is unable to judge the core authenticity attributes such as the authorship of the work.

This example constructs an ordinary differential equation model based on the decay law of radioactive elements — a model that falls into the category of the most basic separable variable differential equation models. By solving this model, we can derive the functional relationship between lead content and time. Further, combining with the actual detected lead content in paintings and calligraphy works, we can calculate the actual survival age of a "famous painting" and ultimately judge whether it is a counterfeit. This is a typical case of integrating theory with practice, which enables students to intuitively understand the application value of ordinary differential equations in solving practical authenticity identification problems and deeply realize the practical significance of mathematical modeling.

2.3 Analysis and Exploration of the Differential Equation Model for Computer Virus Propagation

Case 3: Computer viruses have become a prominent concern in the current field of computer science. Their propagation poses a severe threat to the security of computer systems and the internet ecosystem, and has thus become a key focus of cybersecurity governance for all countries. With the rapid popularization of the internet, computer viruses have gained a more convenient propagation environment: their types are increasing steadily, and their transmission capabilities are continuously enhanced, inflicting severe damage on computer system resources. The propagation process of computer viruses is complex, subject to the constraints and influences of multiple factors. This paper attempts to construct a mathematical model for computer virus propagation based on differential equations, and further analyze the propagation laws and evolutionary characteristics of viruses by virtue of this model.

Analysis: First, the propagation characteristics of computer viruses are briefly explained: their propagation process is highly similar to the transmission mechanism of infectious diseases in the field of biology. In a network environment, if an executable program is infected by a virus and invades a certain computer system, the virus will spread on a large scale within the system once the infected program is run. In view of the characteristics of computer viruses with fast propagation speed and wide influence range, to simplify the process of model construction and facilitate the analysis of core laws, the following assumptions are made for the problem of computer virus propagation:

(1) In the network system, the system files infected by the virus have immunity and will not be infected by the same virus again.

(2) The incubation period of viruses is ignored in the process of model construction, that is, the time-delay effect of virus propagation is not taken into account.

Based on the above assumptions, the executable programs in the network system can be divided into three categories: the first category refers to the susceptible programs that have not been infected by viruses yet, and their quantity is denoted as $S(t)$; the second category refers to the programs that have been infected by computer viruses, and their quantity is denoted as $I(t)$; the third category refers to the programs that have obtained immunity and will no longer be infected by the virus after being treated by antivirus software and other methods due to virus infection, and their quantity is denoted as $R(t)$.

Solution: Assume that the total number of executable programs in the network system remains constant during the propagation of computer viruses, denoted as a constant N, i.e., it satisfies

$$S(t) + I(t) + R(t) = N .$$

Let α be the infection rate of the computer virus and β be the recovery rate, then the system of differential equations can be obtained as

$$\begin{cases} \frac{\mathrm{d}S}{\mathrm{d}t} = -\alpha SI \\ \frac{\mathrm{d}I}{\mathrm{d}t} = \alpha SI - \beta I \\ \frac{\mathrm{d}R}{\mathrm{d}t} = \beta I \end{cases},$$

from

$$\frac{\mathrm{d}S}{\mathrm{d}t} = -\alpha SI, \frac{\mathrm{d}I}{\mathrm{d}t} = \alpha SI - \beta I,$$

we obtain

$$\frac{\mathrm{d}I}{\mathrm{d}S} = \frac{\alpha SI - \beta I}{-\alpha SI},$$

i.e.

$$\frac{\mathrm{d}I}{\mathrm{d}S} = -1 + \frac{\beta}{\alpha S}.$$

This is a simple separable variable equation, and its solution is

$$I(S) = -S + \frac{\beta}{\alpha}\ln S + C,$$

Let the initial condition be $I(t_0) = I_0$ and $S(t_0) = S_0$ for $t = t_0$, and denote $\rho = \frac{\beta}{\alpha}$. Substituting into the above equation, we obtain the constant $C = I_0 + S_0 - \rho \ln S_0$, and finally the particular solution of the equation satisfying the initial condition is

$$I(S) = I_0 + S_0 - S + \rho \ln \frac{S}{S_0},$$

$\rho = \frac{\beta}{\alpha}$ is referred to as the threshold value. Analysis shows that the computer virus will spread if $S_0 > \rho$, i.e., the number of executable programs not yet infected by the virus exceeds the threshold value.

The model constructed above is only a basic simplified model for computer virus propagation. In actual network environments, the virus propagation process is far more complex. On the one hand, people will actively take prevention and control measures such as deploying network firewalls and updating virus databases in real time to intervene in virus propagation. On the other hand, this model does not take into account the virus incubation period — in reality, a system infected with a virus usually does not malfunction immediately; instead, there is a time interval from infection to the onset of failures, which means that virus propagation has a time-delay effect. Therefore, a virus propagation model for real network scenarios needs to take into account complex factors such as prevention and control interventions and time delays, and its construction is far more difficult than that of the simplified model. Generally, it is necessary to

establish a complex delay differential equation model to accurately depict the actual propagation process.

3 Conclusions

This paper discusses several application cases of the case-based teaching method in the differential equations course. The aforementioned cases cover a variety of disciplinary fields such as humanities and arts, and computer science. Introducing and explaining these cases in the teaching of ordinary differential equations enables students to intuitively perceive the application value and immense charm of ordinary differential equations, thereby effectively enhancing their interest in learning this course. It can be seen that the case-based teaching method is different from the traditional conventional teaching mode. Its advantage lies in the ability to help students firmly grasp the core theoretical knowledge of ordinary differential equations while infiltrating relevant knowledge from other fields. This puts forward higher requirements for teachers: it is necessary to continuously delve deep into teaching content, extensively explore specific cases in daily life and various disciplinary fields, and organically integrate these cases into the teaching process. In this way, students can truly experience the unique charm and practical value of ordinary differential equations, so as to improve the teaching effect.

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