SOLUTION OF DIFFERENTIAL EQUATION WITH FINITE DIFFERENCE METHOD OF CARDIOVASCULAR SYSTEM

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Abstract. *This abstract considers the process of solving a parabolic differential equation by the finite difference method. The method is based on discretization of space and time, approximation of derivatives and subsequent calculation of function values on a grid. The solution of the system of equations is performed using an explicit or implicit scheme. In the explicit scheme, the function values in the new time layer are calculated based on the function values in the previous layer. In the implicit scheme, the function values in the new layer are found as a solution to a system of equations. For the implicit scheme, iterative methods are used.*
Keywords*: differential equation, explicit scheme, implicit scheme, mathematical model of blood vessels, discretization of space and time.*

The differential equation used to create a mathematical model of blood vessels is usually of the parabolic type.

Modeling blood vessels using parabolic differential equations allows one to study the dynamics of blood flow, the distribution of nutrients and oxygen in organs and tissues, and also predict the effects of various factors on the circulatory system.

The use of differential equations of parabolic type in modeling blood vessels allows one to study phenomena such as blood transport inside vessels, diffusion of nutrients through capillary walls, gas exchange between blood and tissues, etc.

These equations allow us to take into account various parameters, such as the size and shape of blood vessels, their elasticity, blood pressure, vascular resistance and other factors that affect the functioning of the circulatory system. Modeling blood vessels using differential equations of parabolic type helps to better understand and predict various processes occurring in the body, and can be useful for the development of new methods for diagnosing and treating diseases associated with blood circulation. One of the most common parabolic differential equations used in blood vessel modeling is called the diffusion equation or heat equation. This equation describes the distribution of the concentration of a substance or temperature in space and time.

This equation allows you to model the distribution of nutrients, oxygen or other substances within blood vessels and surrounding tissues. It takes into account the diffusion of a substance along vessels and through their walls, as well as the transfer of a substance under the influence of a concentration gradient.

In addition to the diffusion equation, other parabolic-type differential equations can be used in modeling blood vessels, including equations that describe the propagation of electrical or chemical signals in the nervous system or the propagation of heat in tissue during laser treatment, for example. The choice of a particular equation depends on the specific characteristics of the system and the physical processes that need to be modeled.

Parabolic differential equations are widely used to create mathematical models of blood vessels. These equations describe the distribution and change in the concentration of a substance or parameters in time and space.

Parabolic differential equations are used in various fields to create mathematical models. Some of these areas include:

• Biomedical Engineering: Parabolic differential equations are used in biomedical engineering to model blood vessels and their interactions with tissues. This may include studying the distribution of oxygen, nutrients and drugs in organs and tissues.

• Medical Physics: Parabolic differential equations are used to create models that help understand the physiological processes occurring in the circulatory system. This could include studying thermal conductivity in tissue under laser irradiation or simulating electrical signals in the heart and nervous system.

• Medical Science: Parabolic differential equations are used to study various aspects of blood circulation and their effects on health and disease. This may include modeling the spread of blood clots, the formation of atherosclerotic plaques, the exchange of gases in the lungs and other processes associated with the circulatory system.

• Mathematical Modeling: Parabolic differential equations are used in mathematical modeling of blood vessels to analyze and predict various phenomena and effects. This may include studying the effects of changes in system parameters, such as changes in blood pressure or vascular resistance, on the distribution of nutrients and blood flow in organs and tissues.

In general, parabolic differential equations are used to create mathematical models of blood vessels in various fields to better understand and predict circulatory function, disease effects, and treatment effectiveness.

The finite difference method is one of the numerical methods used to solve differential equations of parabolic type. This method is based on the approximation of derivatives by discrete differences on a grid and subsequent calculation of the function values at the nodes of this grid. To apply the finite difference method to the solution of a parabolic differential equation such as the diffusion equation, an explicit or implicit scheme is usually used. In the explicit scheme, the function values in the new time layer are calculated based on the function values in the previous time layer, and in the implicit scheme, the function values in the new time layer are found as a solution to a system of equations in which the function values are present in both the current and the next time layer .

The process of solving a parabolic differential equation using the finite difference method includes the following steps:

• Specifying a grid: A spatial grid is defined, which is a discrete set of points in space along which the solution is approximated. The time step for sampling the time axis is also specified.

• Derivative approximation: Using difference formulas, the derivatives in the original equation are replaced by difference differences on the grid.

• Construction of a difference scheme: The resulting difference approximations of the derivatives, together with the initial and boundary conditions, lead to the construction of a difference scheme representing a system of equations.

• Solving a system of equations: In the case of an explicit scheme, the values of the function in the new time layer are calculated directly from the values in the previous time layer. In the case of an implicit scheme, the system of equations is solved by iterative methods, such as the simple iteration method or the Gauss-Seidel method.

• Repeating steps 3-4: The process is repeated for each new time layer until the required time interval is reached.

The finite difference method allows one to obtain a numerical approximate solution to a parabolic differential equation on a given grid. However, to achieve accurate results, it is necessary to take into account the choice of time and space steps, as well as the choice of approximation of the derivatives. In the future, we used exactly this method to solve the problem.

Given an equation satisfying the initial $\frac{dy}{dt}=\sum_{\alpha}^{2}\frac{d^2u}{dx_\alpha^2}+f\left(x_1,x_2,t\right),\ \alpha=1,2,$ $0<x_\alpha<1, 0<t\le 0,3$ satisfying the initial

$$u=\left(x_1,x_2,0\right)=1,\ \ 0\le x_\alpha\le 1$$

and boundary condition $u=\left(0,x_{2,}t\right)=1,\ \ u=\left(1,x_{2,}t\right)=e^{-2x_2t},$

$$u=\left(x_{1\,,}0,t\right)=1,\ \ u=\left(x_{1,}1,t\right)=e^{-2x_1t},\ \ 0<t\le 0,3$$

where $f=(x_1,x_2t)=-2\left(2t^2\left(x_1^2+x_2^2\right)+x_1x_2\right)e^{-2x_1x_2t}$

There is a mathematical solution to this problem $u=e^{-2x_1x_2t}$. In order not to make mistakes and not to confuse the values x_1,x_2 we denote x,y, denote u will with P the function for searching for the result and it will turn out $f=(x,y,t)=-2\left(2t^2\left(x^2+y^2\right)+x^2y^2\right)e^{-2x^2y^2t}$.

To numerically solve the boundary value problem in a field, we pass to a discrete field. To do this, we will build a type of difference grid.

$$Q_{ht}=\left\{x=ih,y=ih,i=0,1.\ldots.,N,h=\frac{1}{N},t_j=i\tau,j=0,1\ldots N_0,\tau=\frac{1}{N_0}\right\}$$

The accuracy of the obtained results and the mathematical model for checking the adequacy we obtain an analytical solution to the boundary value problem

To numerically solve the problem in a discrete domain, we use the finite difference method based on the variable orientation scheme. This way you can move from one time layer to another. It is done in two steps with a step $0,5\tau$. The finite-difference equation can be written in operator form, omitting the indices *i* and *j*, as follows:

$$\frac{P^{k+0,5}-P^k}{0,5\tau}=\wedge_1P^{k+0,5}+\wedge_2P^k+f\left(x,y,t\right)$$

$$\frac{P^{k+1}-P^{k+0,5}}{0,5\tau}=\wedge_1P^{k+0,5}+\wedge_2P^{k+1}+f\left(x,y,t\right)$$

In this case, the system of finite-difference equations is equal to *k+0.*5 and *k+1* times the time layer.

$$a_iP_{i-1,j}^{k+0,5}+b_iP_{i,j}^{k+0,5}+cP_{i+1,j}^{k+0,5}=-d_i$$

$$a_iP_{i-1,j}^{k+1}+b_iP_{i,j}^{k+1}+cP_{i+1,j}^{k+1}=-d_i$$

Here in the program the following values are set

$$a_i = 1\,; \qquad b_i = 2(1+\frac{h^2}{\tau};) \qquad c_i = 1$$

$$d_i = \frac{2h^2}{\tau} P_{i,j}^k + \frac{P_{i,j-1}^k - 2P_{i,j}^k + P_{i,j+1}^k}{h^2} + (-2(2t^2(x^2+y^2)+xy)e^{-2xyt}),$$

$$a_j = 1\,; \qquad b_j = 2(1+\frac{h^2}{\tau});) \qquad c_j = 1$$

$$d_j = \frac{2h^2}{\tau} P_{i,j}^{k+0,5} + \frac{P_{i,j-1}^{k+0,5} - 2P_{i,j}^{k+0,5} + P_{i,j+1}^{k+0,5}}{h^2} + (-2(2t^2(x^2+y^2)+xy)e^{-2xyt}),$$

The first equation of the finite difference system (*k+0.5* times layer) is solved using the sweep method. Its numerical solution is based on the sweep method:

$$P_i = A_i P_{i+1} + B_i \qquad (i = n-1....,1)$$

Here A_i и B_i are the run coefficients, and they are determined by the formulas:

$$A_i = \frac{c_i}{b_i - a_i A_{i-1}} \qquad\qquad B_i = \frac{a_i B_{i-1}}{b_i - a_i A_{i-1}};$$

The initial values of the drive coefficients are determined from the boundary conditions A_0, B_0. For the first boundary condition, in general, it is given above.

The program was created in the MathLab programming environment and calculation experiments were carried out. The calculation results are presented in two- and three-dimensional graphs, and the accuracy of the numerical results is compared with the exact solution.

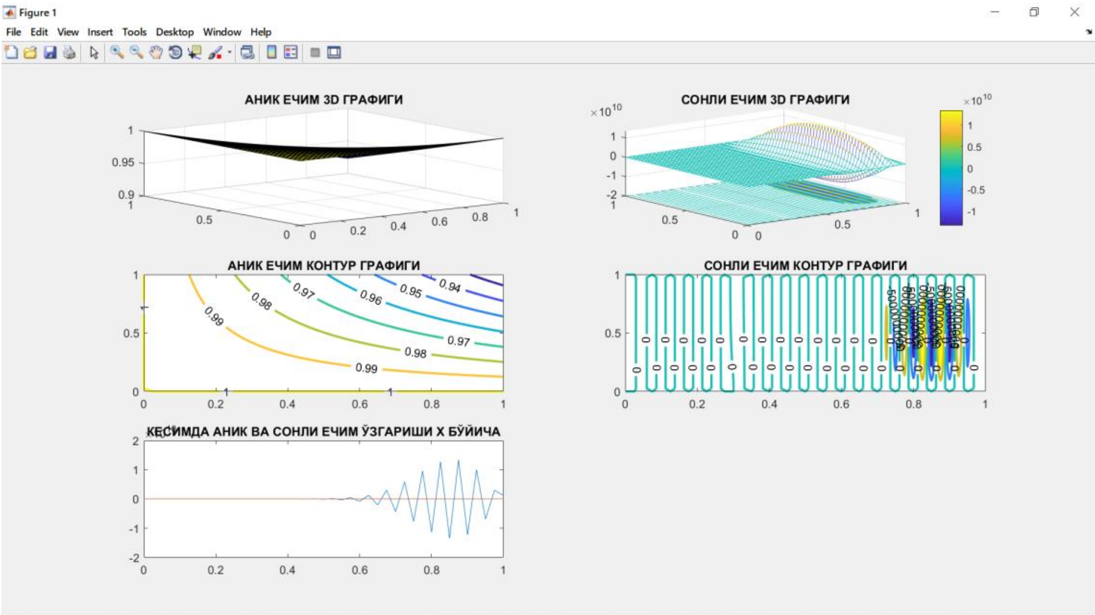

Fig1. Explicit Method and Graph

It is known that the explicit scheme is not constant. From here, computational experiments were carried out to determine the stability of the process. The experiments are mainly based on changing the time step and are presented in the graphs in Fig.1.

The explicit method is based on approximating derivatives at grid points using only known values at the previous time. For each grid point, the solution at the next time step is expressed explicitly in terms of the values at the current time step and/or previous steps. The explicit method is easy to implement and requires less computational resources. However, it has restrictions on the choice of time step to guarantee the stability of the numerical solution. Example of an explicit method: sweep method, Runge-Kutta method.

Implicit finite difference method

The implicit method uses derivative approximations that include the values at the next time step and the unknown values at the current time step. For each grid point, the decision at the next time step is expressed implicitly in terms of the values at the current time step and/or previous steps. The implicit method provides higher stability and allows the use of a larger time step, but requires solving a system of equations at each time step. Solving a system of equations can be quite computationally expensive. Example of an implicit method: backward sweep method, implicit Euler method.

Both finite difference methods are widely used to approximate differential equations of various types, including heat equations, transport equations, and others. The choice between an explicit and an implicit method depends on the requirements for accuracy, robustness, and computational efficiency.

Now we will consider an implicit method for solving this problem. The first equation of the finite difference system (*k+0.5* times layer) is solved using the sweep method. Its numerical solution is based on the sweep method:

$$P_i = A_i P_{i+1} + B_i \qquad (i = n-1....,1)$$

Here A_i and B_i are the run coefficients, and they are determined by the formulas:

$$A_i = \frac{c_i}{b_i - a_i A_{i-1}} \qquad\qquad B_i = \frac{a_i B_{i-1}}{b_i - a_i A_{i-1}};$$

The initial values of the drive coefficients are determined from the boundary conditions A_0, B_0.. For the first boundary condition, in general, it is given above.

$$a_i = 1\,; \qquad b_i = 2(1+\frac{h^2}{\tau};) \qquad c_i = 1$$

$$d_i = \frac{2h^2}{\tau}P_{i,j}^k + \frac{P_{i,j-1}^k - 2P_{i,j}^k + P_{i,j+1}^k}{h^2} + (-2(2t^2(x^2+y^2)+xy)e^{-2xyt}),$$

$$a_j = 1\,; \qquad b_j = 2(1+\frac{h^2}{\tau});) \qquad c_j = 1$$

$$d_j = \frac{2h^2}{\tau}P_{i,j}^{k+0,5} + \frac{P_{i,j-1}^{k+0,5} - 2P_{i,j}^{k+0,5} + P_{i,j+1}^{k+0,5}}{h^2} + (-2(2t^2(x^2+y^2)+xy)e^{-2xyt}),$$

This program is an implementation of a numerical method for solving the heat equation in a two-dimensional domain. The program uses the finite difference method to approximate the differential equation and finds a numerical solution.

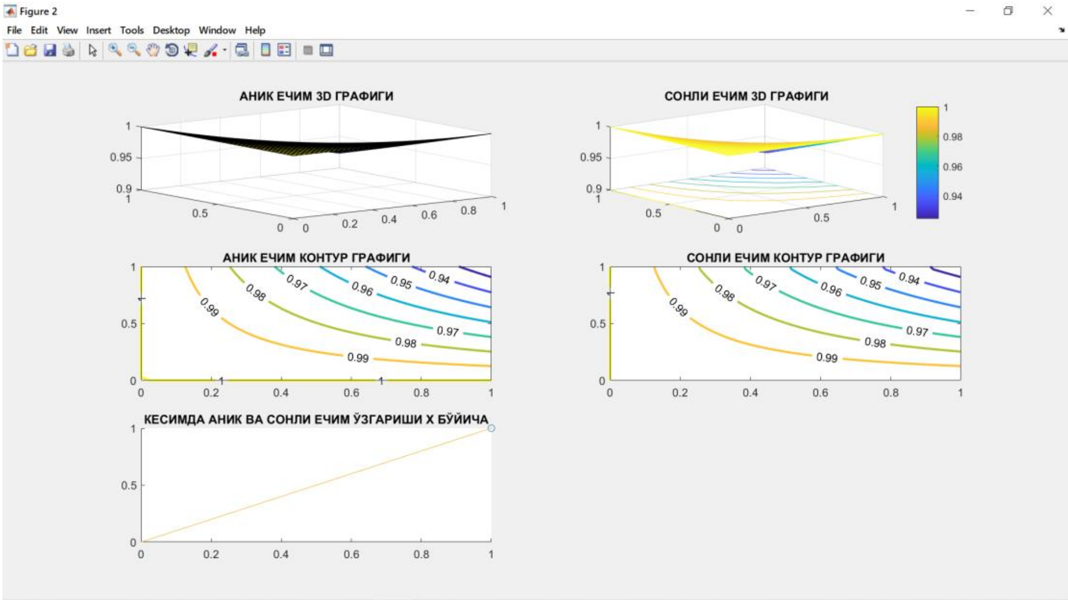

Fig.2. Fig1. Implicit Method and Graph

To check the accuracy of the numerical results, the numerical values p[i,j] and the difference of solutions pt[i,j] were connected to the array b[j] and the difference between them was determined. The difference in the number of points in Table 1 is 41. The largest difference between them is max=0.0120.

№	P	Pt	b
1	1.0000	1.0000	0
2	1.0000	1.0000	0.0000
3	0.9999	1.0000	0.0000
4	0.9999	1.0000	0.0000
5	0.9999	1.0000	0.0000
6	0.9998	1.0000	0.0001
7	0.9998	1.0000	0.0001
8	0.9998	1.0000	0.0001
9	0.9997	1.0000	0.0001
10	0.9997	1.0000	0.0001
11	0.9997	1.0000	0.0001

Table 1. The difference in the number of points in Table 1 is 41. The largest difference between them is max=0.0120.

So, in conclusion, we can say that the greater the number of points, the smaller the error, that is, the numerical solution approaches the exact one, solving problems, as well as presenting its numerical results in the form of 3D graphics, in the process of analysis and forecasting. can be used.

The program displays the values of time steps, numerical and analytical solutions, the differences between them, as well as the maximum difference. It then plots the analytical and numerical solutions in 3D and contour form, as well as a graph of temperature changes along the selected section.

The process of solving a differential equation of parabolic type using the finite difference method requires determining steps in time and space, as well as choosing an approximation of the derivatives. Numerical results of the finite difference method make it possible to obtain an approximate solution of the equation on a given grid and analyze the behavior of the solution depending on changes in the system parameters. Thus, the finite difference method is a useful tool for solving parabolic differential equations and studying various physical processes. The finite difference method is a powerful numerical tool for solving parabolic differential equations. It allows you to approximate derivatives and calculate function values on a discrete grid in space and time. The process of solving a parabolic differential equation by the finite difference method consists of several stages, including specifying a mesh, approximating derivatives, constructing a difference scheme, and solving a system of equations.

The finite difference method has a number of advantages. It is quite simple to implement and can be applied to a variety of geometric and temporal configurations. In addition, it provides fairly high accuracy when choosing sufficiently small steps in space and time. However, when using the finite difference method, some limitations must be taken into account. In particular, the choice of mesh and sampling steps can be computationally intensive, especially for complex 3D geometries. It is also worth noting that the finite difference method may have limitations in regions with sharp fronts and abrupt changes in the solution. In conclusion, the finite difference method provides an effective tool for solving parabolic differential equations. It is widely used in various fields, such as physics, biology, finance and others, where problems arise in modeling and analyzing processes with distribution in time and space. The accuracy and efficiency of the method depend on the choice of approximation, mesh and sampling steps, so appropriate research and testing must be carried out to achieve reliable results.

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