SOLUTION OF THE PRESSURE PARAMETER USING THE DIFFERENTIAL EQUATION OF THE FINITE DIFFERENCE METHOD FOR THE CARDIOVASCULAR AND VISUALIZATION IN COMSOL MULTIPYSICS

[1]Nurjabova D., [2]Abdumominova Sh.R.
[1]Phd student of Tashkent University of Information Technologies
[2]Republican Blood Transfusion Center, Tashkent, Uzbekistan
https://doi.org/10.5281/zenodo.11364407

Abstract*. This abstract discusses how to solve a parabolic differential equation using the finite difference method. The approach involves discretizing space and time, approximating derivatives, and then calculating function values on a grid. The system of equations is solved using one of two methods: explicit or implicit. In the explicit system, the function values in the current temporal layer are derived using the function values from the preceding layer. The implicit strategy finds the function values in the new layer by solving a set of equations. The implicit approach employs iterative procedures. Also, Python was used to build a mathematical model for velocity and pressure components, which was integrated into COMSOL Multiphysics 5.6 for testing and computations.*

Keywords*: differential equation, explicit scheme, implicit scheme, mathematical model of blood vessels, space-time discretization.*

INTRODUCTION

Pressure in blood vessels is determined by the force with which blood presses on the walls of blood vessels. It is usually measured in two values: systolic (upper), which reflects the pressure in the vessels when the heart contracts, and diastolic (lower), which is measured when the heart relaxes between contractions. Typically expressed in mmHg (mmHg). Normal blood pressure is usually around 120 mm Hg. Art. (systolic) at 80 mm Hg. Art. (diastolic). However, normal values may vary slightly depending on age, gender and other factors. High blood pressure (hypertension) can increase your risk of heart disease, while low blood pressure (hypotension) can cause dizziness and other problems.

Researchers *Xushan Huang, Moon-Jin Kang, Jeongho Kim, and Hobin Lee* examined their article about “Asymptotic behavior toward viscous shock for impermeable wall and inflow problems of barotropic Navier-stokes equations” *https://arxiv.org/pdf/2405.03214* the compressible barotropic Navier-Stokes equations in a half-line and analyze the time-asymptotic behavior of the outgoing viscous shock wave. The two boundary issues are impermeable wall and inflow, with a constant velocity at the border. They are demonstrated that when the asymptotic profile of a viscous shock is determined by constant states at the boundary and far-fields, the solution uniformly converges to the shifted viscous shock profiles, provided the initial perturbation is small enough in the H 1 norm. They do not impose the zero mass requirement on initial data, which enhances the prior results by Matsumura and Mei [20] for impermeable case and by Huang, Matsumura and Shi [8] for inflow case. Moreover, for the inflow case, we remove the assumption $\gamma \leq 3$ in [8]. They are results are based on the method of a-contraction with shifts, as the first extension of the method to the boundary value problems.

Froese, V., Gabel, G., Parnell, J. et al. Flow study on a transparent two-phase blood model fluid based on alginate microspheres. *Exp Fluids* 63, 188 (2022). https://doi.org/10.1007/s00348-022-03538-y studied reducing blood damage in medical equipment that conveys blood is still a major challenge. Blood cells are opaque, so in vitro tests are used to study the damage-causing effects, although the results only show near-wall flows. In order to view the rheological behavior of blood, a number of transparent blood models have been developed and investigated. Even so, two-phase blood models with additional particles either don't accurately capture the characteristics of blood or are exceedingly costly and difficult to make. This in vitro study compared the viscosity, flow characteristics, and deformation of human red blood cells to a new, simple-to-manufacture, two-phase blood model fluid that contains microspheres of deformable alginate. A straight and a hyperbolic cone-plate rheometer were used for the comparison converging microchannel. The viscosity of the blood model fluid with a particle fraction of 30% showed a shear-thinning behavior, comparable to that of blood at room temperature and human body temperature within shear rates from 7 to 2000s−1. The alginate microspheres were deformable in an extensional flow and formed a cell free layer comparable to that of blood in a straight microchannel. The experiments showed good optical accessibility of the two-phase flow with traceable movements of individual microspheres in the center of the microchannel. It could be shown that our proposed blood model fluid is a promising tool for the analysis of two-phase flows in complex flow geometries.

Wang, Y., Mu, L. & He, Y. Thermogram-based estimation of foot arterial blood flow using neural networks. *Appl. Math. Mech.-Engl. Ed.* 44, 325–344 (2023). *https://doi.org/10.1007/s10483-023-2959-9* examined one of the most significant markers of early diabetic foot problems is decreased blood flow in the foot. Measuring blood flow on the full foot scale is difficult, though. This article describes a method for measuring the blood flow via the foot arteries by utilizing an artificial neural network and the temperature distribution. A bioheat transfer model of voxel-meshed foot tissue with discrete blood vessels is built based on computed tomography (CT) sequential images and the anatomical information of the vascular structure in order to assess the link between the temperature distribution and the blood flow. Our model fully accounts for the heat transfer from tissue and blood vessels as well as the inter-domain heat exchange between them, and the calculated temperatures agree with the findings of the experiments. Analytical data are then used to train a neural network to determine foot arterial blood flow. The trained network is able to estimate the objective blood flow for various degrees of stenosis in multiple blood vessels with an accuracy rate of more than 90%. Compared with the Pennes bioheat transfer equation, this model fully describes intra- and inter-domain heat transfer in blood vessels and tissue, closely approximating physiological conditions. By introducing a vascular component to an inverse model, the blood flow itself, rather than blood perfusion, can be estimated, directly informing vascular health.

Pandit, A.B., Abbas, S. & Mukherjee, J. Mathematical Modelling of Pulsatile Blood Flow in Straight Rigid Artery System. Trans Indian Natl. Acad. Eng. 7, 1043–1057(2022*),https://doi.org/10.1007/s41403-022-00349* observed about developing diagnostic instruments for the management of cardiovascular disorders greatly benefits from an understanding of the blood flow characteristics in the circulatory system. Analytical and numerical efforts are offered in the research to illustrate how blood flows in a stiff, straight artery system behave under a creeping flow regime. Blood is seen as a viscoelastic fluid substance in this context and is also regarded as a functioning fluid system. The Casson fluid model is used to characterize the rheological behavior of blood flow. Analytical solutions are carried out utilizing the zeroth-order Bessel function technique in the context of the hydrodynamic analysis of pulsatile blood flow in the straight rigid artery system, and they are confirmed using the derived numerical findings based on the finite difference method (FDM).

The explicit technique of FTCS scheme is employed in FDM to numerically solve the non-dimensional essential governing equations. For a given value of the Womersley number (α = 3), there is a good agreement between the analytical and numerical methods based on FDM, with ± 10% and ± 7% for the operating Reynolds number (Re = 5) respectively. The finite element approach is used using COMSOL Multiphysics software to display the dimensional velocity and pressure distribution inside the straight stiff artery system. The acquired numerical research indicates that the radial velocity considerably varies as a function of transverse position and progressively achieves fully developed flow, whereas the pressure distribution decreases as a function of axial position of the straight rigid artery system. The next stage we should looking through using the pressure parameter using the differential equation of the finite difference method for the cardiovascular and design in Comsol Multipysics.

Body pressure includes not only blood pressure, which we talked about earlier, but also other types of pressure that play an important role in the normal functioning of the body.

Blood pressure: This is the pressure that blood exerts on the walls of the arteries. It is measured as systolic (upper) and diastolic (lower) pressure.

Intravascular pressure: This is the pressure inside the blood vessels that is exerted on their walls by the blood. It changes throughout the circulatory circuit and plays a key role in maintaining blood circulation.

Intracellular pressure: This is the pressure inside the cells of the body. Intracellular pressure is regulated by various processes, such as osmotic equilibrium, cellular pump activity, and transport mechanisms.

Hydrostatic pressure: This is the pressure created by fluids in the body such as blood and lymph. It plays a role in maintaining fluid balance and interacting with tissues and organs.

Pressure in the lungs: In the lungs, pressure plays a role in the exchange of gases and ensures the delivery of oxygen to the blood and the removal of carbon dioxide from the body.

All these types of pressure are closely related and interact with each other to maintain the vital functions of the body. Their balance and regulation play a critical role in maintaining the health and functioning of all body systems.

Blood pressure (BP) is the force that blood exerts on the walls of the arteries as it circulates through the circulatory system. It is measured in millimeters of mercury (mmHg) and is usually expressed in two numbers: upper (systolic) and lower (diastolic) pressure. Systolic pressure describes the maximum force with which blood is squeezed from the heart into the arteries during contraction of the heart muscle (systole), while diastolic pressure describes the force with which blood pushes against the arterial walls between contractions of the heart (diastole), when the heart relaxed and filled with blood.

Blood pressure is maintained by a complex interaction between the heart, blood vessels, and nervous system, which is regulated by various mechanisms, including the contraction and relaxation of vascular walls, circulating blood volume, and the activity of the sympathetic and parasympathetic nervous systems. Disturbances in this regulation can lead to hypertension (high blood pressure) or hypotension (low blood pressure), which can have serious health consequences such as cardiovascular disease or organ failure.

Vascular pressure, also known as intravascular pressure or internal vascular pressure, is the force that blood exerts on the walls of blood vessels within the circulatory system. This pressure is created by the blood flow, which is provided by the heart pump and controlled by the body's vascular system.

Vascular pressure can vary depending on various factors, including cardiac output, peripheral vascular resistance (how the vessels respond to blood flow), the amount of blood reserve, and the total volume of circulating blood. It is important to note that vascular pressure varies throughout the blood circuit: it is highest in the arteries, decreases in the arterioles and capillaries, and then increases in the veins.

The pressure in the blood vessels is regulated not only by the heart, but also by the vessels of the body itself, as well as by the nervous system. Various regulatory mechanisms, such as contraction and relaxation of vascular walls and changes in peripheral resistance, help maintain optimal pressure in the vessels to ensure blood delivery to the organs and tissues of the body.

Intravascular pressure (or intravascular pressure) is the pressure that blood exerts on the walls of blood vessels such as arteries, veins and capillaries. This pressure is created by blood flow and is a key factor determining blood flow and perfusion of the body's organs and tissues.

Intravascular pressure is regulated by various mechanisms, including the contraction and relaxation of vascular walls, the level of vascular resistance, the volume of blood in the circulatory system, and the activity of the heart muscle. Disturbances in this regulation can lead to various pathological conditions such as hypertension (high blood pressure), hypotension (low blood pressure) or circulatory complications. Measuring intravascular pressure can be performed by a variety of methods, such as invasive procedures such as arterial catheterization, or noninvasive methods such as the use of noninvasive blood pressure machines on the wrist or upper arm. These data are often used to diagnose cardiovascular diseases and monitor patient conditions in clinical practice.

In COMSOL Multiphysics software, which is designed to model and simulate physical processes in various fields of science and technology, the term "pressure" is used to describe the pressure in liquids or gases within a specific region of the model.Within COMSOL Multiphysics, pressure is typically defined as a parameter that can be included in the Navier-Stokes equations (for liquids) or the Navier-Stokes equations along with an equation of state (for gases) to describe the flow of a liquid or gas within a modeled system. For example, if you are creating a model of fluid flow through a pipeline, pressure can be used to describe the differences in pressure along the pipe or at different points in the system. COMSOL Multiphysics lets you model complex physics phenomena such as heat transfer, electromagnetic fields, chemical reactions, and more, including their interactions with pressure.

Using pressure in COMSOL Multiphysics allows you to analyze and optimize a variety of engineering systems and processes, such as fluid flow in pipes, gas dynamics, aerodynamic characteristics, and more. Intravascular pressure can be described in COMSOL Multiphysics using appropriate physics models and equations. The following is a general approach to modeling intravascular pressure:

Geometry Selection: Create a model geometry that reflects the vascular system you want to study. This may be a model of arteries, veins or capillaries, or a combination of them.

Definition of physical equations: Include appropriate physical equations in the model to describe the flow of blood in the vessels. In particular, these are the Navier-Stokes equations for liquids, which describe the dynamics of blood movement inside the vessels. An equation of state may also be required to account for changes in blood density with pressure and temperature.

Defining Boundary Conditions: Define boundary conditions for your model, such as initial pressure, blood velocity, or temperature at the vessel inlet, as well as conditions at the outlet and on the vessel walls. Setting parameters: Set model parameters such as blood viscosity, blood density, vessel sizes and other physical properties. Solving the Model: Solve the model equations using numerical methods available in COMSOL Multiphysics to obtain the pressure distribution within the vascular system.

Analysis of results: Examine your results to understand how intravascular pressure changes depending on various factors such as vessel geometry, blood velocity, vessel diameter, and other model parameters.

By following these steps, you will be able to describe intravascular pressure in COMSOL Multiphysics and explore its influence on various physiological and pathological conditions in the body.

METHODS

The state of a moving fluid is determined by setting five values: three components of velocity *V(x; y; z; t)* pressure *p*(*x; y; z; t*) and density _(*x; y; z; t*) In fluid mechanics, its molecular structure is not considered, it is assumed that the fluid fills the space entirely, instead of the fluid itself, its model is studied, a fictitious continuous medium with the property of continuity. This approach simplifies the researching, all mechanical and hemodynamics characteristics of the liquid medium (velocity, pressure, density) are assumed to be continuous and differentiable.

The equations of motion of a viscous incompressible fluid (Navier-Stokes equations) in projections on the coordinate axis by velocity components have the form [1,6]

$$\frac{\partial \upsilon_x}{\partial t} = F_x - \frac{1}{\rho}\frac{\partial p}{\partial x} - \upsilon_x \frac{\partial \upsilon_x}{\partial x} - \upsilon_y \frac{\partial \upsilon_x}{\partial y} - \upsilon_z \frac{\partial \upsilon_x}{\partial z} + \nu\left(\frac{\partial^2 \upsilon_x}{\partial x^2} + \frac{\partial^2 \upsilon_x}{\partial y^2} + \frac{\partial^2 \upsilon_x}{\partial z^2}\right), \quad (1)$$

$$\frac{\partial \upsilon_y}{\partial t} = F_y - \frac{1}{\rho}\frac{\partial p}{\partial y} - \upsilon_x \frac{\partial \upsilon_y}{\partial x} - \upsilon_y \frac{\partial \upsilon_y}{\partial y} - \upsilon_z \frac{\partial \upsilon_y}{\partial z} + \nu\left(\frac{\partial^2 \upsilon_y}{\partial x^2} + \frac{\partial^2 \upsilon_y}{\partial y^2} + \frac{\partial^2 \upsilon_y}{\partial z^2}\right), \quad (2)$$

$$\frac{\partial \upsilon_z}{\partial t} = F_z - \frac{1}{\rho}\frac{\partial p}{\partial z} - \upsilon_x \frac{\partial \upsilon_z}{\partial x} - \upsilon_y \frac{\partial \upsilon_z}{\partial y} - \upsilon_z \frac{\partial \upsilon_z}{\partial z} + \nu\left(\frac{\partial^2 \upsilon_z}{\partial x^2} + \frac{\partial^2 \upsilon_z}{\partial y^2} + \frac{\partial^2 \upsilon_z}{\partial z^2}\right), \quad (3)$$

We assume that $F_x(x,y,z,t), F_y(x,y,z,t),\ F_z(x,y,z,t)$ are given continuous functions domain of $\Omega\times T$.

$$F_x(x,y,z,t)\in C_{x,y,z,t}(\Omega\times T), F_y(x,y,z,t)\in C_{x,y,z,t}(\Omega\times T), F_z(x,y,z,t)\in C_{x,y,z,t}(\Omega\times T)$$

and functions $p_x(x,y,z,t)=\frac{\partial \rho}{\partial x}\in C_{x,y,z,t}$, $p_y(x,y,z,t)=\frac{\partial \rho}{\partial y}\in C_{x,y,z,t}$, $p_z(x,y,z,t)=\frac{\partial \rho}{\partial z}\in C_{x,y,z,t}$

.

DISCUSSION

The findings are based on theoretical investigations of mathematical hydrodynamics and show the uniqueness of solutions of the Navier-Stokes equations for a viscous incompressible fluid under specified beginning circumstances as well as with the condition of continuation. The Navier-Stokes equations have a unique solution with regard to the fluid velocity components under certain beginning circumstances and continuity requirements, according to a theorem on the existence and uniqueness of solutions. This implies that there is a single solution for any hydrodynamics issue that the Navier-Stokes equations can represent.

The Navier-Stokes equations, a mathematical hydrodynamics model, have unique solutions for viscous incompressible fluids under specific conditions and continuation conditions. This implies a single solution for any hydrodynamics issue that these equations can represent.

For every velocity component, algorithmic of mathematical model were generated using the defined finite algorithmic method. These algorithmic methods were used to create a software utility. The Python programming language was utilized to build the algorithmic logic based on the algorithmic flowcharts. The Python file that was produced was then included into the COMSOL Multiphysics 5.6 setup. Thus, in the COMSOL Multiphysics 5.6 environment, tests and computations were carried out utilizing the implemented file based on the Python programming language.

RESULTS

An analytical solution to the Navier-Stokes equations may not always be possible, despite the existence and uniqueness theorem of solutions being a significant outcome. As a result, numerical techniques that enable computer-based Navier-Stokes equation solution become crucial for real-world applications.

Algorithmic of mathematical model were generated for velocity components using finite algorithmic block diagrams, forming a software utility. Python was used to build logic, which was integrated into COMSOL Multiphysics 5.6 for testing and computations.

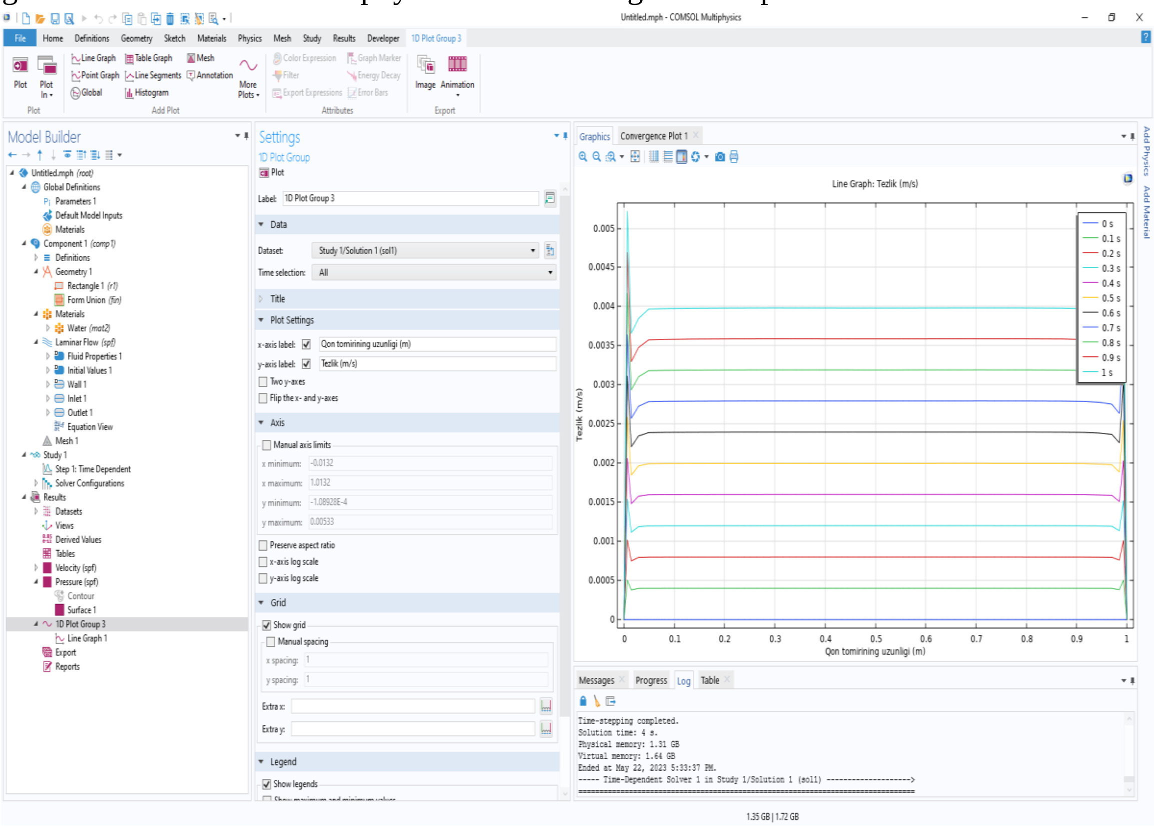

Fig.1.When the blood pressure P=1 Pa, the variation of blood velocity along the blood vessel

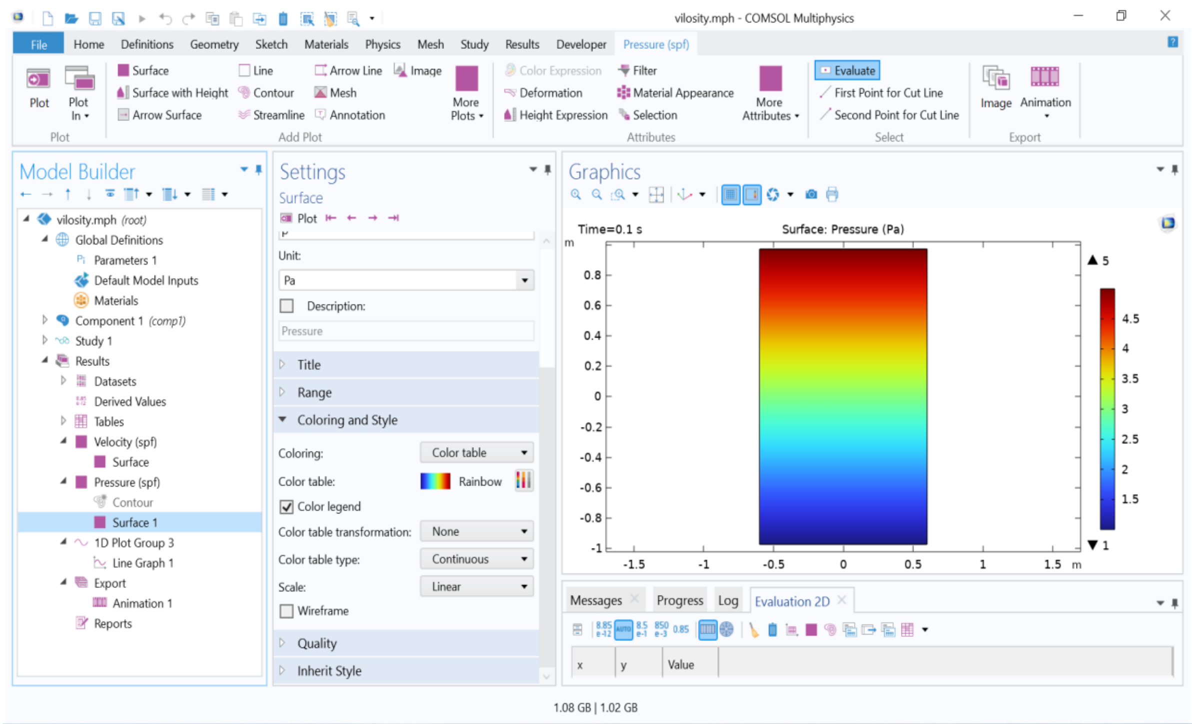

Fig.2.When the blood pressure P=1 Pa, the variation of blood pressure along the blood vessel

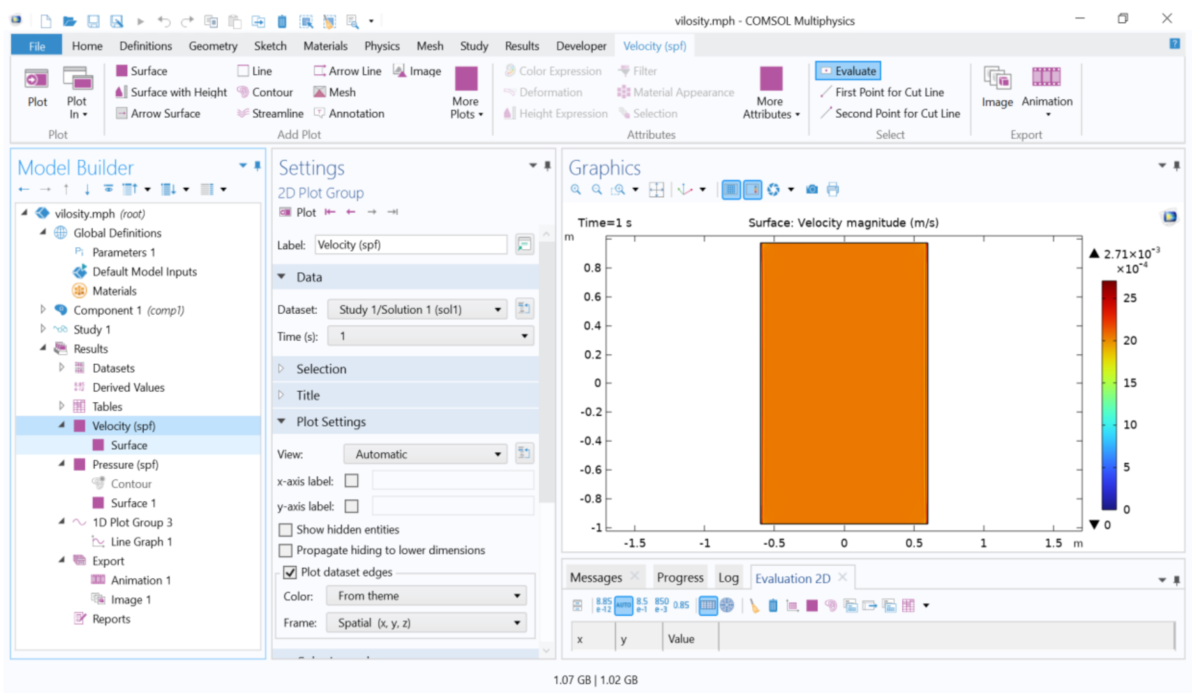

Fig.3.When the blood pressure P=1 Pa, the variation of blood velocity magnitude along the blood vessel

REFERENCES

1. Froese, V., Gabel, G., Parnell, J. *et al.* Flow study on a transparent two-phase blood model fluid based on alginate microspheres. *Exp Fluids* 63, 188 (2022). https://doi.org/10.1007/s00348-022-03538-y
2. Wang, Y., Mu, L. & He, Y. Thermogram-based estimation of foot arterial blood flow using neural networks. *Appl. Math. Mech.-Engl. Ed.* 44, 325–344 (2023). https://doi.org/10.1007/s10483-023-2959-9

3. Yeo, E.F., Oliver, J.M., Korin, N. *et al.* A continuum model for the elongation and orientation of Von Willebrand factor with applications in arterial flow. *Biomech Model Mechanobiol* (2024). https://doi.org/10.1007/s10237-024-01840-8
4. Pandit, A.B., Abbas, S. & Mukherjee, J. Mathematical Modelling of Pulsatile Blood Flow in Straight Rigid Artery System. *Trans Indian Natl. Acad. Eng.* 7, 1043–1057 (2022). https://doi.org/10.1007/s41403-022-00349-9
5. Awan, A.U., Shafique, S., Shahzad, M.H. *et al.* Non-Newtonian rheology of blood in elliptical cross-section artery affected by several stenosis: Prandtl fluid model. *Arch Appl Mech* 94, 943–960 (2024). https://doi.org/10.1007/s00419-024-02557-x
6. Leoni, M., Szasz, J., Meier, J. *et al.* Blood flow but not cannula positioning influences the efficacy of Veno-Venous ECMO therapy. *Sci Rep* 12, 20950 (2022). https://doi.org/10.1038/s41598-022-23159-z
7. Godefroy, G., Arnal, B. & Bossy, E. Full-visibility 3D imaging of oxygenation and blood flow by simultaneous multispectral photoacoustic fluctuation imaging (MS-PAFI) and ultrasound Doppler. *Sci Rep* 13, 2961 (2023). https://doi.org/10.1038/s41598-023-29177-9
8. Nurjabova D., Sultonov M. USING THE VISCOSITY PARAMETER ON THE NAVIER-STOKES EQUATIONS FOR A VISCOUS INCOMPRESSIBLE FLUID IN AN UNBOUNDED DOMAIN //Евразийский журнал математической теории и компьютерных наук. – 2023. – Т. 3. – №. 5. – С. 64-73.
9. Nurjabova, D., & Muxammadbobir, S. (2023). PARAMETER ON THE NAVIER-STOKES EQUATIONS FOR A VISCOUS INCOMPRESSIBLE FLUID IN AN UNBOUNDED DOMAIN. *Наука и технология в современном мире*, *2*(15), 56-58.
10. Nurjabova, D. (2024). CALCULATING THE VELOCITY OF GLOBAL BLOOD FLOW WITH COMSOLE MULTYPYSICS IN THE CARDIOVESSEL. *Евразийский журнал математической теории и компьютерных наук*, *4*(2), 25-29.
11. Nurjabova, D. (2024). USING COMSOLE MULTYPYSICS IN THE CARDIOVESSEL. *"TRENDS OF MODERN SCIENCE AND PRACTICE"*, *2*(1), 44-48. academicsresearch.ru
12. Nurjabova, D., Sayyora, Q., & Gulmira, P. (2022, December). Using Discretization and Numerical Methods of Problem 1D-3D-1D Model for Blood Vessel Walls with Navier-Stokes. In *International Conference on Next Generation Wired/Wireless Networking* (pp. 73-82). Cham: Springer Nature Switzerland.
13. Dilafruz Nurjabova, Qulmatova Sayyora, Pardayeva Gulmira: Artificial Intelligence Software Architecture in the Field of Cardiology and Application in the Cardio Vessel Project Using CJM and Customer Development Methods. NEW2AN 2022: 57-72