

# ANALYSIS OF THE STEADY MODE OF SUBHARMONIC OSCILLATIONS IN THREE-PHASE FERRORESONANCE CIRCUITS

M. I. Ibadullaev

Professor, doctor of technical sciences, Department of Electrical Engineering, Tashkent State Technical University named after Islam Karimov=

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**Abstract.** *It is known that the occurrence and existence of ferroresonant oscillations at the subharmonic frequency (SHF) in power transmission lines (PTLs) and in power supply systems is extremely undesirable, since they cause ferroresonant overvoltages at various frequencies. At the same time, there is a wide class of nonlinear electrical circuits in which the excitation of autoparametric oscillations (APO) at the SGC frequency forms the basis of frequency-converting devices that serve as secondary power sources. It is shown that three-phase nonlinear systems are, to one degree or another, equivalent circuits for power lines, the main elements of which are: longitudinal compensation capacitors, transverse compensation reactors, and transformers with a nonlinear characteristic. To study the pattern of excitation and maintenance of SGC at frequency  $\left(\frac{\omega}{3}\right)$  in three-phase electroferromagnetic circuits (EFMC), theoretical and experimental studies were carried out on an equivalent model of a three-phase circuit with nonlinear inductance. To analyze the steady-state mode of the SGC at frequency  $\left(\frac{\omega}{3}\right)$ , the method of frequency-energy relations was applied. An algebraic equation is obtained that characterizes the steady-state mode of the SGC at frequency  $\omega/3$  in a three-phase nonlinear circuit. The frequency-converting properties of nonlinear inductance are also shown. The obtained results of the theoretical study were confirmed by experimental studies.*

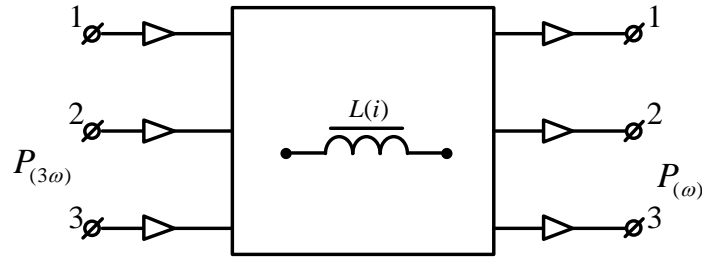
**Keywords:** *ferroresonance, self-oscillations, subharmonic, approximation, lowest harmonic, frequency-energy relationship, ferromagnetic element.*

Research into the processes of excitation of subharmonic oscillations (SHA) in three-phase ferroresonant circuits, first of all, seems relevant from the point of view of the development of new types of three-phase frequency dividers, switching devices with phase-discrete, converting properties, etc. In addition, the study of the physics of SHA in three-phase systems makes it possible to establish some patterns of overvoltage in power lines (power lines) with capacitive compensation caused by harmonic oscillations, and, if possible, take measures to prevent such anomalous conditions or mitigate their negative consequences.

The works are devoted to experimental and theoretical study of the processes of excitation of SGC in three-phase EFMCs and to the identification of the main patterns of their manifestation. [1, 2, 3, 4, 5, 6, 7, 8, 9]

According to [1, 6], in three-phase systems, depending on the amplitude of the applied voltage, initial conditions and parameters of the SGC circuit, they are excited with different amplitude-phase relationships and phase alternation.

In this work, the analysis of the steady-state mode of third-order SGC in a symmetrical three-phase circuit (Fig. 1) is considered using frequency-energy relationships.



**Pic. 1. Block diagram of a three-phase frequency converter.**

To simplify the analysis, we approximate the Weber-ampere characteristic of nonlinear inductance (identical in all phases) by a cubic polynomial of the form

$$i_{\lambda} = a\Psi_{\lambda}^3 + b\Psi_{\lambda}^2, \quad (1)$$

where  $\lambda$  – is the order of the phase sequence (1, 2, 3).

Let us present the law of changes in fluxes created by nonlinear inductance current in magnetic circuits in the form:

$$\Psi_{\lambda} = \Psi_{1\lambda} \cos\left[\omega t + \varphi_1 + \left(\frac{1-\lambda}{9}\right)2\pi\right] + \Psi_{3\lambda m} \cos\left[3\omega t + \varphi_3 + \left(\frac{1-\lambda}{3}\right)2\pi\right] \quad (2)$$

To analyze the excitation of third-order SGCs in three-phase EFMCS, we take the subharmonic phase shifts of individual phases to be 0, 40°, 80°, and the fundamental harmonic flows are symmetrical and have a direct sequence.

Putting (2) into (1), we get:

$$i_{\lambda} = A_{1\lambda} \cos\left[\omega t + \varphi_1 + \left(\frac{1-\lambda}{9}\right)2\pi\right] + B_{1\lambda} \cos\left[3\omega t + \varphi_3 + \left(\frac{1-\lambda}{3}\right)2\pi\right] + A_{2\lambda} \sin\left[\omega t + \varphi_1 + \left(\frac{1-\lambda}{9}\right)2\pi\right] + B_{2\lambda} \sin\left[3\omega t + \varphi_3 + \left(\frac{1-\lambda}{3}\right)2\pi\right], \quad (3)$$

where

$$A_{1\lambda} = \left[ a\Psi_{1\lambda} + \frac{3b}{4}\Psi_{1\lambda}^3 + \frac{3b}{2}\Psi_{1\lambda} \cdot \Psi_{3\lambda}^2 \right] \cos\left[\left(\frac{1-\lambda}{9}\right)2\pi\right] + \frac{3b}{4}\Psi_{1\lambda}^2 \Psi_{3\lambda} \cos(3\varphi_1 - \varphi_3)$$

$$A_{2\lambda} = \left[ a\Psi_{1\lambda} + \frac{3b}{4}\Psi_{1\lambda}^3 + \frac{3b}{2}\Psi_{1\lambda} \cdot \Psi_{3\lambda}^2 \right] \sin\left[\frac{1-\lambda}{9} \cdot 2\pi\right] - \frac{3b}{4}\Psi_{1\lambda}^2 \Psi_{3\lambda} \sin(3\varphi_1 - \varphi_3)$$

$$B_{1\lambda} = \left[ a\Psi_{3\lambda} + \frac{3b}{4}\Psi_{3\lambda}^3 + \frac{3b}{2}\Psi_{1\lambda}^2 \cdot \Psi_{3\lambda} \right] \cos\left[\frac{1-\lambda}{3} \cdot 2\pi\right] + \frac{b}{4}\Psi_{1\lambda}^3 \cos(3\varphi_1 - \varphi_3)$$

$$B_{2\lambda} = \left[ a\Psi_{3\lambda} + \frac{3b}{4}\Psi_{3\lambda}^3 + \frac{3b}{2}\Psi_{1\lambda}^2 \cdot \Psi_{3\lambda} \right] \sin\left[\frac{1-\lambda}{3} \cdot 2\pi\right] + \frac{b}{4}\Psi_{1\lambda}^3 \sin(3\varphi_1 - \varphi_3).$$

Moving on to the complex form of expressing currents and voltages (assuming this is acceptable for a single harmonic), we have:

$$\dot{I}_{1\lambda} = (jA_{1\lambda} - A_{2\lambda})e^{j\left[\varphi_1 + \frac{1-\lambda}{9} \cdot 2\pi\right]}$$

$$\dot{I}_{3\lambda} = (jB_{1\lambda} - B_{2\lambda})e^{j\left[\varphi_3 + \frac{1-\lambda}{3} \cdot 2\pi\right]}$$

$$\dot{U}_{1\lambda} = -\omega\Psi_{1\lambda}e^{j\left[\varphi_1 + \frac{1-\lambda}{9} \cdot 2\pi\right]} \quad \dot{U}_{3\lambda} = -3\omega\Psi_{3\lambda}e^{j\left[\varphi_3 + \frac{1-\lambda}{3} \cdot 2\pi\right]} \quad (4)$$

Next, using Tellegen's theorem [7, 10, 11], we write the third-order total power complex of the SGC for three-phase systems as:

$$\tilde{S}_{1\lambda} = P_{1\lambda} + jQ_{1\lambda} \text{ и } \tilde{S}_{3\lambda} = P_{3\lambda} + jQ_{3\lambda}. \quad (5)$$

The joint solution and (taking into account the condition) allows us to obtain algebraic equations characterizing the stationary mode of third-order SGC in three-phase EFMC  
 The algebraic equations for each phase of a three-phase system are:

$$36\Psi_{31}^4 + 27\Psi_{31}^2\Psi_{11}^2 + 9\Psi_{11}^4 - 48\left(\frac{a-k_q}{b}\right)\Psi_{32}^2 - 24\left(\frac{a-k_q}{b}\right)\Psi_{32}^2 + 16\left[\frac{K_p^2 + (a+k_q)}{b^2}\right] = 0 \quad (6)$$

$$36\Psi_{31}^4 + 27\Psi_{32}^2\Psi_{21}^2 + 9\Psi_{12}^4 - 48\left(\frac{a-0.643k_p-0.766k_q}{b^2}\right)\Psi_{32}^2 -$$

$$- 24\left(\frac{a-0.643k_p-0.766k_q}{b^2}\right)\Psi_{12}^2 + 16\left[\frac{k_p^2 + (a-0.643k_p-0.766k_q)^2}{b^2}\right] = 0 \quad (7)$$

$$36\Psi_{33}^4 + 27\Psi_{33}^2\Psi_{13}^2 + 9\Psi_{13}^4 + 48\left(\frac{a-0.984k_p-0.173k_q}{b^2}\right)\Psi_{33}^2 -$$

$$- 24\left(\frac{a-0.984k_p-0.173k_q}{b^2}\right)\Psi_{13}^2 + 16\left[\frac{k_p^2 + (a-0.984k_p-0.173k_q)^2}{b^2}\right] = 0 \quad (8)$$

where

$$k_p = -\alpha \sin x \quad (9)$$

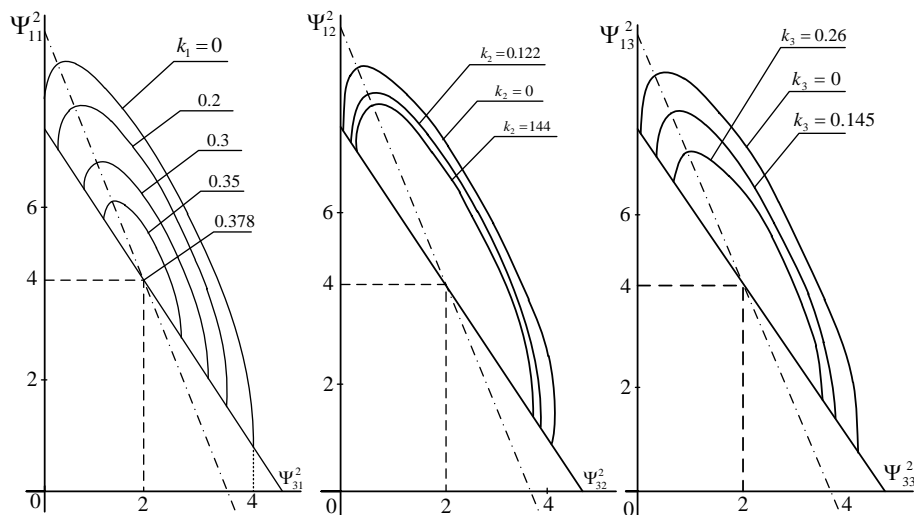
$$k_q = \beta + \alpha \cos x \quad (10)$$

$$x = 3\varphi_1 - \varphi_3 \quad (11)$$

$$\alpha = \frac{3b}{4} \Psi_{1\lambda} \Psi_{3\lambda} \quad (12)$$

$$\beta = a + \frac{3b}{4} \Psi_{1\lambda}^2 + \frac{3b}{2} \Psi_{3\lambda}^2 \quad (13)$$

Equations (6), (7) and (8) are in the plane and can be represented as second-order curves.



**Pic. 2. Dependencies for different values and**

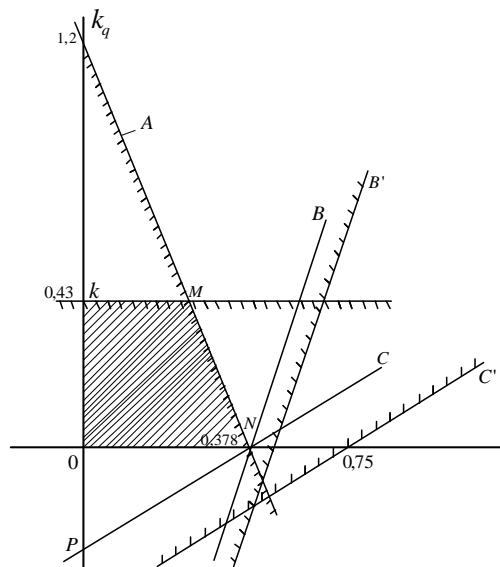
When the conditions are met

$$K_1 = \left| \frac{k_p}{a-k_q} \right| \leq 0.378 \quad (14)$$

$$K_2 = \left| \frac{0.766k_p - 0.643k_q}{a - 0.643k_p - 0.766k_q} \right| \leq 0.378 \quad (15)$$

$$K_3 = \left| \frac{0.173k_p - 0.984k_q}{a - 0.984k_p - 0.173k_q} \right| \leq 0.378 \quad (16)$$

equations (6), (7) and (8) will describe real ellipses (Fig. 2), the family of which indicates different conditions for the existence of the SGC for each phase depending on the circuit parameters. From the joint mode of the system of inequalities (14), (15) and (16) it follows that the simultaneous existence of third-order SGCs in all phases is possible only for values and limited by the polygon OKMNP0 (Fig. 3); in other cases, SGCs can only be excited in one or two phases. [1, 7].



**Pic. 3. Region of simultaneous existence of SGC at frequency  $\omega/3$  in all phases of a three-phase ferroresonant circuit**

In Fig. Figure 3 shows the calculated dependences of the squared amplitude of the SGC on the squared amplitude of the input influence for various values of and. It can be seen that, at the same values, the regions of existence of the SGC are different, and the flow changes in one phase and decreases in other phases. A certain region of existence of the SGC is also visible, determined by the circuit parameters and the magnitude of the supply voltage.

The obtained results of the theoretical study confirm the experimental data given in [1, 7].

### **Conclusion**

1. Using frequency-energy relationships, a system of equations was obtained that characterizes the steady-state mode of existence of the SGC at a frequency of  $\omega/3$  in a three-phase ferroresonant circuit.

2. A dependence was obtained characterizing the amount of power converted by the ferromagnetic element at the frequency of the SGK  $\omega/3$  on the power consumption.

3. Dependency analysis also makes it possible to determine the critical values of the circuit parameters that characterize the region of existence of the SGC of a particular frequency.

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