COMPARATIVE ANALYSIS OF ELEMENTARY AND LAGRANGIAN METHODS FOR DETERMINING THE PERIOD OF FREE ELECTROMAGNETIC OSCILLATIONS ON AN IDEAL OSCILLATORY CIRCUIT

[1]Badalov Kodir, [2]Saloydinov Mirjalol, [3]Shuhratov Ogabek

[1]Assistant of the Department of Physics and Astronomy of the Uzbek-Finnish Pedagogical Institute

[2,3]Students of the Faculty of Applied Mathematics and Physics of the Uzbek-Finnish Pedagogical Institute

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***Abstract**. The purpose of this article is to show and compare two different approaches to determine the period of free electromagnetic oscillations in an ideal oscillation circuit - elementary and Lagrange method. While the elementary method relies on traditional electrical circuit analysis methods, the Lagrange method approaches the problem from the point of view of classical mechanics. Emphasis is placed on how the relationship between classical electromagnetism and the Lagrangian formalism can be better explained by revealing the basic principles and computational techniques associated with both methodologies.*

***Keywords**: electrical oscillations, ideal oscillatory circuit, Lagrange method, classical electromagnetism.*

Introduction

We know that periodic changes in charge, current and voltage are called electrical oscillations. The simplest electric vibrations occur in the vibration circuit. As a simple circuit, we can say a circuit consisting of a capacitor and an inductive coil connected to these capacitor covers.

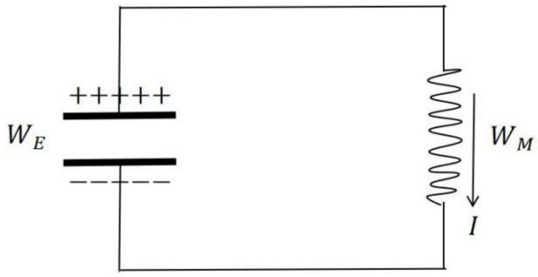

Figure 1.

In this scheme, the vibration is generated in such a way that the energy of the electric field in the capacitor and the energy of the magnetic field in the inductive coil rotate periodically. However, the total electromagnetic field energy does not change. More precisely, the total electromagnetic field is equal to the sum of the energies of the electric field and the magnetic field.

$$W_{total} = W_{el} + W_{mg} = const \qquad (1)$$

Electrical vibrations have some similarities to mechanical vibrations. The aforementioned electric field energy can be compared to potential energy, and magnetic field energy can be compared to kinetic energy [1].

We know that the potential energy reaches a large value when the mathematical pendulum deviates from the equilibrium state to the extreme point ($t = 0$). Similarly, in a pendulum with a spring, the potential energy reaches its maximum value when it moves the greatest distance x_m. Now let's compare it to the vibration contour. The capacitor in the circuit is fully charged, and when the switch is turned on, the energy of the electric field reaches its maximum value. It is at this moment that the amount of charge in the capacitor is at its maximum value. In the inductive coil, no current is generated [2].

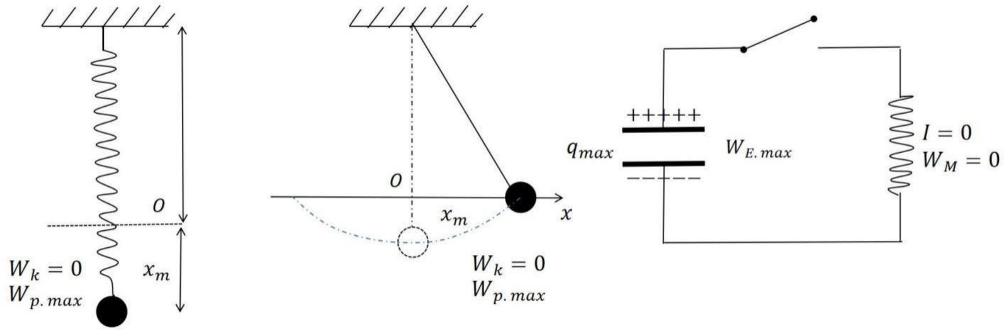

Figure 2.

In mechanical oscillations, when a quarter of the period of oscillation has passed ($t = T/4$), the mathematical and spring pendulum passes through the state of equilibrium. That is, the kinetic energy reaches the greatest value in pendulums. At the same time, the energy of the magnetic field reaches its maximum value in the oscillation circuit. At this moment, the amount of charge in the capacitor is equal to zero, and the current generated in the inductive coil reaches its maximum value.

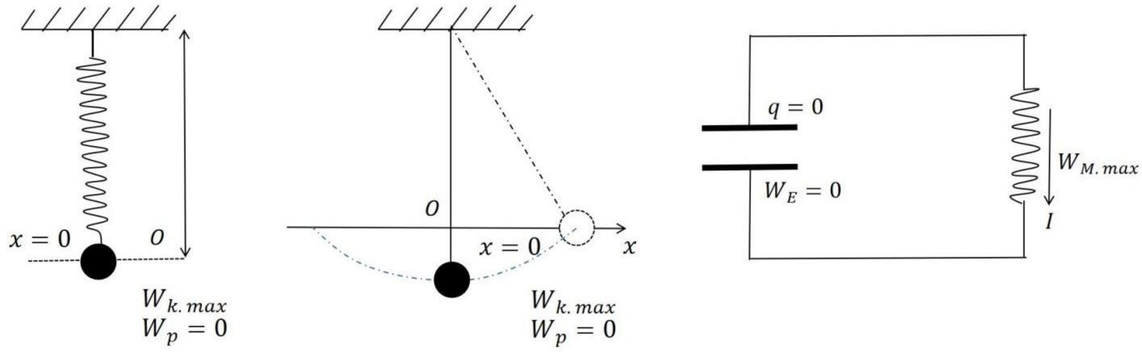

Figure 3.

Now let's look at the situation at the moment of time $t = T/2$. In this case, the capacitor is fully charged. But it will have the opposite charge. The current in the inductive coil is zero. That is, the energy of the electric field reaches its maximum value, and the energy of the magnetic field reaches zero. If we compare this situation with mechanical vibrations, it can be compared to a mathematical pendulum swinging to its extreme point on the opposite side of the starting point. A pendulum with a spring is in the same position.

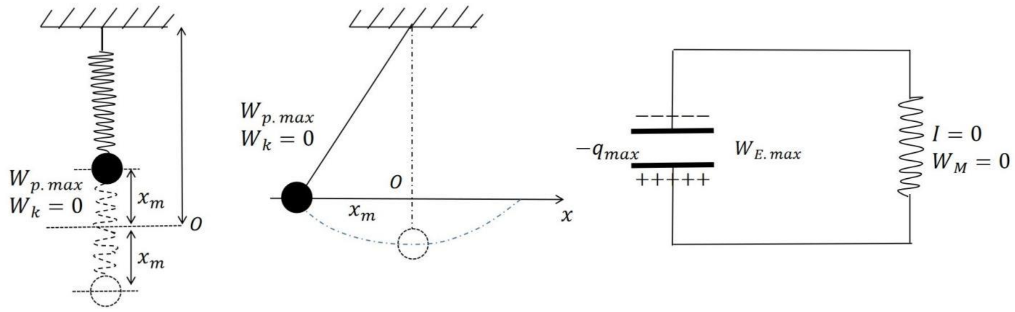

Figure 4.

In general, the total electromagnetic energy at an arbitrary instant of time can be written as:

$$W_{total} = W_{el} + W_{mg} \tag{2}$$

$$W_{total} = \frac{q^2}{2C} + \frac{LI^2}{2} \tag{3}$$

It can be seen from the above that if the amount of charge in the oscillating capacitor increases, the current in the inductive coil decreases. Conversely, if the charge decreases, the current increases. If we interpret these transitions in terms of phase, current and charge oscillate with a phase difference of $\pi/2$.

The time dependence of the charge on the oscillation circuit is as follows:

$$q = q_{max} \cos \omega t \tag{4}$$

If we take the first-order derivative with respect to time, the current is derived:

$$I = (q_{max} \cos \omega t)'_t = -q_{max} \omega \sin \omega t \tag{5}$$

$$I = I_{max} \cos(\omega t + \frac{\pi}{2}) \tag{6}$$

Methodology

Below is a methodology table describing the steps of comparing the elementary method and the Lagrangian method to find the period of free electromagnetic oscillations in an ideal oscillation circuit:

Stage	Brief review
Literature review	A comprehensive review of the literature on the elementary method of analysis of oscillation periods and the Lagrangian method. Collect relevant textbooks and scientific articles to understand the principles and applications of both methods.
Learning the elementary method	To gain a thorough understanding of the elementary method used in the analysis of oscillatory cycles. Learning to derive the governing equations of free electromagnetic oscillations in the cycles of a simple circuit using the basic principles of electromagnetism and differential equations.
Learning the Lagrangian method	Study of the Lagrangian method and its application to the analysis of physical systems, including electrical circuits. Learn how to apply Lagrange's formula to oscillating cycles. Understanding physical interpretation and mathematical derivations.
Analysis and comparison	Use both methods to analyze various types of oscillatory cycles, including cycles of a series circuit. Compare the obtained results in terms of accuracy, required calculations and suitability for different scenarios.
Summary	Summarizing the conclusions, citing the strengths and weaknesses of each method.
List of references	Provide full reference to all sources given in the literature review and cited throughout the thesis. Acknowledgment of previous work in this area.

An elementary (classical electromagnetism) method of determining the period of free electromagnetic oscillations in an ideal oscillation circuit.

The derivation of Thomson's formula for the oscillation circuit is based on the analysis of Kirchhoff's law, characteristics of inductors and capacitors. To do this, it is necessary to write a differential equation describing the movement of current in an electric circuit and solve it to find the oscillation frequency or oscillation period.

For this, the following sequence should be performed:

An electrical circuit is analyzed to determine the capacitance of the capacitor and the inductance of the coil;

Equations for an electric circuit are written based on Kirchhoff's law;

The flowing current is expressed by the charge, and a differential equation is written that expresses its movement;

The frequency of oscillation is found by solving the differential equation;

The period of oscillation is expressed by the frequency of oscillation.

We will dwell on this in detail. As we noted above, the energy of the full electromagnetic field does not change at any instant of time. The derivative of the first order in time is equal to zero, like the derivative of a constant number:

$$\left(\frac{q^2}{2C}+\frac{LI^2}{2}\right)'_t=0 \qquad (7)$$

$$\frac{q}{2C}2q'_t+\frac{LI}{2}2I'_t=0. \qquad (8)$$

Here, taking $q'_t=I;\ I'_t=q''_t$ into account,

$$\frac{q}{C}I+LIq''_t=0 \qquad (9)$$

is generated. If we divide both sides of the equation by current I, we get the following result:

$$\frac{q}{C}+Lq''_t=0 \qquad (10)$$

From this equation we find q''_t :

$$q''_t=-\frac{q_{max.}}{CL} \qquad (11)$$

The dependence of the charge on time has the form of formula (4). The first-order time derivative from it is given in equation (5). Now let's take the time derivative from equation (5):

$$q''_t=(-q_{max}\omega\,sin\,\omega t)'_t=-q_{max}\omega^2\,cos\,\omega t \qquad (12)$$

Since the phase difference between current and voltage is $\pi/2$:

$$q''_t=-q_{max}\omega^2 \qquad (13)$$

is generated. Equations (11) and (13) represent the same quantity, so we equate them:

$$-\frac{q_{max}}{CL}=-q_{max}\omega^2 \qquad (14)$$

If we make mathematical simplifications from equation (14) and find ω, the following equation is formed:

$$\omega=\frac{1}{\sqrt{LC}} \qquad (15)$$

We know that equation (15) represents the cyclic frequency. If we use the relationship between the oscillation period and the cyclic frequency, we get the Thomson formula:

$$T=\frac{2\pi}{\omega} \qquad (16)$$

$$T = 2\pi\sqrt{LC} \tag{17}$$

Lagrangian method

The derivation of the Thomson formula for the oscillation contour based on the Lagrangian formalism is carried out as follows:

Generalized coordinates for the system are defined (usually q and $\dot{q}$ for charge and magnetic flux) [3-4];

Lagrange's function is usually written as the difference between kinetic and potential energies;

The Euler-Lagrange equation is used to derive the equations of motion;

These equations are solved to find the motion of the system, including the period of oscillation.

Based on the above sequence, we find the period of free electromagnetic oscillations in the ideal oscillation circuit. To do this, we start by finding the Lagrangian to find the period of free electromagnetic oscillations in an ideal oscillating chain using Lagrangian mechanics [5].

An ideal oscillating circuit is an electrical circuit consisting of an inductive coil (L) and a capacitor (C), usually connected in series or parallel [6]. Let's consider a simple series LC circuit. The total energy in the circuit is equal to the sum of the energies stored by the capacitor and the inductive coil.

When the capacitor is connected to the network, the energy stored in it is given by $\frac{1}{2}CU^2$, where U is the voltage across the capacitor. The energy stored in the inductive coil is $\frac{1}{2}LI^2$, where I is the current through the inductive coil.

Based on the difference of energies, this gives us the Lagrangian function $\mathcal{L}$ of the system:

$$\mathcal{L} = \frac{1}{2}LI^2 - \frac{1}{2}CU^2 \tag{18}$$

Now we need to express U and I in generalized coordinates. We denote the charge of the capacitor as q, and the current through the inductor as I.

Voltage in the capacitor: $U = \frac{q}{C}$.

Current through an inductive coil $I = \frac{dq}{dt} = \dot{q}$, where q is the charge on the capacitor.

Substituting these into the Lagrangian, we get:

$$\mathcal{L} = \frac{1}{2}L\left(\frac{dq}{dt}\right)^2 - \frac{1}{2}C\left(\frac{q}{C}\right)^2 = \frac{1}{2}L\dot{q}^2 - \frac{1}{2C}q^2 \tag{19}$$

Now we can find the equations of motion of the system from the following Euler-Lagrange equation:

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}}\right) - \frac{\partial \mathcal{L}}{\partial q} = 0 \tag{20}$$

we get a derivative with respect to $\dot{q}$:

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}}\right) = \frac{d}{dt}\frac{\partial}{\partial \dot{q}}\left(\frac{1}{2}L\dot{q}^2 - \frac{1}{2C}q^2\right) = \frac{d}{dt}(L\dot{q}) = L\ddot{q} \tag{21}$$

and the derivative with respect to q is equal to the following:

$$\frac{\partial \mathcal{L}}{\partial q} = \frac{\partial}{\partial q}\left(\frac{1}{2}L\dot{q}^2 - \frac{1}{2C}q^2\right) = -\frac{1}{C}q \tag{22}$$

We transfer the obtained expressions to the Lagrange-Euler equation:

$$L\ddot{q}+\frac{1}{C}q=0 \tag{23}$$

Taking into account that the coil inductance L is always a positive value, we divide both sides of the equation by L and get the following differential equation of the second order:

$$\ddot{q}+\frac{1}{LC}q=0 \tag{24}$$

This is the equation of a harmonic oscillator oscillating with a cyclic frequency of $\omega=\sqrt{\frac{1}{LC}}$. The definition of cyclic frequency is as follows: "The time taken to turn the angle 2π is called cyclic frequency." That is, $\omega=\frac{2\pi}{T}$. If we find the period and use the cyclic frequency expression above, we get Thomson's formula

$$T=\frac{2\pi}{\omega}=2\pi\sqrt{LC} \tag{25}$$

for the period of free electromagnetic oscillations in an ideal oscillatory chain.

Discussion of the obtained result

Approach: The elementary method relies on circuit analysis techniques and differential equations, while the Lagrange method treats the circuit as a physical system and applies the principles of classical mechanics.

Complexity: The Lagrangian method may seem a bit more complicated because it involves concepts from classical mechanics and calculus of variations. However, for some systems Kirchhoff's laws and differential equations lead to simpler and more accurate solutions compared to direct application.

Generalization: The Lagrangian method is more generalizable to more complex systems and provides a deeper physical explanation than the elementary method, which is often tailored to specific electrical circuit configurations.

Educational perspective: The elementary method is often taught first in circuit analysis courses because of its direct application to circuit problems. The Lagrangian method can be introduced later in advanced courses or in courses involving classical mechanics.

Conclusion

In summary, although both methods can be used to find the period of free electromagnetic oscillations in an ideal oscillating circuit, they differ in approach, complexity, generalization, and educational content.

The choice between them depends on the specific problem, the required level of detail and the level of knowledge of the person performing the analysis.

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