PRORERTIES OF SOLUTION SET OF CAUCHY PROBLEM FOR CONTROLLED DIFFERENTIAL INCLUSION WITH DELAY ARGUMENT

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Abstract. *In this paper we consider the controlled differential inclusion with a delay argument. For such models of dynamical systems, the Cauchy problem is studied. Some properties of the set of solutions and the ensemble of trajectories of the considered class of differential inclusions are studied. Conditions for continuous dependence and convexity of multivalued mappings related to the studied properties of solution of differential inclusions are obtained.*
Keywords: *differential inclusion, delay argument, Cauchy problem, solution set, ensemble of trajectories, continuous dependence, convexity.*

INTRODUCTION

Differential inclusions are of great interest in the study of optimal control problems and other areas of applied mathematics and modeling of dynamic systems. made a significant contribution to the development of the theory of differential inclusions and their applications. A.F.Filippov, T.Wazewski., J.P.Aubin, C.Castaing, N.Kikuchi, A.Cellina, V.I.Blagodatskix, A.B.Kurzhansky, S.M.Aseev, V.A.Plotnikov, E.S.Polovinkin etc. [1–7]. It should also be noted that research on differential inclusions has numerous applications in the theory of differential games [8], which is an important area of modern mathematics.

To date, the theory of differential inclusions has been developing in various directions. Differential-functional and integro-differential inclusions, differential inclusions with delays, differential inclusions in partial derivatives, differential inclusions in Banach spaces, differential inclusions with a fuzzy right-hand side, controlled differential inclusions, etc. are studied [9–13]. In the theory of differential inclusions, differential inclusions with a retarded argument are of great interest [9,12,13].

Controlled differential switching, i.e. differential inclusions with a control parameter constitute an important class of such models. Studies devoted to the properties of a family of solutions of controlled differential inclusions, issues of control and optimization of the ensemble of trajectories of such systems according to various criteria are developed in the works of N. Kikuchi, N.S. Papageorgio, T.F. Filippova, A.V. Plotnikova and others [7,11,13–20]. Problems of controlling an ensemble of trajectories of differential inclusions with a retarded argument constitute one of the developing areas of the theory of controlled differential inclusions. Certain aspects of issues relating to the properties of an ensemble of trajectories and problems of their control for some particular classes of controlled differential inclusions with delays are considered in [14–16,19,20].

OBJECT OF STUDY AND METHODS

In what follows we use the following notation: $\Omega(G)$ – the collection of all compacta from the space G; $co\Omega(G)$ – the collection of all convex compacta from G; $\rho^*(X,Y)=\sup\{\rho(x,Y): x\in X\}$; $h(X,Y)=\max\{\rho^*(X,Y),\ \rho^*(Y,X)\}$ – Hausdorff deviation of sets $X\subset G$ и $Y\subset G$; R^n – n- dimensional Euclidean vector space $x=(x_1,...,x_n)$; $C(X,\psi)=\sup\{(x,\psi):\ \ x\in X\}$ – set support function $X\subset R^n$; $C^n(T)$- space of continuous on an interval $T\subset R^1$ n- dimensional vector functions; $L_1(T)$- space of integrable (Lebesgue) T functions.

Let us consider a control object whose state at each moment of time is $t\in T=[t_0,t_1]$ is expressed n-dimensional vector $x=x(t)$, y satisfying differential inclusion with delays

$$\dot{x}\in F(t,x(t),x(t-h_1(t)),...,x(t-h_k(t)),u), \qquad (1)$$

where $F(t,x,y,u)\subset R^n$, $x\in R^n$, $y=(y_1,...,y_k)$, $y_i\in R^n$, $i=\overline{1,k}$, $u\in U\subset R^m$. Here $u=u(t)$ plays the role of control influences, a $h_i(t)$, $i=\overline{1,k}$, – given non-negative and continuous functions. A model of type (1) is usually called controlled differential inclusion with delays.

For the control system model (1), we choose measurable limited ones as the class of admissible controls m - vector functions $u=u(t)$, $t\in T$. Let us denote by $U(T)$ set of all measurable bounded controls $u=u(t)$, $t\in T$, accepting almost everywhere on T values from a convex compact set $U\subset R^m$.

Here $t_*=\min\limits_{i-1,x}\min\limits_{t\in T}[t-h_i(t)]$, $T_0=[t_*,t_0]$, $u(\cdot)\in U(T)$, $D^0\subset C^n(T_0)$. Consider the initial Cauchy problem for the system (1):

$$\dot{x}\in F(t,x(t),x(t-h_1(t)),..,x(t-h_k(t)),u(t)), \quad x_{T_0}(\cdot)\in D^0, \qquad (2)$$

here $x_{T_0}(\cdot)$ – narrowing of function $x(t), t\in T_1=[t_*,t_1]$, for a segment T_0, т.е. $x_{T_0}(t)=x(t)$ at $t\in T_0$. By solving this problem we will understand a continuous T_0 and absolutely continuous for T n- vector function $x=x(t)$, satisfying almost everywhere on T the differential inclusion (1) and the initial condition $x_{T_0}(\cdot)\in D^0$.

Let $H(u,D^0)$ the set of all solutions to the Cauchy problem (2). Let's say $X(\tau,u,D^0)=\{\xi\in R^n:\xi=x(\tau),x(\cdot)\in H(u,D^0)\}$, $\tau\in T_1$. Multivalued mapping $t\to X(t,u,D^0)$, $t\in T$, is called the ensemble of system trajectories (2).

Lemma 1. [9] Let the following conditions be satisfied:

a1) for any $(t,x,y,u)\in T\times R^n\times R^{nk}\times U$ a bunch of $F(t,x,y,u)$ convex compact of R^n;

б1) multivalued mapping $(t,x,y,u)\to F(t,x,y,u)$ let's measure by t at $\forall(x,y,u)\in R^n\times R^{nk}\times U$ and continuously (x,y,u) with almost everyone $t\in T$;

c1) there are functions $g_i(t,u)$, $i=\overline{1,2}$, such that the functions $g_i(t,u(t))$, $i=\overline{1,2}$, summable for any $u(\cdot)\in U(T)$ and

$$\|f\| \le g_1(t,u)\left(\|x\| + \sum_{i=1}^{k}\|y_i\|\right) + g_2(t,u), \qquad (3)$$
$$\forall f \in F(t,x,y,u),\ (t,x,y,u) \in T \times R^n \times R^{nk} \times U$$

Then for anyone $u(\cdot) \in U(T)$ and $D^0 \in \Omega(C^n(T_0))$ sets $H(u,D^0)$ and $X(t,u,D^0)$, $t \in T_1$, are non-empty compact sets of $C^n(T_1)$ и R^n, respectively.

Lemma 2. [13] Let the conditions be met a1) – в1) lemms 1. Let's also assume that:

г1) support function $c(F(t,x,y,u),\psi) = \max\limits_{f \in F(t,x,y,u)}(f,\psi)$ sets $F(t,x,y,u)$ concave along (x,y) with almost everyone $t \in T$ and all $(u,\psi) \in U \times R^n$.

Then for anyone $u(\cdot) \in U$ and $D^0 \in co\Omega(C^n(T_0))$ sets $H(u,D^0)$ and $X(t,u,D^0)$, $t \in T_1$, nonempty convex compacta of $C^n(T_1)$ и R^n, respectively.

Comment. Let

$$F(t,x,y,u) = A(t)x + \sum_{i=1}^{k} A_i(t)y_i + b(t,u), \qquad (4)$$

here $A(t)$, $A_i(t)$, $i = \overline{1,k}$, $-$ $n \times n$ - matrices whose elements are summable to T, $b(t,u) \in co\Omega(R^n)$, multivalued mapping $(t,u) \to b(t,u)$ измеримо по $t \in T$ matrices whose elements are summable to $u \in V$, and $\sup\limits_{\beta \in b(t,u)} \|\beta\| \le \beta_1(t)\|u\| + \beta_2(t)$, $\beta_i(\cdot) \in L_1(T)$, $i = \overline{1,2}$. Then all statements of Lemma 2 remain valid.

RESEARCH RESULTS

We will study some properties of multivalued mappings $(u,D^0) \to H(u,D^0)$, $(t,u,D^0) \to X(t,u,D^0)$.

We will impose the following conditions on the right-hand side of differential inclusion (1):

а2) for each $(t,x,y,u) \in T \times R^n \times R^{nk} \times U$ multiple $\mathrm{F}(t,x,y,u)$ – convex compact of R^n;

б2) multivalued mapping $(t,x,y,u) \to F(t,x,y,u)$ let's measure by $t \in T$ at $\forall\ (x,y,u) \in R^n \times R^{nk} \times U$ and satisfies the Lipschitz condition according to (x,y,u) ,т.е.

$$h(F(t,x,y,u),F(t,\xi,\eta,v)) \le l(t)[\|x-\xi\| + \sum_{i=1}^{k}\|y_i - \eta_i\| + \|u-v\|],$$

here $l(t)-$ square-integrable function on T ($l(\cdot) \in L_2(T)$);

в2) there are an element $(\xi^0,\eta^0,v^0) \in R^n \times R^{nk} \times U$ and function summable on T $l_1(t)$ such that $\|F(t,\xi^0,\eta^0,v^0)\| \le l_1(t), t \in T$;

г2) support function $c(F(t,x,y,u),\psi) = \max\{\ (f,\psi) : f \in F(t,x,y,u)\ \}$ concave along (x,y,u) with almost everyone $t \in T$.

If conditions a2) – b2) are met, all conditions a1) – b1) of Lemma 1 are satisfied. Therefore, it is fair

Lemma 3. For any $u \in U(T)$, $D^0 \in \Omega(C^n(T_0))$ and $t \in T$ the relations are valid: $H(u,D^0) \in \Omega(C^n(T_1))$, $X(t,u,D^0) \in \Omega(R^n)$.

Using the results of [9], we can show that

Lemma 4. Let (u_1, D_1) ,(u_2, D_2) – arbitrary points from $U \times \Omega(C^n(T_0))$. Then for anyone $x_1(\cdot) \in H(u_1, D_1)$ is $x_2(\cdot) \in H(u_2, D_2)$, that

$$\|x_1(\cdot) - x_2(\cdot)\|_{C^n(T_1)} \le \left[h(D_1, D_2) + \|l(\cdot)\|_{L_2(T)} \|u_1(\cdot) - u_2(\cdot)\|_{L_2^m(T)}\right] M,$$

here $M = \exp\left[(k+1)\int_{t_0}^{t_1} l(s)ds\right]$.

Theorem 1. Let the conditions be met a2) – в2). Then the multivalued mappings $(u, D) \to H(u, D)$ и $(t, u, D) \to X(t, u, D)$ continuous for $U(T) \times \Omega(C^n(T_0))$ and $T \times U(T) \times \Omega(C^n(T_0))$ respectively.

Proving. Let $u_k(\cdot) \in U,\ D_k \in \Omega(C^n(T_0)),\ u_k(\cdot) \to u^*(\cdot)$ (in metric $L_2(T)$), $D_k \to D$ (in the Hausdorff metric) at $k \to \infty$, $u^*(\cdot) \in U$, $D^* \in \Omega(C^n(T_0))$.

Let's take an arbitrary element $x_k(\cdot) \in H(u_k, D_k)$. Then, by Lemma 4, there exists $x^*(\cdot) \in H(u^*, D^*)$ that

$$\rho(x_k(\cdot), H(u^*, D^*)) \le \|x_k(\cdot) - x^*(\cdot)\|_{C^n(T_1)} \le \Delta_k, \qquad (5)$$

$$\Delta_k = \left[h(D_k, D^*) + \|l(\cdot)\|_{l_2(T)} \|u_k(\cdot) - u^*(\cdot)\|_{l_2^m(T)}\right] M. \qquad (6)$$

From (5) we get

$$\sup_{\xi(\cdot) \in H(u_k, D_k)} \rho(\xi(\cdot), H(u^*, D^*)) \le \Delta_k. \qquad (7)$$

Similarly we have

$$\sup_{\xi(\cdot) \in H(u^*, D^*)} \rho(\xi(\cdot), H(u_k, D_k)) \le \Delta_k. \qquad (8)$$

From (7) and (8) it follows that

$$h(H(u_k, D_k), H(u^*, D^*)) \le \Delta_{k.}. \qquad (9)$$

Since according to (6) $\Delta_k \to 0,\ k \to \infty$, then from (9) we obtain the relation $\lim_{k \to \infty} h(H(u_k, D_k), H(u^*, D^*)) = 0$,

which shows the continuity of the multivalued mapping $(u, D) \to H(u, D)$ на $U(T) \times \Omega(C_n(T_0))$.

Let us now consider an arbitrary sequence of points $(t_k, u_k, D_k) \in T \times U(T) \times \Omega(C^n(T_0))$, сходящуюся к $(t^*, u^*, D^*) \in T \times U(T) \times \Omega(C^n(T_0))$. Let's take an arbitrary trajectory $x^*(\cdot) \in H(u^*, D^*)$. Using formula

$$x^*(t) = x^*(t_0) + \int_{t_0}^{t} \dot{x}^*(s)ds, \quad t \in T,$$

it is easy to show that

$$\left\| x^*(t_k) - x^*(t^*) \right\| \le \delta_k, \qquad (10)$$

where

$$\delta_k = \left| \int_{t_k}^{t^*} [g_1(t)(k+1)M + g_2(t)]dt \right|.$$

It's clear that $\rho(x^*(t_k), X(t^*,u^*,D^*) \le \left\| x^*(t_k) - x^*(t^*) \right\| \le \delta_k$. Therefore, given the arbitrariness $x^*(\cdot) \in H(u^*,D^*)$, it is easy to verify that

$$h(X(t_k,u^*,D^*), X(t^*,u^*,D^*) \le \delta_k. \qquad (11)$$

Due to the properties of the Hausdorff metric, the following inequalities are true:

$$\begin{aligned} h(X(t_k,u_k,D_k), X(t^*,u^*,D^*)) &\le h(X(t_k,u_k,D_k), X(t_k,u^*,D^*)) + \\ &+ h(X(t_k,u^*,D^*), X(t^*,u^*,D^*)), \end{aligned} \qquad (12)$$

$$h(X(t_k,u_k,D_k), X(t_k,u^*,D^*)) \le h(H(u_k,D_k), H(u^*,D^*)). \qquad (13)$$

Now, using relations (9), (11), (12), (13), we have

$$h(X(t_k,u_k,D_k), X(t^*,u^*,D^*)) \le \Delta_k + \delta_k.$$

So, $\Delta_k \to 0, \delta_k \to 0, k \to \infty$, then the last inequality implies the continuity of the multivalued mapping $(t,u,D) \to X(t,u,D)$ на $T \times U(T) \times \Omega(C_n(T_0))$. The theorem has been proven.

Consider the graph of a multivalued mapping $(u,D) \to H(u,D)$, i.e. multitude $\Gamma_H = \{(\omega,x) : \omega = (u,D) \in U \times \Omega(C^n(T_0)), x = x(\cdot) \in H(u,D)\}$.

Similarly, consider the graph of a multivalued mapping $(t,u,D) \to X(t,u,D)$, i.e. multitude

$$\Gamma_X = \{(t,\omega,\xi) : t \in T, \omega = (u,D) \in U \times \Omega(C^n(T_0)),\ \xi \in X(t,u,D)\}.$$

Theorem 2. When conditions a2) – b2) are met, multivalued mappings $(u,D) \to H(u,D)$, $u \in U$, $D \in \Omega(C^n(T_0))$ и $(t,u,D) \to X(t,u,D)$, $t \in T$, $u \in U$, $D \in \Omega(C^n(T_0))$ closed, i.e. their schedules are closed.

Proving. Let us prove the closedness of the set Γ_H. Closedness Γ_X is proved in a similar way. Let $(\omega_k, x_k) \in \Gamma_H$, $(\omega_k, x_k) \to (\omega^*, x^*)$, so $\omega_k = (u_k, D_k)$, $x_k = x_k(\cdot) \in H(u_k,D_k)$, $u_k = u_k(\cdot) \in U(T)$, $D_k \in \Omega(C^n(T_0)), x_k \to x^*$ (in methric $C^n(T_1)$), $u_k \to u^*$ (in methric $L_2^m(T)$), $D_k \to D^*$ (in the Hausdorff metric) with $k \to \infty$, $u^* = u^*(\cdot) \in U(T)$, $D^* \in \Omega(C^n(T_0))$. Let's show that $x^* \in H(u^*,D^*)$.

It is clear that

$$\rho(x_k, H(u^*,D^*)) \le h(H(u_k,D_k), H(u^*,D^*)) \qquad (14)$$

Let $\xi_k^*(\cdot) \in H(u^*,D^*)$ such that

$$\rho(x_k, H(u^*,D^*)) = \left\| x^*(\cdot) - \xi_k^*(\cdot) \right\|_{C^n(T_1)}. \qquad (15)$$

By Lemma 3, the set $H(u^*,D^*)$ compact and therefore such an element $\xi_k^*(\cdot)$ exists. Now, using (14) and (15), we have

$$\rho(x^*, H(u^*, D^*)) \le \left\| x^*(\cdot) - x_k(\cdot) \right\|_{C^n(T_1)} + \rho(x_k, H(u^*, D^*)) \le$$
$$\le \left\| x^*(\cdot) - x_k(\cdot) \right\|_{C_n(T_1)} + \rho^*(H(u_k, D_k), H(u^*, D^*)).$$

Passing here to the limit at $k \to \infty$ and taking into account the continuity of the multivalued mapping $(u, D) \to H(u, D)$, we get that $\rho(x^*, H(u^*, D^*)) = 0$. So $H(u^*, D^*)$ – is a closed set, then this implies the relation $x^* \in H(u^*, D^*)$. The theorem has been proven.

Theorem 3. Let the conditions be met a2) – г2). Then the multivalued mappings $(u, D) \to H(u, D)$ and $(u, D) \to X(t, u, D)$,$u \in U$, $D \in co\Omega(C^n(T_0))$ $(t \in T)$ convex and closed.

Proving. By Theorem 4, it suffices for us to show the convexity of the following sets:
$\Gamma_H^0 = \{(\omega^0, x) : \omega^0 = (u, D^0) \in U(T) \times co\Omega(C^n(T_0)), x = x(\cdot) \in H(u, D^0)\}$,
$\Gamma_X^0(t) = \{(\omega^0, \xi) : \omega^0 = (u, D^0) \in U(T) \times co\Omega(C^n(T_0)),\ \xi^0 \in X(t, u, D^0)\}$, $t \in T$.

Let's prove convexity Γ_H^0. Convex $\Gamma_X^0(t)$ follows from the convexity of the set Γ_H^0. Let's take arbitrary points

$$(u_i, D_i^0) \in U \times \Omega^0(C^n(T_0)),\ x_i(\cdot) \in H(u_i, D_i^0), \qquad \alpha_i \ge 0,\ i = \overline{1,2},\ \alpha_1 + \alpha_2 = 1.$$

Consider the function $z(t) = \alpha_1 x_1(t) + \alpha_2 x_2(t),\ t \in T_1$. Using condition d2), we have: $(\dot z(t), \psi) \le c(F(t, z(t), z(t - h_1(t)), ..., z(t - h_k(t)), \alpha_1 u_1(t) + \alpha_2 u_2(t)), \psi)$.

Due to the properties of support functions [2], it follows that

$$\dot z(t) \in F(t, z(t), z(t - h_1(t)), ..., z(t - h_k(t)), \alpha_1 u_1(t) + \alpha_2 u_2(t)) \qquad (16)$$

almost everywhere on T. Further, since $x_{1T_0}(\cdot) \in D_1^0$, $x_{2T_0}(\cdot) \in D_2^0$, so

$$\alpha_1 x_{1T_0}(\cdot) + \alpha_2 x_{2T_0}(\cdot) \in \alpha_1 D_1^0 + \alpha_2 D_2^0 \in co\Omega(C^n(T_0)). \qquad (17)$$

Relations (16) and (17) show that

$$z(\cdot) \in H(\alpha_1 u_1 + \alpha_2 u_2, \alpha_2 D_1^0 + \alpha_2 D_2^0),$$

i.e. multitude Γ_H^0 – convex. The theorem has been proven.

DISCUSSION OF RESULTS AND CONCLUSION

The results obtained in Theorem 1 provide conditions for the continuous dependence of the set of solutions and the ensemble of trajectories on the parameters of the Cauchy problem (2). Theorems 2 and 3 give conditions for the closedness and convexity of multivalued mappings $(u, D) \to H(u, D)$ and $(t, u, D) \to X(t, u, D)$. Regarding the results obtained, we present the following as comments:

1. The statements of Theorem 1 are preserved if we replace condition b2) with the following condition:

b2) multivalued mapping $(t, x, y, u) \to F(t, x, y, u)$ let's measure by $t \in T$ at $\forall\ (x, y, u) \in R^n \times R^{nk} \times U$, continuously on u at $\forall\ (t, x, y) \in T \times R^n \times R^{nk}$ and satisfies the Lipschitz condition for almost all $t \in T$,т.е.

$$h(F(t, x, y, u), F(t, \xi, \eta, v)) \le l(t)[\|x - \xi\| + \sum_{i=1}^{k} \|y_i - \eta_i\|],\ \forall\ (x, y, u), (\xi, \eta, v) \in R^n \times R^{nk} \times V,$$

where $l(t)-$ square-integrable function on T ($l(\cdot) \in L_2(T)$);

2. From the statement of Theorem 3 regarding the convexity of the graph of a multivalued mapping $(u, D) \to X(t, u, D)$ $(t \in T)$ concavity of the support function follows

$$c(X(t,u,D^0),\psi) \text{ по } (u, D^0) \in U(T) \times \Omega^0(C^n(T_0)) \text{ at } t \in T,\ \psi \in R^n.$$

In this work, the cauchy problem is considered for a class of controlled differential inclusions with delays. This problem was studied using the methods of the theory of differential inclusions, multivalued and convex analysis. Some properties of the set of solutions to the Cauchy problem and the ensemble of trajectories are studied depending on the control parameters and the initial state. Sufficient conditions for the continuity of such multivalued mappings are found. The conditions for their closedness and convexity are indicated.

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