IMPROVEMENT OF THE PROJECT ACTIVITY OF FUTURE TEACHERS OF MATHEMATICS USING THE MAPLE COMPUTER MATHEMATICS SYSTEM

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Abstract. *The processes of globalization and the emergence of a post-industrial, information society have posed new challenges for higher education. The role of higher education in solving these problems is defined in modern educational standards and standard educational programs. In the current situation, it is necessary to develop in future mathematics teachers not only systemic knowledge, but also the quality of a young specialist who can apply the knowledge and skills acquired in practical activities and everyday life. Therefore, it is necessary to study the features of improving students' project activities. Over the past ten years, design method as a new pedagogical technology has become the subject of numerous studies. A lot of information is contained in the pedagogical, psychological and methodological literature on the organization of student project activities.*

This article discusses the issues of organizing students' project activities when studying the discipline "Mathematical Analysis" and the use of one of its most effective methods - the Maple computer mathematics system (SCM). It provides for the improvement of students' design work using this program. But when improving students' project activities using the Maple computer mathematics system, the principles for selecting topics and the requirements for them are not defined. It is necessary to determine what type of design work is best performed using the Maple computer mathematics system. In particular, this problem has still not been sufficiently studied in the psychological and pedagogical literature: these are a) the principles of choosing educational materials that organize project work, the problem of determining the levels of project activity of students; b) a complete model for preparing project assignments has not been developed; c) the content-structural features of performing project activities using the Maple computer mathematics system are not considered.

The content-structural model and didactic features of the development of project activities of future mathematics teachers using the Maple computer mathematics system are outlined .

Keywords: *training, student project activities, Maple computer mathematics system, subject of mathematical analysis, level design work.*

History of the study

A study of the methodological system for teaching the course “Mathematical Analysis” in higher educational institutions preparing future teachers (O. Satybaldiev) [1], a study of the development of a methodological system for teaching mathematical disciplines to students of economic universities (M. E. Isin) [2]. Improving the professional and methodological training of mathematics teachers in the university system (K. Bumazhbaeva) [3], Formation of research skills of teachers (G. P. Skamnitskaya) [4], formation of cognitive activity of students (Karataev, A., Ibragimov, R., Kalimbetov, B., Kerimbekov, T.) [5], formation of future specialists of personal and professional competence and the organization of their creative work (A. E. Abylkasymova) [6], the formation of scientific and methodological foundations for preparing a future mathematics teacher for work on improving the logical and methodological knowledge of students (D. Rakhimbek) [7], etc. problems are studied in the works of modern scientists, and in addition, ways to improve the quality of students' knowledge using developmental teaching methods are proposed.

It has been established that the scientific and theoretical foundations of design in the management of pedagogical systems (Bakhisheva S. M.) [10], the formation of the ecological culture of future specialists during educational and project activities (Sagyndykova E. U.) [11] make it possible to improve the quality of mathematical knowledge.

In the works of such leading teachers as J. Dew [12], V. Kilpatrick [13], E. Collings [14], T. A. Novikova [15], V. I. Slobodchikov [16], philosophical and methodological basics of organizing students' project activities. In the works of E.S. Polat [17], N.Yu. Pakhomova [18], I.D. Chechel [19] and others analyzed and developed the basics of project activities, and it was devoted to the use of project activities in the study of various disciplines.

When studying the discipline "Mathematical Analysis", the presence of various ways to define the basic concepts of organizing students' design activities prevents the application of the design method in practice. In particular, there is a lack of consistency and some specific concepts in the definition of concepts such as creative activity, project activity and research activities of students. For example, it can be noted that the essence and content of the concept "development of students' project activities" have not been disclosed, and its characteristic features have not been analyzed.

Despite the large volume of psychological, pedagogical, methodological and research work of the above scientists, they have not lost their relevance, since when studying the discipline "mathematical analysis" the conditions and methods for developing students' project activities were not systematically studied .

Based on the results of the study, we can be convinced of the need to develop students' project activities. Contradictions arise here due to the fact that, methodologically, the problems of developing students' project activities at the appropriate levels (flexibility, revival, search-executive and creative activities) have not yet found their solution.

However, in the process of studying the discipline "mathematical analysis" on the topic of our research, one can notice the presence of contradictions between the possibilities for developing students' project activities, the needs of its application and insufficient methodological support.

These contradictions are:

there is no classification of the content of topics in the subject of mathematical analysis corresponding to various project activities of students ;

the issue of the need in universities for a research specialist involved in project activities of students has not been sufficiently resolved, and the problem of its pedagogical, psychological and methodological support has not been resolved;

Research methods:

To solve these problems, the following methods are used:

- analysis of psychological, pedagogical, mathematical and methodological literature on the research topic;
- analysis of the results of mastering knowledge on the project activities of future mathematicians on the research problem;
- conducting questionnaires and interviews, tests among teachers, students and analyzing their results;
- analysis of university documents and familiarization with the work experience of teachers.

Analysis

To begin with, let us analyze the definitions given to the concept of student project activity. The literature gives different interpretations of the concepts project, design method and design activity.

G.V. Narykova [20] defines design as an activity that covers the period from the emergence of an idea in the student's mind to the implementation of this idea as a full-fledged independent creative work.

According to T.N. Ivochkina [21] design is a teaching method that can be used in the study of any subject; it can be used in lessons and in extracurricular activities. It is aimed at developing the skills of students and achieving their goals. It is emphasized that design gives students the necessary experience of creative activity.

O. V. Rybina [22] considers design as a set of actions specially organized by the teacher and performed by students independently. In this regard, they are responsible for the decisions they make and the results of their creative work.

D. Chechel [23] considers the design method as a pedagogical technology. It is not aimed at integrating specific knowledge, but at applying it and acquiring new knowledge (sometimes through self-education).

T.N.Ivochkina and O.V.Rybina consider educational technology as a design method, the purpose of which is to develop skills in students by supporting their searches aimed at obtaining new knowledge and solving a specific problem.

Thus, the authors select topics in accordance with the discipline being studied, compose project work (including level project work), and students complete problem-based assignments. It is stated that their research activities can be shaped by teaching them the design method. But, when studying the discipline "Mathematical Analysis", the issues of organizing students' project activities have not been sufficiently studied, because the complex of problematic tasks has not been studied to organize students' project activities in this discipline. As a result, numerous shortcomings are discovered in the assessment of students' project work.

Results

In the methodological literature, there are two approaches regarding the basics of students' project activities.

According to the first approach, the basis of the design method is the idea that the student's educational and cognitive activity is achieved by solving problems that have practical or theoretical significance.

According to the second approach, the design method is based on the problems of developing students' cognitive abilities, developing their critical and creative thinking skills, and the ability to formulate and solve a problem problem. That is, we are talking about solving a particular problem, which involves, on the one hand, the use of various approaches, and on the other, the integration of knowledge and skills from the fields of science, technology, and creativity.

From our point of view, the main task should be to organize and improve students' project activities. Therefore, we examined various types of design work, which are designed to develop the project activities of students, to determine their essence and content. As a result, we determined the feasibility of organizing student project activities on the following topics:

1st project work

Topic of project work: Application of operations to irrational numbers

Guide: the operations of addition, subtraction, multiplication and division can be applied to irrational numbers, as well as to whole and fractional numbers, however, operations on irrational numbers have their own characteristics.

First: it is necessary to define the concept of approximate values of irrational numbers.

$\sqrt{5}$ =2.2 or $\sqrt{5 =}$2.3, therefore, $2{,}2^2$ =4.84 or $2{,}3^2$ =5.29 , respectively

Complete tasks:

The area of the square is 16.7 cm^2 . Calculate its edges with an accuracy of 0.01 cm.

The circumference area is 5.6 m^2 . Calculate the diameter of the circle with an accuracy of 0.01 m (let's say $\pi = \frac{22}{7}$).

Find the meaning of the numbers $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$. Calculate the resulting smallest and largest values with an accuracy of 0.1; 0.01; 0.001; 0.0001; 0.00001; 0.000001; 0.0000001

$\sqrt{2} = 1{,}4$
$\sqrt{2} = 1{,}41$
$\sqrt{2} = 1{,}414$
$\sqrt{2} = 1{,}4142$
$\sqrt{2} = 1{,}41421$
$\sqrt{2} = 1{,}414213...$

$\sqrt{2} = 1{,}5$
$\sqrt{2} = 1{,}42$
$\sqrt{2} = 1{,}415$
$\sqrt{2} = 1{,}4143$
$\sqrt{2} = 1{,}41422$
$\sqrt{2} = 1{,}414214...$

Follow this action:

$\sqrt{2} + \sqrt{3}$ =1.414+ 1.732 =3.146 (guide: addition to 0.001 precision).

$\sqrt{5} - \sqrt{3}$ =2.236- 1.732 =0.504 (guide: subtract to 0.001 precision).

$\sqrt{3}$ x $\sqrt{2} = 1.732$ x $1{,}414 = 2.449$

Draw a segment of length $\sqrt{3}$см.

Draw the following numbers on the number line: - $\sqrt{2}$; $\sqrt{3}$;$\sqrt{2} + \sqrt{3}$; $\sqrt{5} - \sqrt{3}$.

Follow these steps: 1.1414123...+3.1213456...;

2.71+2.7123; 2.71...+3.14...

Try to answer the questions:

Give examples of irrational numbers.

What new numbers do we add to the fractional numbers to get the set of irrational numbers? If we add irrational numbers to fractional numbers, what set of numbers do we get?

What other name can be given to infinite decimals?

What name is given to the set that contains infinite periodic decimal fractions?

Give examples of pure and mixed periodic decimal fractions. Is it possible to represent them as an ordinary fraction? Is it possible to convert infinite decimals to fractions?

2nd project work

In order to become familiar with the practical application of determining the limits of a function, it was compiled for all members of the student group. When performing this work, students are asked to recall the initial concepts of the theory of limits learned in high school, and independently analyze and study information about the increment of a function and the increment of an argument. The final result is expected to be completed by writing a report.

Design topic : **Limit of a function** .

Design goal: Find limits, calculate function limits, solve applied problems.

Design content: Tasks of such practical content are given to all student groups so that they independently solve these problems:

The linear motion of a point is given by the equation $S = 3t^2 - 2t + 5$, where t it is measured in seconds and S measured in meters. Find the speed of the point at the moment $t = 5$ seconds.

Expected result : Using limits, calculate the limit of the function.

Example 1. Students are expected to solve a problem in the following way after conducting an analysis:

Definition (Heine)

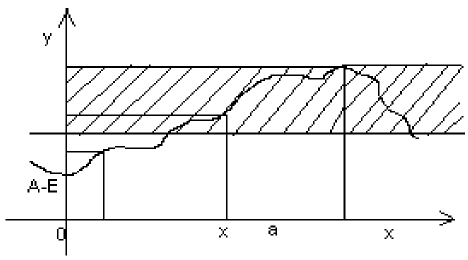
If: 1) given a function $y = f(x)$ defined in the area E ,

2) a sequence of numbers $x_1, x_2, x_3, ..., x_n, ...$ (1) is any sequence made up of numbers from the set E,

3) size a (1) is the limit of the sequence, and also not a single member of that sequence is equal to a , the sequence $f(x_1)$, $f(x_2)$,…, $f(x_n)$,… (2) function values, specified in accordance with the values of the argument in sequence, always sums up into one value, x tends to a , let's say the function $f(x)$ tends to a , then the value *And* let's call the limit of the function $f(x)$ at point a and write down $\lim_{x \to a} f(x) = A$.

Definition: (Cauchy)

for any predetermined smaller number ε > 0, the number $\delta > 0$ *is found,* for any values satisfying the inequalities $x < a$ and $|x - a| < \delta$ in the domain of definition of the function $f(x)$, the inequality



holds $\left| f(x) - a \right| < \varepsilon$, the quantity A is called the limit of the function f(x), when it tends to $x \to a$.

Let the geometric meaning of the limit of a function be written as $\lim\limits_{x \to a} f(x) = A$, that is, with the fulfillment of the inequality $\forall \varepsilon > 0$, $\exists \delta > 0$, $0 < |x - a| < \delta$ the inequality $\left| f(x) - A \right| < \varepsilon$. In other words, all values of the function *f(x),* corresponding to all values that satisfy the inequality $\alpha - \delta < x < \alpha + \delta$of the argument x , must satisfy the inequality $A - \varepsilon < f(x) < A + \varepsilon$ (4).

Statement that the number A is the limit of the function $f(x)$ in $x \to a$ can be explained by the method geometry as follows. Whatever (small) number is in the array (strip) defined by the straight lines $A - \varepsilon$, $A + \varepsilon$, one can construct neighborhoods $a - \delta$, $a + \delta$ around the point A , i.e. $\exists\ \delta > 0$. If this is so, then all points of the function $y = f(x)$ whose abscissa satisfy the inequality $a - \delta < x < A + \delta$ are located inside the array bounded by the straight lines $A - \varepsilon$, $A + \varepsilon$. Only a point whose abscissa is equal *to a* may not be included in the array if this point is on the graph of the function. Then the existence or non-existence of a point (*a,) f(a*) inside an array whose width is 2ε will not have any effect on the existence of the limit of the function.

The number $\delta > 0$ in defining the limit of a function depends on the number *E*. As a result, if ε decreases, then δ also decreases.

Using Cauchy's definition, we will show what number should be obtained for δ > 0 in accordance with ε > 0, or how the number δ will change when ε changes.

Example 2. Prove that $\lim\limits_{x \to 1}(5x - 1) = 4$.

Function $f(x) = 5x - 1$defined on the entire number line

For proving the limit $\lim\limits_{x \to 1}(5x - 1) = 4$ let's arbitrarily set a number $\forall \varepsilon > 0$, we must find the number $\delta > 0$, which will satisfy the inequality $0 < |x - 1| < \delta$ of the argument x or inequality holds $\left|(5x - 1) - 4\right| < \varepsilon$ for values not equal to 1.

lim

Let, |(5x – 1) – 4| = |5x – 5| = 5 |x – 1|, then it should be 5|x – 1| < ε or $|x - 1| < \frac{\varepsilon}{5}$.

Therefore $\delta = \frac{\varepsilon}{5}$, then for all values satisfying the inequality $0 < |x - 1| < \delta$ of the argument x , the inequality $\left|(5x - 1) - 4\right| < \varepsilon$, i.e. $\lim\limits_{x \to 1}(5x - 1) = 4$

3rd project work

Compiled for all members of the student group in order to familiarize themselves with the practice of applying the method of determining the asymptotes of a function. During this work, students independently study data on the increment of a function and the increment of an argument, asymptotes of a function. The final result is expected to be completed by writing an abstract.

Design Topic: Asymptotes

Design goal: Consideration of the possibilities of using limits in determining asymptotes

1. Study of theoretical materials .

1. What is an asymptote?

2. What are the horizontal asymptotes of a curve?
3. Specify methods for calculating asymptote:
4. Characteristics of asymptote:
a) determination of horizontal asymptotes of functions;
b) determination of the vertical asyptotes of the curve.

1. **Problem tasks corresponding to practical significance and level of calculation of limits.**

Consider the curve given by the equation *y =f(x*). When the argument *x* tends to plus infinity *(x⟶+.)* or minus infinity *(x⟶–.)*, then the function *y=f (x)* can tend to the linear function *y=kx+b*. We know that a linear function cuts the line *y=kx+b* .

If

$$\lim_{x\to\pm\infty}\left[f(x)-(kx+b)\right]=0,$$

then the straight line *y=kx+b* is called the asymptote of the curve *y=f(x)* as *x tends* to plus or minus infinity. In this way, only oblique asymptotes can be found . A If

$$\lim_{x\to\pm\infty} f(x)=b,$$

then the straight line *y = b* is called *the horizontal asymptote* of the curve *y =f (x)* .

For example, let's look at the curve given by the equation $y=\dfrac{1}{x}$. This curve is an equilateral hyperbola. Now let’s find its asymptote, why

$$\lim_{x\to\pm\infty}\frac{1}{x}=0.$$

From here we come to the following conclusion: Axis *OX* , i.e. *y =0,* is the horizontal asymptote of the example curve $y=\dfrac{1}{x}$.

If the straight line $y=kx+b$ is an asymptote of the curve $y=f(x)$, then

$$\lim_{x\to\infty}\left[f(x)-kx-b\right]=0$$

or

$$\lim_{x\to\infty}\left[\frac{f(x)}{x}-k-\frac{b}{x}\right]=0,$$

from here

$$\lim_{x\to\infty}\left[\frac{f(x)}{x}-k\right]=0,$$

or

$$\lim_{x\to\infty}\frac{f(x)}{x}=k$$

the equation $\lim\limits_{x\to\infty}\left[f(x)-kx-b\right]=0$ can be written like this:

$$\lim_{x\to\infty}\left[f(x)-kx\right]=b$$

From the execution of the equation we come to the following conclusions: if the straight line $y = kx + b$ is an inclined asymptote of the curve $y = f(\text{x})$, then the equalities $\lim\limits_{x\to\infty}\frac{f(\text{x})}{x} = k$ and are satisfied $\lim\limits_{x\to\infty}\left[f(\text{x}) - \text{kx}\right] = b$. On the contrary, if the equations $\lim\limits_{x\to\infty}\frac{f(\text{x})}{x} = k$ are $\lim\limits_{x\to\infty}\left[f(\text{x}) - \text{kx}\right] = b$ satisfied, then the straight line $y = kx + b$ is an oblique asymptote of the curve.

Take for example a curve defined by an equation such as $y = \frac{x^2 + 3x + 5}{x+1}$:

$$k = \lim_{x\to\pm\infty}\frac{f(\text{x})}{x} = \lim_{x\to\pm\infty}\frac{x^2 + 3x + 5}{x\left(x+1\right)} = 1,$$

$$b = \lim_{x\to\pm\infty}\left[f(\text{x}) - kx\right] = \lim_{x\to\pm\infty}\left(\frac{x^2 + 3x + 5}{x\left(x+1\right)} - x\right) = 2.$$

So, the asymptote of the curve, taken for example $y = 2x + 1$.

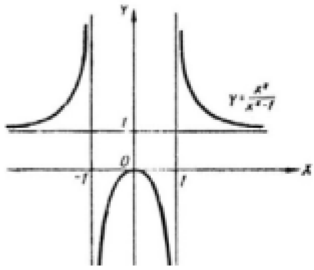
when the argument *x* tends to a constant number x_o (from whatever direction it tends), the function *f(x)* can tend to plus and minus infinity, i.e.

$\lim\limits_{x\to x_0} f(\text{x}) = \infty$, $\lim\limits_{\substack{x\to x_0\\ x<x_0}} f(\text{x}) = \infty$, $\lim\limits_{\substack{x\to x_0\\ x>x_0}} f(\text{x}) = \infty$, in such cases, firstly, the corresponding point of the curve $y = f(\text{x})$ goes up or down to infinity, and secondly, the vertical $x = x_0$ infinitely approaches the straight line, this subsequent straight line is called the vertical asymptote of the curve $y = f(\text{x})$.

To find the vertical asymptote of a given curve, $y = f(\text{x})$ it is necessary to find the points where the function $y = f(\text{x})$ becomes infinite in its absolute value. For example, if these are points $x_1, x_2, \ldots$, then these straight lines $x = x_1, x = x_2, \ldots$ vertical asymptote s .

For example, if we take a curve projected by the equation $y = \frac{x^2}{x^2 - 1}$, this curve has both vertical asymptotes $x = 1, x = -1$, and horizontal asymptotes, in order to find it, we need to find the limit of a given function $\frac{x^2}{x^2 - 1}$ with *x* tending to plus and minus infinity , i.e.

$$\lim_{x\to\pm\infty}\frac{x^2}{x^2 - 1} = 1.$$



Thus, the straight line $y=1$ is the horizontal asymptote of the function we take as an example. This curve is shown in the figure above.

So, $\lim\limits_{x,\Delta x\to 0}\frac{f(x)}{g(x)}=\frac{f`(x_0)}{g`(x_0)}$

For example, the resulting uncertainty is: $f(x)=x^2-5x+6,\ \ g(x)=x^2-3x+2,\ \ \lim\limits_{x\to 2}\left(\frac{f(x)}{g(x)}\right)=\frac{0}{0}=$

Solution:

$$\lim_{x\to 2}\frac{(x^2-5x+6)}{(x^2-3x+2)}=\frac{(x^2-5x+6)`}{(x^2-3x+2)`}=\frac{2x-5}{2x-3}=-1$$

Theorem 2 . $\left(\frac{\infty}{\infty}\right)$If there are functions $f(x)$and $g(x)$and are differentiable in the neighborhood of $x=x_0$,
$\lim\limits_{x\to 0}f(x)=\lim\limits_{x\to 0}g(x)=\infty$and is satisfied again $f`(x)\neq 0, gf(x)\neq 0$, then this equality is satisfied $\lim\limits_{x\to 0}\frac{f(x)}{g(x)}=\lim\limits_{x\to 0}\frac{f`(x_0)}{g`(x_0)}$.

If the types of uncertainties look like $(0\cdot\infty), 0^0, (\infty^0), (\infty,-\infty), (1^\infty)$, then by algebraic operations they are reduced to the form $\left(\frac{0}{0}\right)$or$\left(\frac{\infty}{\infty}\right)$

If $f(x)\to 0, g(x)\to\infty$given in the form $f(x), g(x)$, then reduced to the form $(0\cdot\infty)$, we transform it in this way:

$f\cdot g=\frac{f}{1\backslash g}\left(\frac{0}{0}\right)$ or$f\cdot g=\frac{g}{1\backslash f}\left(\frac{\infty}{\infty}\right)$

B) If the types of uncertainties look like $(1^\infty), 0^0, (\infty^0)$, then they are transformed in this way:

$$f^e=e^{xlnf}, \text{агар } \lim_{x\to 0}f^k=k, \text{онда } \lim_{x\to 0}f^k=e^k, f>0$$

B) If $(\infty,-\infty)$, for example$f(x)-g(x),\ x\to x_0$ Then

$$f-g=\frac{-1}{\frac{1}{g}}+\frac{1}{\frac{1}{f}}=\left[\frac{\frac{1}{g}-\frac{1}{f}}{\frac{1}{g}\cdot\frac{1}{f}}\right]\left(\frac{0}{0}\right)$$

If lower limits exist, the slant line *y=kx+b* will become the center of the slope of the function *y=f(x)*,

$$\lim_{x\to 0}\frac{f(x)}{x}=k;\ \lim_{x\to 0}[f(x)=b]=b_2\ b\neq 0.$$

Proof. Let the expression of a straight line be given $y_{ox} = kx + b$. Given $f(x) - kx - b = 0$and center of a point on a line and functionx → 0.
$\Delta = y - y_0 \to 0$; отсюда $\lim_{x\to 0}[kx + b] = 0$, divide by x .

$$\lim_{x\to\infty}\left[\frac{f(x)}{x} - k - \frac{b}{x}\right] = \lim_{x\to\infty}\left[\frac{f(x)}{x} - k\right].$$

Plotting a graph of a function consists of the following steps:
Find the domain of definition.
Checking a function for even and odd parity.
Find vertical asymptotes.
Checking the change of a function at infinity, determining horizontal and oblique asymptotes.
Testing a function for extemum and determining monotonic intervals.
Finding bending points with the study of convexity and concavity.
Determining the intersection points of the graph with the axes.

Example 1. It is necessary to compose a function $y = \frac{1+x^2}{1-x^2}$.
Solution: 1) Scope:

$$1 - x^2 \neq 0, \quad 1 - x^2 > 0, \quad 1 - x^2 < 0, 0 \to (1 + x) \neq 0;$$

$$1 - x > 0, \quad 1 > x, 1 + x > 0, \quad x > -1, \quad 1 - x < 0, \quad 1 + x < 0, x < -1, x > 1,$$
$$(-\infty, -1) \cup (-1,1) \cup (1, +\infty), x \neq \pm 1.$$

2) The function is even, since$y(-x) = y(x)$.
3) Vertical asymptotes. The two vertical asymptotes $x > 1, x = -1,$ of the graph intersect the *x-axis* :

$$\lim_{x\to 1+0}\frac{1+x^2}{1-x^2} = \infty, \quad \lim_{x\to 1-0}\frac{1+x^2}{1-x^2} = \infty,$$

4) Situation at infinity

$$\lim_{x\to\infty}\frac{1+x^2}{1-x^2} = -1, \quad \lim_{x\to-\infty}\frac{1+x^2}{1-x^2} = -1,$$

$y = -1$- horizontal asymptote.

4th project work

Project work topic : Geometric illustration of limits.

Before talking about the geometric illustration of limits, the student must fully master the concept of limits. When designing a topic for geometric illustration of limits, the student must master the ability to study, copy, compare, systematize and analyze materials from the presented sources.

Purpose of the project work : Presentation of theoretical materials relevant to the topic “Geometric illustration of limits” in the form of tasks that students must study independently.

Contents of the project work:

Students independently look for answers to the following questions and, to draw up definitions, regulations, etc., carry out design work.
Problematic issues:
What is the limit of a function? Signs of the existence of function limits.
A necessary and sufficient sign of the existence of a function. Cauchy criterion.

A sufficient condition for the existence of a limit of a function. Theorem (about the limit of an intermediate function).
Necessary and sufficient condition for the existence of the limit of a monotone function.
5. How do you understand the geometric illustration of limits (meaning)?

Student Analysis . We study different ways to define the concept of geometric illustration of limits .

Example 1: Calculate the limit: $\lim\limits_{n\to\infty}\frac{n+1}{n}$.

Solution. Direct calculation of the limit leads to uncertainty of the form $\left(\frac{\infty}{\infty}\right)$.

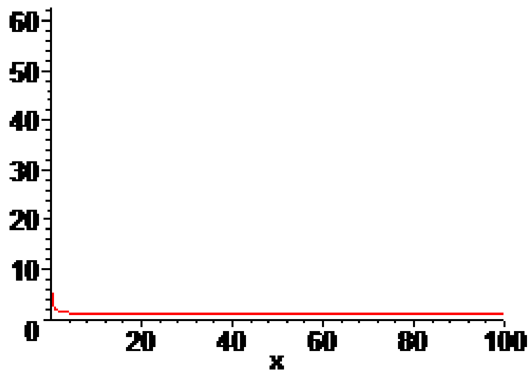
a) calculate in the Maple computer mathematics system:

```
> restart;
> Limit( (n+1)/ n,n =infinity);
```

$$\lim_{n\to\infty}\frac{n+1}{n}$$

```
> limit( (n+1)/ n,n =infinity);
```

b) draw a graph of the function $y=\frac{x+1}{x}$:



As can be seen from the graph, as the argument value tends to infinity, the function value approaches 1.

c) from the following definition $\forall\varepsilon>0\ \exists\delta=\delta(\varepsilon)>0:\forall x\in X\ \ |x|<\delta\Rightarrow|f(x)-A|<\varepsilon.$ in the language "ε – δ"

we use.

$$\forall\varepsilon>0\ \ \exists\delta=\delta(\varepsilon)>0:\ \forall x\neq0\ \ |x|<\delta\ \Rightarrow\left|\frac{x+1}{x}-1\right|<\varepsilon.$$

The last inequality is equivalent to the following inequality $\left|\frac{1}{x}\right|<\varepsilon,\ |x|>\frac{1}{\varepsilon},$ from which you can get $\delta=\frac{1}{\varepsilon}$.

Example 2: Limit calculation: $\lim\limits_{n\to\infty}(\sqrt{n^2+n}-n)$.

Solution: Direct calculation of the limit leads to uncertainty of the form $(\infty-\infty)$.

a) calculate in the Maple computer mathematics system:

```
> restart;
```

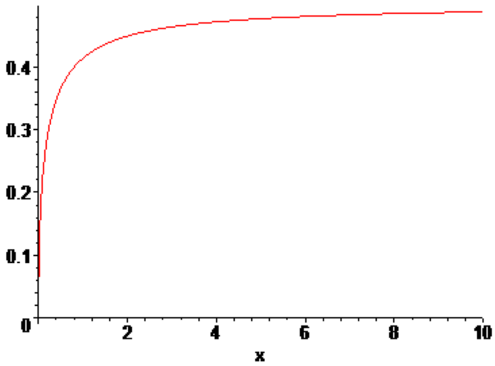
> **Limit(((n^2+n)^(1/2)-n),n=infinity);**

$$\lim_{n\to\infty} \sqrt{n^2+n} - n$$

> **limit(((n^2+n)^(1/2)-n),n=infinity);**

$$\frac{1}{2}$$

b) draw a graph of the function $y = \sqrt{x^2+x} - x)$:



As can be seen from the graph, as the value of the argument tends to infinity, the value of the function approaches 0.5.

c) from the following definition $\forall\varepsilon > 0\ \exists\delta = \delta(\varepsilon) > 0 : \forall x \in X\ \ |x| < \delta \Rightarrow |f(x) - A| < \varepsilon.$ in the language "ε – δ"

we use

$$\forall\varepsilon > 0\ \ \exists\delta = \delta(\varepsilon) > 0 :\ \forall x \neq 0\ \ |x| < \delta\ \Rightarrow \left|\sqrt{x^2+x} - x - \frac{1}{2}\right| < \varepsilon.$$

The last inequality is equivalent to the following inequality $\left|\frac{2\sqrt{x^2+x} - 2x - 1}{2}\right| < \varepsilon$, from which we can obtain $\delta = \frac{1}{\varepsilon}$.

5th project work

Topic of the project work : Functionality tizbekter men qatarlar.

The purpose of the project work : Functionality qatarlardyn zhinaktylygyn zertteuge uyretu. Student oz betinshe үyrenuine tiisti tapsyrmalar turinde usnu.

Contents of the project work:

Students independently look for answers to the following questions and, to draw up definitions, regulations, etc., carry out design work.

Problematic issues:

1. What is a functional series?
2. Give the definition of convergence of a functional series.
3. Area of accumulation of a functional series.
4. Interval of convergence of the functional series.
5. Determination of the functional sequence.
6. Name the conditions for the convergence of a functional sequence.
7. Weierstrass test

Fragment of project work execution .

A functional series is a sequence whose members are composed of functions:

$$u_1(x)+u_2(x)+\ldots+u_n(x)+\ldots=\sum_{n=1}^{\infty}u_n(x).$$

The convergence of functional series is formulated as follows:

any x_0 in its numerical value is transformed into $a\ u_n(x_0)$ number, therefore the functional series is transformed into a numerical series, and convergence at a point is studied by the signs of convergence of numerical series.

Definition 1. The set of all X that converge in a series is called the region of convergence of the functional series.

If the members of the functional series are defined in a certain interval, then the following definition is accepted.

Definition 2 . When a function series (a,b) converges as a number series at each point, it is called the convergence of an intermediate function series.

Between the functional series (a,b), the partial sums give the functional sequence.

$$S_1(x)=u_1(x),\quad S_2(x)=u_1(x)+u_2(x)\ldots,\ S_n(x)=\sum_{n=1}^{\infty}u_n(x).$$

Definition 3. If in the interval (a, b) at each value x the numbers converge as a sequence, we speak of the convergence of a functional sequence $\sum_{n=1}^{\infty}u_n(x)$.

If all numbers converge as a sequence $x\in(a,b)$- those $\sum_{n=1}^{\infty}u_n(x)$, then this limit is satisfied

$$\lim_{n\to\infty}S_n(x)=S(x),$$

Where $S(x)$- $\sum_{n=1}^{\infty}u_n(x)$ is called the limiting function of a sequence of functions and it is equal to the sum of the functional series:

$$\sum_{n=1}^{\infty}u_n(x)=S(x)$$

Example. The segment $[0,1]$ gives a functional series: $\sum_{n=1}^{\infty}\frac{1}{x+n}$

Define the functional sequence and the limit function.

Solution: 1) if n = 1,2 .... then We write the functional sequence in the following form:

$$\frac{1}{x+1},\frac{1}{x+2},\ldots,\frac{1}{x+n},\ S_n(x)=\sum_{n=1}^{\infty}\frac{1}{x+n}.$$

2) We find the limit function from the above expression.

$$S(x)=\lim_{n\to\infty}S_n(x)=\sum_{n=1}^{\infty}\frac{1}{x+\infty}=0,\quad S(x)=0.$$

Definition 4 . in an interval, (a,b) the functional sequence $S_n(x)$ converges to the limit function uniformly if in any $\varepsilon > 0$ number, in any interval $n \geq N(x)$ $x \in (a,\text{b})$ the following condition is met $|S_n(x) - S(x)| < \varepsilon$.
it is called the normal convergence condition.

We write this condition as follows:

$$S(x) - \varepsilon < S_n(x) < S(x) + \varepsilon, \quad (n = 1, 2, \ldots)$$

If a functional sequence $S_n(x)$ uniformly converges to a limit function $S(x)$ in the interval (a,b), then the sequence $S_n(x)$ will be located between $[S(x) - \varepsilon]$ and $[S(x) + \varepsilon]$.
Definition 5. If in the interval (*a*, *b*) the functional sequence $S_n(x)$ uniformly converges to the limit function $S(x)$, then the functional series $\sum_{n=1}^{\infty} u_n(x)$ converges uniformly in this interval.

Theorem 1 . (Weierstrass sign). Let the functional series $\sum_{n=1}^{\infty} u_n(x)$ be given in the interval (a,b). If we find a convergent series $\sum_{n=1}^{\infty} b_n(x)$ with a positive sign, and the condition is satisfied

$|u_n(x) \leq |b_n, \quad n = 1, 2, 3, \ldots$

then the functional series $\sum_{n=1}^{\infty} u_n(x)$ converges absolutely and uniformly, and $\sum_{n=1}^{\infty} b_n$ is called a majorant.

Example . Prove uniform convergence of the series $\sum_{n=1}^{\infty} \frac{\sin nx}{n^2}$.

Solution: Since therefore $|\sin nx| \leq 1$,, the majorant $\sum_{n=1}^{\infty} \frac{1}{n^2}$ (it is convergent), as $\sum_{n=1}^{\infty} \left| \frac{\sin nx}{n^2} \right| < \sum_{n=1}^{\infty} \frac{1}{n^1}$, in Theorem 1, is uniformly convergent in the series $(-\infty, \infty)$.
Determine the region of convergence of the series in the problem below:

$\sum_{n=1}^{\infty} e^{-nx}$. Solution : Apply Cauchy's test

$$\lim_{n\to\infty}\sqrt[n]{e^{-nx}} = e^{-x} = \begin{Bmatrix} <1, if\ x>0, converge \\ >1, if\ x<0, doesnt\ converge \end{Bmatrix}$$

at point x =0 the series is transformed into a number series, $1+1+...+1+...$ does not converge, so the region of convergence $(0<x<+\infty)$.

$\sum_{n=1}^{\infty}\frac{1}{n}\left(\frac{|x|}{x}\right)^n$. Solution : members of the functional series are not defined at the point $x=0$, but at other points they are defined and available.

If $x<0$, then $\frac{|x|}{x}=\frac{-x}{x}=-1,$ hence in negative x characters:

$$\sum_{n=1}^{\infty}\frac{1}{n}\left(\frac{|x|}{x}\right)^n=\sum_{n=1}^{\infty}(-1)^n\frac{1}{n}.$$

converge according to Leibniz's criterion.

If $x>0,$ Then $\frac{|x|}{x}=\frac{x}{x}=1,$ hence in positive x symbols:

$$\sum_{n=1}^{\infty}\frac{1}{n}\left(\frac{|x|}{x}\right)^n=\sum_{n=1}^{\infty}\frac{1}{n}.$$

The harmonic series does not converge, so the region of conditional convergence $(-\infty,0)$.

Solution: Based on Cauchy's criterion

$$\lim_{n\to\infty}\sqrt[n]{\left(3-x^2\right)^n}=\lim_{n\to\infty}\left(3-x^2\right)=3-x^2<1,\quad 3-x^2<1,\quad 2<x^2,\quad \pm\sqrt{2}<x,$$

$$\left(-2,-\sqrt{2}\right)\quad\left(2,\sqrt{2}\right).$$

$$\sum_{n=1}^{\infty}\frac{1}{x^n};5.\ \sum_{n=1}^{\infty}\ln^2 x;6.\sum_{n=1}^{\infty}\left(2-x^2\right)^n;$$

$$\sum_{n=1}^{\infty}\left[\frac{x(x+n)}{n}\right]^n;8.\sum_{n=1}^{\infty}\frac{1}{x^n+1}.$$

During our control, we were convinced that the involvement of future mathematicians in project activities allows us to form them as professional, social and creative individuals. Such activities allow the student to reveal their own potential; to develop knowledge, research abilities, independence, activity and creativity, the ability to strategically plan one’s activities, achieve expected results, and work in a team.

Organizing the project activities of future mathematicians requires solving a complex of methodological, psychological and pedagogical problems. If teachers have a high level of scientific and methodological training, and have mastered the design method in teaching the subject and the stages of students’ design activities, then these issues can be resolved in any educational institutions.

To develop students' abilities in studying the theory of limits, we used the following system of project and research activities.

- systematization of information;
- identification of cause-and-effect relationships (identification of causes, arguments, patterns, connections between elements of the system);
- planning and implementation of practical actions (drawing up a plan for completing tasks, analyzing the plan for carrying out activities).

In the process of project activities, students learn to independently master new knowledge, analyze non-standard situations, systematize the search for new solutions, consolidate acquired knowledge, pose specific questions, reason about a problem, understand, and comprehend the current situation. In addition, they gain experience in implementing project activities, deeply assimilate educational material, they will develop professionally significant qualities (application of skills acquired in lessons on mathematical analysis, active integration into cognitive activity, familiarity with the realities of our time, increasing motivation and interest in learning .

So, in the process of developing students' project activities, the following are formed:

reflective, search, communicative, visual, cognitive (working with a textbook, scientific literature, scheduling, ability to express one's thoughts, self-control and assessment), special (mastering materials), research (setting a goal, drawing up a program and research plan, describing a phenomenon, formulation of hypotheses, integration of data, use of mathematical apparatus to describe patterns, formulation of conclusions) and other competencies..

Using the Maple SCM allows you to track any processes on a computer screen during the research process, create graphs of functions in two-dimensional and three-dimensional space, and quickly carry out calculations.

We have found that high-quality implementation of project activities using the Maple SCM, compatibility in the formulation of problematic tasks and their solution, is a fairly effective way to enhance the educational and cognitive activity of students. The use of educational software changes the lesson system, pace, and sometimes their structure. All this contributes to the activation of the training system and creates conditions for organizing project activities .

It is known that it is beneficial to use special mathematical programs, such as Mathematica, Maple, MathCAD, MathLAB, etc., as tools for performing design work related to mathematical modeling, developing graphic skills. The package of these programs is an indispensable tool for students when performing scientific research. research work, creation of scientific projects and mathematical models. Because here you can quickly achieve results using special computer programs, as well as quickly complete the work of constructing graphs of the desired functions using a special graphical environment, and here, along with algorithmization, much attention is paid to modeling.

During the research, we used the Maple package of applied mathematical programs to solve problems related to design work and the theory of limits. Maple is used to graph complex problems.

When studying the theory of limits, we will consider examples concerning the effectiveness of using the Maple SCM in improving the project activities of students.

Effective use of modern mathematical packages in the field of education and science requires changes in this direction. They simplify the solution of complex mathematical problems and help eliminate psychological barriers in students' acquisition of basic mathematical concepts. As interest in practical exercises in the learning process grows, their intensity also increases; The software package also helps expand the teacher's ability to solve problems of practical importance. Currently, the educational process is associated not only with memorizing concepts and images. Lately, questions remain related to how to find information and, most importantly, how and where to find it. Students must be able to assimilate concepts and theories not only from those of others, but also from their own knowledge and life experiences.

As one of the ways to develop students' project activities, we will demonstrate the progress of using the Maple computer mathematics system to calculate the limits of functions. As a result of the project work, the knowledge, skills and abilities acquired during the study of mathematical analysis and information technology are analyzed. The materials in the project work will be useful to students of the specialty "Mathematics" for systematizing and deepening the knowledge they have acquired in the subject of mathematical analysis.

Let's consider problems designed to calculate specific limits:

> **restart;**

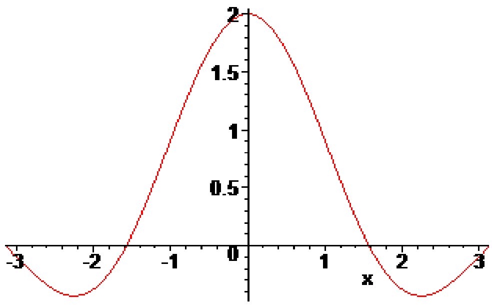
> **Limit(sin(2*x)/ x,x =0);**

$$\lim_{x \to 0} \frac{\sin(2\,x)}{x}$$

> **limit(sin(2*x)/ x,x =0);**

$$2$$

> **plot(sin(2*x)/ x,x =- Pi..Pi);**



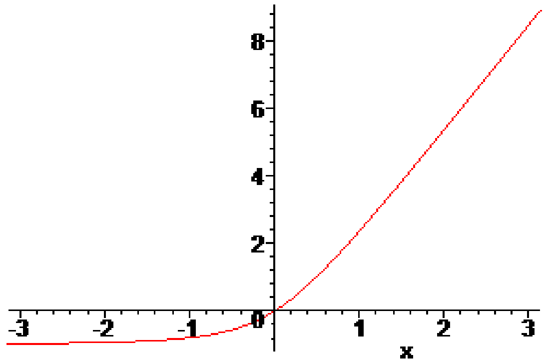
> **Limit(x*(Pi/2+arctan(x)) ,x =-infinity);**

$$\lim_{x \to (-\infty)} x\left(\frac{\pi}{2} + \arctan(x)\right)$$

> **limit(x*(Pi/2+arctan(x)), x=-infinity);**

$$-1$$

>**plot(x*(Pi/2+arctan(x)),x=-Pi..Pi);**



>Limit(1/(1+exp(1/x)),x=0,left);

$$\lim_{x \to 0-} \frac{1}{1 + \mathbf{e}^{\left(\frac{1}{x}\right)}}$$

>limit(1/(1+exp(1/x)),x=0,left);

$$1$$

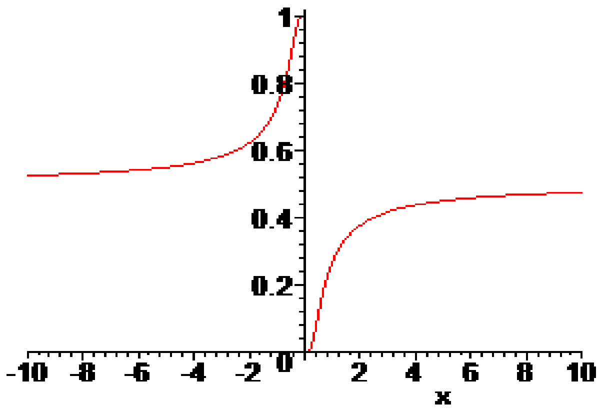
>Limit(1/(1+exp(1/x)),x=0,right);

$$\lim_{x \to 0+} \frac{1}{1 + \mathbf{e}^{\left(\frac{1}{x}\right)}}$$

>limit(1/(1+exp(1/x)),x=0,right);

$$0$$

>plot(1/(1+exp(1/x)),x=-10..10);



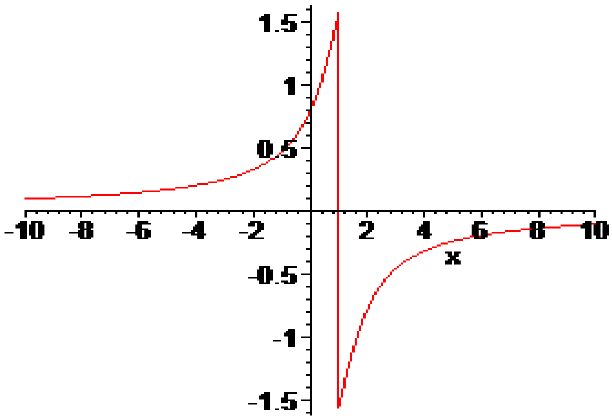
>Limit(arctan(1/(1-x)),x=1,left)=limit(arctan(1/(1-x)),x=1,left);

$$\lim_{x \to 1-} \arctan\left(\frac{1}{1-x}\right) = \frac{\pi}{2}$$

> Limit(arctan(1/(1-x)),x=1,right)= limit(arctan(1/(1-x)),x=1,right);

$$\lim_{x \to 1+} \arctan\left(\frac{1}{1-x}\right) = -\frac{\pi}{2}$$

>plot(arctan(1/(1-x)),x=-10..10);



>Limit(((1+x^2)^(1/2)-1)/x^2,x=0);

$$\lim_{x \to 0} \frac{\sqrt{1+x^2}-1}{x^2}$$

>limit(((1+x^2)^(1/2)-1)/x^2,x=0);

$$\frac{1}{2}$$

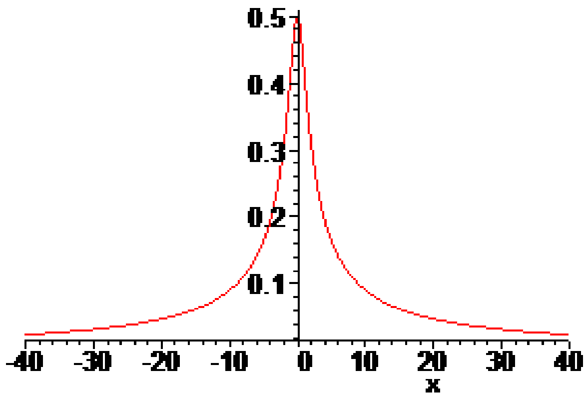
>Limit(((1+x^2)^(1/2)-1)/x^2,x=0,right)=limit(((1+x^2)^(1/2)-1)/x^2,x=0,right);

$$\lim_{x \to 0+} \frac{\sqrt{1+x^2}-1}{x^2} = \frac{1}{2}$$

>Limit(((1+x^2)^(1/2)-1)/x^2,x=0,left)=limit(((1+x^2)^(1/2)-1)/x^2,x=0,left);

$$\lim_{x \to 0-} \frac{\sqrt{1+x^2}-1}{x^2} = \frac{1}{2}$$

>plot(((1+x^2)^(1/2)-1)/x^2,x=-5..5);



As we can see, in the process of teaching mathematical analysis, the effective use of the Maple computer mathematics system (the possibility of using it in calculating limits) gives a positive result. In turn, the organization by the teacher of independent project work of future mathematicians contributes to the improvement of their project activities, their in-depth study of mathematical analysis, and the solution of problems of applied and practical training.

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