THE STUDY OF THE MOTION OF A SINKING OR RISING BODY IN A LIQUID, TAKING INTO ACCOUNT THE RESISTANCE OF THE MEDIUM AND SOLVING RELATED PROBLEMS

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Abstarct. *This article examines the laws of motion of a body in a vertical plane under the action of the fluid resistance force, the Archimedan force and gravity, and develops issues related to them.*
Keywords: *acceleration; the equation of motion; the equation velocity; the equation of motion; force of resistance; Archimedean force; the force of gravity; to take an integral; limits of integral.*

It is known that subjects related to the study of the movement of a body moving in a liquid are not fully covered for students of upper classes of general secondary schools and academic lyceum. In this case, either the resistance of the fluid is ignored or it is taken into account as an average constant force.

Depending on the type of higher education, such a topic is either not included in the curriculum at all or is not studied more seriously.

Only some techniques are studied in depth in the special science of "Hydraulics" from the higher education institutions.

In the fields of "Physics" specialty of Pedagogical Higher Education Institutions, the topics of hydrostatics are abandoned, and only topics such as Bernoulli's law, continuity of flow, types of flow and Puzeil's equation are studied as elements of hydrodynamics.

However, the case where the resistance of the liquid changes depending on the speed is almost not studied.

That's why it's important to study fluid resistance and body motion that takes into account gravity and Archimedean forces, and it brings students much closer to the real situation.

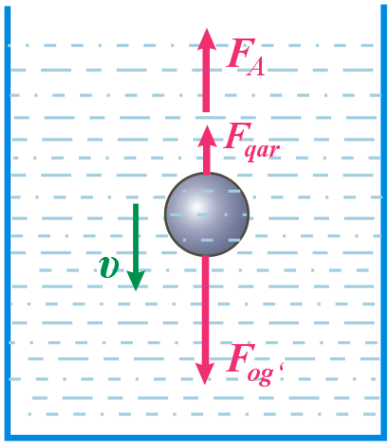
We know that the resistance of the medium can be directly proportional to the first power or the second power of the velocity.

For example, in liquid phenomena (studying the movement of a boat or ship), the resistance force is proportional to the first degree of speed, while in air phenomena (in studying the movement of a car, in aviation, in military equipment, a parachute, q, in the study of projectiles, etc.) the resistance force is proportional to the second power of the velocity. In this article, we will study the vertical movement of a body that sinks down or floats up while completely immersed in a liquid, connecting the resistance force proportional $\left(F_{qar} \sim \vartheta\right)$ to the speed, and we will work on related problems.

Let's consider separately the cases of the body sinking in the liquid and rising from the bottom of the liquid.

I.

If an object is submerged in a liquid, the resistance force acting on the object is upward because the motion is downward. In addition, the body is affected by the upward Archimedean force and downward gravity (Fig. I.1). Whether the body is moving downwards under the influence of these three forces, either accelerating or decelerating. Using the basic equation of dynamics, we create a differential equation, and by solving the resulting differential equation, we can create connections between time, speed, and path.



I.1-rasm

$$ma = F_{og'} - F_{qar} - F_A, \rightarrow \; m\frac{d\vartheta}{dt} = mg - \alpha\vartheta - F_A, \; \rightarrow \; dt = \frac{md\vartheta}{mg - \alpha\vartheta - F_A} = -\frac{m}{\alpha}\cdot\frac{d\vartheta}{\vartheta - \dfrac{mg - F_A}{\alpha}}$$

$$\int_0^t dt = -\frac{m}{\alpha}\cdot\int_{\vartheta_0}^{\vartheta}\frac{d\vartheta}{\vartheta - \dfrac{mg-F_A}{\alpha}}, \; \rightarrow \; t = -\frac{m}{\alpha}\cdot\ln\left|\vartheta - \frac{mg-F_A}{\alpha}\right|\Bigg|_{\vartheta_0}^{\vartheta} = -\frac{m}{\alpha}\cdot\ln\left|\frac{\vartheta - \dfrac{mg-F_A}{\alpha}}{\vartheta_0 - \dfrac{mg-F_A}{\alpha}}\right| =$$

$$= \frac{m}{\alpha}\cdot\ln\left|\frac{\vartheta_0 - \dfrac{mg-F_A}{\alpha}}{\vartheta - \dfrac{mg-F_A}{\alpha}}\right| = \frac{m}{\alpha}\cdot\ln\left|\frac{\alpha\vartheta_0 + F_A - mg}{\alpha\vartheta + F_A - mg}\right| = \frac{m}{\alpha}\cdot\ln\left|\frac{mg - (\alpha\vartheta_0 + F_A)}{mg - (\alpha\vartheta + F_A)}\right|$$

Thus, we created an equation in the form of $t=t(\vartheta)$ which determines the time it takes for a body immersed in a liquid to reach a certain speed.

$$\boxed{t = \frac{m}{\alpha}\cdot\ln\left|\frac{mg - (\alpha\vartheta_0 + F_A)}{mg - (\alpha\vartheta + F_A)}\right|} \qquad \textbf{(I.1)}$$

Using the formula (I.1), the following specific formula can be obtained for a body that begins to sink from rest:

$$\boxed{t = \frac{m}{\alpha}\cdot\ln\left|\frac{mg - F_A}{mg - (\alpha\vartheta + F_A)}\right|} \qquad \textbf{(I.1a)}$$

If (I.1) is in the formula $mg < F_A$, the sedimentation process is decelerating, otherwise $mg > F_A$, the sedimentation process is accelerated. If an object is moving with deceleration, then the object will stop after some time. The downtime is as follows:

$$\boxed{t_{to'xt} = \frac{m}{\alpha}\cdot\ln\left|1 + \frac{\alpha\vartheta_0}{F_A - mg}\right|} \qquad \textbf{(I.1b)}$$

Now let's create the formula of speed dependence on time.

$$t=-\frac{m}{\alpha}\cdot\ln\left|\frac{\vartheta-\frac{mg-F_A}{\alpha}}{\vartheta_0-\frac{mg-F_A}{\alpha}}\right|,\ \rightarrow\ \frac{\vartheta-\frac{mg-F_A}{\alpha}}{\vartheta_0-\frac{mg-F_A}{\alpha}}=e^{-\frac{\alpha}{m}t},\ \rightarrow\ \vartheta-\frac{mg-F_A}{\alpha}=$$

$$=\left(\vartheta_0-\frac{mg-F_A}{\alpha}\right)e^{-\frac{\alpha}{m}t},\ \rightarrow\ \vartheta=\left(\vartheta_0-\frac{mg-F_A}{\alpha}\right)e^{-\frac{\alpha}{m}t}+\frac{mg-F_A}{\alpha}$$

$$\boxed{\vartheta=\left(\vartheta_0-\frac{mg-F_A}{\alpha}\right)e^{-\frac{\alpha}{m}t}+\frac{mg-F_A}{\alpha}} \qquad \textbf{(I.2)}$$

The formula (I.2) above leaves one term when t→∞ and this gives the maximum achievable speed.

$$\boxed{\vartheta_{\max}=\frac{mg-F_A}{\alpha}} \qquad \textbf{(I.3)}$$

can be derived from the basic equation of dynamics

$$ma=F_{og'}-F_{qar}-F_A=0,\ \rightarrow\ F_{qar}=F_{og'}-F_A,\ \rightarrow\ \alpha\vartheta=mg-F_A,\ \rightarrow\ \vartheta_{\max}=\frac{mg-F_A}{\alpha}$$

If we consider the formula (I.3), then we can write the formulas (I.1) and (I.2) through the maximum speed as follows

$$\boxed{t=\frac{m}{\alpha}\cdot\ln\left|\frac{\vartheta_{\max}-\vartheta_0}{\vartheta_{\max}-\vartheta}\right|} \qquad \textbf{(I.1')}$$

$$\boxed{\vartheta=\vartheta_{\max}-\left(\vartheta_{\max}-\vartheta_0\right)e^{-\frac{\alpha}{m}t}} \qquad \textbf{(I.2')}$$

Now let's create the relationship between the path and the speed, that is, the equation s=s(ϑ) for the situation in Fig. I.1. To do this, we create a differential equation using the basic equation of dynamics, such as the equations derived above, and solve it.

$$m\frac{d\vartheta}{dt}=mg-\alpha\vartheta-F_A,\ \rightarrow\ m\frac{\vartheta d\vartheta}{ds}=mg-\alpha\vartheta-F_A,\ \rightarrow\ ds=\frac{m\vartheta d\vartheta}{mg-\alpha\vartheta-F_A}=$$

$$=-\frac{m}{\alpha}\cdot\frac{\vartheta d\vartheta}{\vartheta-\frac{mg-F_A}{\alpha}}=-\frac{m}{\alpha}\cdot\left(1+\frac{\frac{mg-F_A}{\alpha}}{\vartheta-\frac{mg-F_A}{\alpha}}\right)d\vartheta.$$

$$s=\int_0^s ds=-\frac{m}{\alpha}\cdot\int_{\vartheta_0}^{\vartheta}\left(1+\frac{\frac{mg-F_A}{\alpha}}{\vartheta-\frac{mg-F_A}{\alpha}}\right)d\vartheta=-\frac{m}{\alpha}\cdot\left[\vartheta+\frac{mg-F_A}{\alpha}\cdot\ln\left|\vartheta-\frac{mg-F_A}{\alpha}\right|\right]\Bigg|_{\vartheta_0}^{\vartheta}=$$

$$=-\frac{m}{\alpha}\cdot\left[\vartheta-\vartheta_0+\frac{mg-F_A}{\alpha}\cdot\ln\left|\frac{\vartheta-\frac{mg-F_A}{\alpha}}{\vartheta_0-\frac{mg-F_A}{\alpha}}\right|\right]=\frac{m}{\alpha}\cdot\left[\vartheta_0-\vartheta+\frac{mg-F_A}{\alpha}\cdot\ln\left|\frac{\alpha\vartheta_0+F_A-mg}{\alpha\vartheta+F_A-mg}\right|\right].$$

$$\text{yoki}\quad s=\frac{m}{\alpha}\cdot\left[\vartheta_0-\vartheta+\vartheta_{\max}\cdot\ln\left|\frac{\vartheta_{\max}-\vartheta_0}{\vartheta_{\max}-\vartheta}\right|\right]$$

Thus, we created the equation s=s(ϑ).

$$s=\frac{m}{\alpha}\cdot\left[\vartheta_0-\vartheta+\frac{mg-F_A}{\alpha}\cdot\ln\left|\frac{\alpha\vartheta_0+F_A-mg}{\alpha\vartheta+F_A-mg}\right|\right] \quad \textbf{(I.3)}$$

$$s=\frac{m}{\alpha}\cdot\left[\vartheta_0-\vartheta+\vartheta_{\max}\cdot\ln\left|\frac{\vartheta_{\max}-\vartheta_0}{\vartheta_{\max}-\vartheta}\right|\right] \quad \textbf{(I.3')}$$

The above formulas (I.3) and (I.3') are the formulas that determine the distance s a body must travel to reach an arbitrary speed ϑ .

Using the above formulas (I.3) and (I.3'), the following specific formula can be obtained for a body that starts to sink from rest (ϑ_0=0):

$$s=\frac{m}{\alpha}\cdot\left[\frac{mg-F_A}{\alpha}\cdot\ln\left|\frac{F_A-mg}{\alpha\vartheta+F_A-mg}\right|-\vartheta\right] \quad \textbf{(I.3a)}$$

$$s=\frac{m}{\alpha}\cdot\left[\vartheta_{\max}\cdot\ln\left|\frac{\vartheta_{\max}}{\vartheta_{\max}-\vartheta}\right|-\vartheta\right] \quad \textbf{(I.3'a)}$$

If $mg<F_A$ so, the process of sinking will be slow, the body will stop after covering a certain distance and will start to rise again under the influence of Archimedes' force. In this case, it will be possible to determine the stopping time using the above formulas (I.3) and

$$s_{to'xt}=\frac{m}{\alpha}\cdot\left[\vartheta_0+\frac{mg-F_A}{\alpha}\cdot\ln\left|1-\frac{\alpha\vartheta_0}{mg-F_A}\right|\right] \quad \textbf{(I.3b)}$$

$$s_{to'xt}=\frac{m}{\alpha}\cdot\left[\vartheta_0+\vartheta_{\max}\cdot\ln\left|1-\frac{\vartheta_0}{\vartheta_{\max}}\right|\right] \quad \textbf{(I.3'b)}$$

It should be noted that in the above 2 formulas $\vartheta_{\max}=\frac{mg-F_A}{\alpha}<0$, that is, the negative sign ϑ_{max} of the magnitude of thmax should not be forgotten

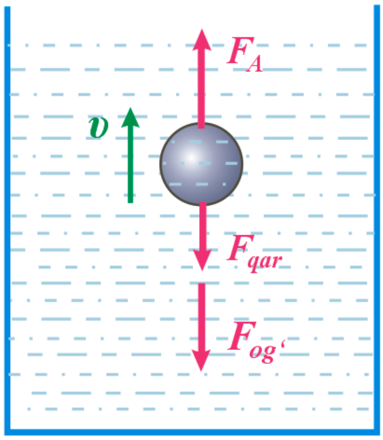
II.

Now let the body rise from the bottom of the liquid under the influence of the Archimedean force and the initial upward velocity.

In this case, since the movement is directed upwards, the force of resistance acting on the body is directed downwards. In addition, the body is affected by the upward Archimedean force and downward gravity (Fig. II.1).

Let the body move upward under the influence of these 3 forces with acceleration.

Using the basic equation of dynamics, we create a differential equation, and by solving the resulting differential equation, we can create connections between time, speed, and path.



II.1-fig

$$ma = F_A - F_{og'} - F_{qar}, \rightarrow \ m\frac{d\vartheta}{dt} = F_A - mg - \alpha\vartheta, \rightarrow \ dt = \frac{md\vartheta}{F_A - mg - \alpha\vartheta} = -\frac{m}{\alpha}\cdot\frac{d\vartheta}{\vartheta - \frac{F_A - mg}{\alpha}}$$

$$\int_0^t dt = -\frac{m}{\alpha}\cdot\int_{\vartheta_0}^{\vartheta}\frac{d\vartheta}{\vartheta - \frac{F_A - mg}{\alpha}}, \rightarrow \ t = -\frac{m}{\alpha}\cdot\ln\left|\vartheta - \frac{F_A - mg}{\alpha}\right|\Bigg|_{\vartheta_0}^{\vartheta} = -\frac{m}{\alpha}\cdot\ln\left|\frac{\vartheta - \frac{F_A - mg}{\alpha}}{\vartheta_0 - \frac{F_A - mg}{\alpha}}\right| =$$

$$= \frac{m}{\alpha}\cdot\ln\left|\frac{\vartheta_0 - \frac{F_A - mg}{\alpha}}{\vartheta - \frac{F_A - mg}{\alpha}}\right| = \frac{m}{\alpha}\cdot\ln\left|\frac{\alpha\vartheta_0 + mg - F_A}{\alpha\vartheta + mg - F_A}\right|$$

Thus, we created an equation in the form of $t=t(\vartheta)$ which determines the time it takes for a body rising from the liquid bottom to reach a certain speed.

$$\boxed{t = \frac{m}{\alpha}\cdot\ln\left|\frac{\alpha\vartheta_0 + mg - F_A}{\alpha\vartheta + mg - F_A}\right|} \qquad \textbf{(II.1)}$$

Using the formula (II.1), the following specific formula can be obtained for a body starting to rise from rest:

$$\boxed{t = \frac{m}{\alpha}\cdot\ln\left|\frac{mg - F_A}{\alpha\vartheta + mg - F_A}\right|} \qquad \textbf{(II.1a)}$$

If (II.1) is $mg > F_A$ in the formula, the ascent process is decelerating, and if it is the opposite, then the ascent process is $mg < F_A$ accelerated. If the body is moving with deceleration, then the body will stop after some time and start to sink again. The downtime is as follows:

$$\boxed{t_{to'xt} = \frac{m}{\alpha}\cdot\ln\left|1 + \frac{\alpha\vartheta_0}{mg - F_A}\right|} \qquad \textbf{(II.1b)}$$

Now let's create the formula of speed dependence on time.

$$t=-\frac{m}{\alpha}\cdot\ln\left|\frac{\vartheta-\dfrac{F_A-mg}{\alpha}}{\vartheta_0-\dfrac{F_A-mg}{\alpha}}\right|,\ \rightarrow\ \frac{\vartheta-\dfrac{F_A-mg}{\alpha}}{\vartheta_0-\dfrac{F_A-mg}{\alpha}}=e^{-\frac{\alpha}{m}t},\ \rightarrow\ \vartheta-\frac{F_A-mg}{\alpha}=$$

$$=\left(\vartheta_0-\frac{F_A-mg}{\alpha}\right)e^{-\frac{\alpha}{m}t},\ \rightarrow\ \vartheta=\left(\vartheta_0-\frac{F_A-mg}{\alpha}\right)e^{-\frac{\alpha}{m}t}+\frac{F_A-mg}{\alpha}$$

$$\boxed{\vartheta=\left(\vartheta_0-\frac{F_A-mg}{\alpha}\right)e^{-\frac{\alpha}{m}t}+\frac{F_A-mg}{\alpha}} \qquad \textbf{(II.2)}$$

The formula (II.2) above leaves one term when t→∞ and this gives the maximum achievable speed.

$$\boxed{\vartheta_{\max}=\frac{F_A-mg}{\alpha}} \qquad \textbf{(II.3)}$$

The formula (II.3) can be derived from the basic equation of dynamics

$$ma=F_A-F_{og'}-F_{qar}=0,\ \rightarrow\ F_{qar}=F_A-F_{og'},\ \rightarrow\ \alpha\vartheta=F_A-mg,\ \rightarrow\ \vartheta_{\max}=\frac{F_A-mg}{\alpha}$$

If we consider the formula (II.3), then we can write the formulas (II.1) and (II.2) through the maximum speed as follows:

$$\boxed{t=\frac{m}{\alpha}\cdot\ln\left|\frac{\vartheta_{\max}-\vartheta_0}{\vartheta_{\max}-\vartheta}\right|} \qquad \textbf{(II.1')}$$

$$\boxed{\vartheta=\vartheta_{\max}-\left(\vartheta_{\max}-\vartheta_0\right)e^{-\frac{\alpha}{m}t}} \qquad \textbf{(II.2')}$$

Now let's create the relationship between path and speed for the situation in Figure II.1, that is, the equation s=s(ϑ). To do this, we create a differential equation using the basic equation of dynamics, such as the equations derived above, and solve it.

$$m\frac{d\vartheta}{dt}=F_A-mg-\alpha\vartheta,\ \rightarrow\ m\frac{\vartheta d\vartheta}{ds}=F_A-mg-\alpha\vartheta,\ \rightarrow\ ds=\frac{m\vartheta d\vartheta}{F_A-mg-\alpha\vartheta}=$$

$$=-\frac{m}{\alpha}\cdot\frac{\vartheta d\vartheta}{\vartheta-\dfrac{F_A-mg}{\alpha}}=-\frac{m}{\alpha}\cdot\left(1+\frac{\dfrac{F_A-mg}{\alpha}}{\vartheta-\dfrac{F_A-mg}{\alpha}}\right)d\vartheta.$$

$$s=\int_0^s ds=-\frac{m}{\alpha}\cdot\int_{\vartheta_0}^{\vartheta}\left(1+\frac{\dfrac{F_A-mg}{\alpha}}{\vartheta-\dfrac{F_A-mg}{\alpha}}\right)d\vartheta=-\frac{m}{\alpha}\cdot\left[\vartheta+\frac{F_A-mg}{\alpha}\cdot\ln\left|\vartheta-\frac{F_A-mg}{\alpha}\right|\right]\Bigg|_{\vartheta_0}^{\vartheta}=$$

$$=-\frac{m}{\alpha}\cdot\left[\vartheta-\vartheta_0+\frac{F_A-mg}{\alpha}\cdot\ln\left|\frac{\vartheta-\dfrac{F_A-mg}{\alpha}}{\vartheta_0-\dfrac{F_A-mg}{\alpha}}\right|\right]=\frac{m}{\alpha}\cdot\left[\vartheta_0-\vartheta+\frac{F_A-mg}{\alpha}\cdot\ln\left|\frac{\alpha\vartheta_0+mg-F_A}{\alpha\vartheta+mg-F_A}\right|\right].$$

$$\text{yoki}\quad s=\frac{m}{\alpha}\cdot\left[\vartheta_0-\vartheta+\vartheta_{\max}\cdot\ln\left|\frac{\vartheta_{\max}-\vartheta_0}{\vartheta_{\max}-\vartheta}\right|\right]$$

So, we create equalitys=s(ϑ).

$$s=\frac{m}{\alpha}\cdot\left[\vartheta_0-\vartheta+\frac{F_A-mg}{\alpha}\cdot\ln\left|\frac{\alpha\vartheta_0+mg-F_A}{\alpha\vartheta+mg-F_A}\right|\right] \quad \textbf{(II.4)}$$

$$s=\frac{m}{\alpha}\cdot\left[\vartheta_0-\vartheta+\vartheta_{\max}\cdot\ln\left|\frac{\vartheta_{\max}-\vartheta_0}{\vartheta_{\max}-\vartheta}\right|\right] \quad \textbf{(II.4')}$$

The above formulas (II.4) and (II.4') are formulas that determine what distance s the body must travel to reach an arbitrary speed ϑ .

Using the formulas (II.4) and (II.4') above, the following specific formula can be obtained for a body starting to rise from rest (ϑ_0=0) $s=\frac{m}{\alpha}\cdot\left[\frac{F_A-mg}{\alpha}\cdot\ln\left|\frac{mg-F_A}{\alpha\vartheta+mg-F_A}\right|-\vartheta\right]$

(II.4a)

$$s=\frac{m}{\alpha}\cdot\left[\vartheta_{\max}\cdot\ln\left|\frac{\vartheta_{\max}}{\vartheta_{\max}-\vartheta}\right|-\vartheta\right] \quad \textbf{(II.4'a)}$$

If so $mg>F_A$, the ascent will be slow, the body will stop after covering a certain distance and begin to sink down under the influence of gravity. In this case, it will be possible to determine the stopping time using the above formulas (II.4) and (II.4').

$$s_{to'xt}=\frac{m}{\alpha}\cdot\left[\vartheta_0+\frac{F_A-mg}{\alpha}\cdot\ln\left|1-\frac{\alpha\vartheta_0}{F_A-mg}\right|\right] \quad \textbf{(II.4b)}$$

$$s_{to'xt}=\frac{m}{\alpha}\cdot\left[\vartheta_0+\vartheta_{\max}\cdot\ln\left|1-\frac{\vartheta_0}{\vartheta_{\max}}\right|\right] \quad \textbf{(II.4'b)}$$

It should be noted that in the above 2 formulas $\vartheta_{\max}=\frac{F_A-mg}{\alpha}<0$, the negative sign of ϑ_{max} max should not be forgotten.

III. Now we will strengthen our knowledge by working on several problems related to all the above-mentioned formulas.

Problem 1: If an aluminum ball with a mass of m=10 g reached the bottom of the container at time *t=0,5 s*, with what speed did it fall? What fraction of the maximum possible speed is this speed? What is the depth of the container? The density of aluminum is equal to $\rho_{al}=2700\,\frac{kg}{m^3}$, and that of water is equal to $\rho_{suv}=1000\,\frac{kg}{m^3}$. Let be the coefficient of resistance $\alpha=3\cdot10^{-4}\,\frac{kg}{s}$ of water on a given aluminum sphere and let be the acceleration of free fall.

Solution: Using the condition of the problem, we determine the Archimedean force acting on the sphere.

$$F_A = \rho_{suv} V g = \frac{\rho_{suv}}{\rho_{al}} mg = \frac{1000 \frac{kg}{m^3}}{2700 \frac{kg}{m^3}} \cdot 0{,}01 kg \cdot 9{,}8 \frac{m}{s^2} = 0{,}0363 N$$

We use the formula (I.2) derived above $\vartheta_0 = 0$. Let's call it the initial speed.

$$\vartheta = \left(\vartheta_0 - \frac{mg - F_A}{\alpha} \right) e^{-\frac{\alpha}{m}t} + \frac{mg - F_A}{\alpha} = \frac{mg - F_A}{\alpha} \left(1 - e^{-\frac{\alpha}{m}t} \right) =$$

$$= \frac{0{,}01 kg \cdot 9{,}8 \frac{m}{s^2} - 0{,}0363 N}{3 \cdot 10^{-4} \frac{kg}{s}} \cdot \left(1 - e^{-\frac{3 \cdot 10^{-4} \frac{kg}{s}}{0{,}01 kg} \cdot 0{,}5 s} \right) = 207 \frac{m}{s} \cdot (1 - 0{,}985) = 3{,}1 \frac{m}{s}$$

This speed is a part of the maximum speed that can be reached $\vartheta_{\max} = \frac{mg - F_A}{\alpha} = 207 \frac{m}{s}$ part $(1 - 0{,}985) = 0{,}015 = 1{,}5\%$

We calculate the depth of the container using the formula (I.3') derived above. Let's call it the initial speed.

$$s = \frac{m}{\alpha} \cdot \left[\vartheta_0 - \vartheta + \vartheta_{\max} \cdot \ln \left| \frac{\vartheta_{\max} - \vartheta_0}{\vartheta_{\max} - \vartheta} \right| \right] = \frac{m}{\alpha} \cdot \left[\vartheta_{\max} \cdot \ln \left| \frac{\vartheta_{\max}}{\vartheta_{\max} - \vartheta} \right| - \vartheta \right] =$$

$$= \frac{0{,}01 kg}{3 \cdot 10^{-4} \frac{kg}{s}} \cdot \left[207 \frac{m}{s} \cdot \ln \left| \frac{207 \frac{m}{s}}{207 \frac{m}{s} - 3{,}1 \frac{m}{s}} \right| - 3{,}1 \frac{m}{s} \right] = 33{,}33 s \cdot \left[207 \frac{m}{s} \cdot 0{,}015089 - 3{,}1 \frac{m}{s} \right] =$$

$$= 33{,}33 s \cdot 0{,}2345 \frac{m}{s} = 0{,}78 m = 78\ cm.$$

Problem solving 2: What is the maximum speed a lead ball with a mass of *m=100 mg* can reach in glycerol liquid? Calculate the force of resistance of the liquid to the sphere from the Stokes formula $F_{qar} = 6\pi \eta r \vartheta$. The density of lead is equal to $\rho_q = 11300 \frac{kg}{m^3}$, and that of glycerin is equal to $\rho_s = 1260 \frac{kg}{m^3}$. Assume the dynamic viscosity of glycerol at 20^0C and the acceleration of free fall $\eta = 1{,}5 Pa \cdot s$.

Solution: To solve the problem, first of all, it is necessary to determine the radius of the sphere. Body size

$$V = \frac{m}{\rho} = \frac{10^{-4} kg}{11300 \frac{kg}{m^3}} = 8{,}85 \cdot 10^{-9} m^3$$

Using the formula for finding the volume of a sphere, we determine the radius of the sphere.

$$V=\frac{4}{3}\pi r^3, \rightarrow \ r=\sqrt[3]{\frac{3V}{4\pi}}=\sqrt[3]{\frac{3\cdot 8,85\cdot 10^{-9}\,m^3}{4\cdot 3,14}}=0,001283\,m=1,283\,mm$$

Now let's calculate the resistance coefficient *α*. The resistance of the liquid is $F_{qar}=\alpha\vartheta$ calculated from the formula on the one hand $F_{qar}=6\pi\eta r\vartheta$, and from the Stokes formula on the other hand. By equating them, we can determine the coefficient *α*.

$$\alpha=6\pi\eta r=6\cdot 3,14\cdot(1,5\,Pa\cdot s)\cdot\left(1,283\cdot 10^{-3}\,m\right)=36,26\cdot 10^{-3}\,\frac{kg}{s}$$

We will focus our next calculations on determining the Archimedean force.

$$F_A=\rho_s Vg=\frac{\rho_s}{\rho_q}mg=\frac{1260\frac{kg}{m^3}}{11300\frac{kg}{m^3}}\cdot 10^{-4}\,kg\cdot 9,8\frac{m}{s^2}=1,093\cdot 10^{-4}\,N$$

Now we will determine the maximum speed that can be reached during the sinking of the given lead ball in glycerol.

$$\vartheta_{\max}=\frac{mg-F_A}{\alpha}=\frac{10^{-4}\,kg\cdot 9,8\frac{m}{s^2}-1,093\cdot 10^{-4}\,N}{3,626\cdot 10^{-4}\,\frac{kg}{s}}=2,4\,\frac{m}{s}.$$

Problem solving 3: A rubber ball with a radius *r=5 sm* fully immersed in a liquid surface is given an initial downward $\vartheta_0=20\frac{m}{s}$ velocity. How deep will this ball fall and how long will it take for it to come back up? The density of the rubber material is equal to $\rho_r=600\,\frac{kg}{m^3}$, and that of water is equal to $\rho_s=1000\,\frac{kg}{m^3}$, and the dynamic viscosity for water is equal to $\eta=1\,mPa\cdot s$. Consider the acceleration of free fall $g=9,8\,m/s^2$.

Solution: First of all, we determine the mass of the rubber ball.

$$m=\rho V=\frac{4}{3}\pi\rho r^3=\frac{4}{3}\cdot 3,14\cdot\left(600\frac{kg}{m^3}\right)\cdot\left(5\cdot 10^{-2}\,m\right)^3=0,314\,kg$$

We determine the Archimedean force

$$F_A=\rho_s Vg=\rho_s\frac{4}{3}\pi\rho r^3 g=1000\frac{kg}{m^3}\cdot\frac{4}{3}\cdot 3,14\cdot\left(5\cdot 10^{-2}\,m\right)^3\cdot 9,8\frac{m}{s^2}\approx 5,13\,N$$

α we determine the resistance coefficient

$$\alpha=6\pi\eta r=6\cdot 3,14\cdot\left(10^{-3}\,Pa\cdot s\right)\cdot\left(5\cdot 10^{-2}\,m\right)=9,42\cdot 10^{-4}\,\frac{kg}{s}$$

Since the Archimedean force is greater than the force of gravity, a rubber ball thrown downward slows down to a certain height and stops and rises again under the influence of Archimedean force. We use the formula (I.3) to determine the depth $\vartheta=0$ of immersion of this rubber ball. In this case, we call it the final speed.

$$s_{to'xt}=\frac{m}{\alpha}\cdot\left[\vartheta_0-\vartheta+\frac{mg-F_A}{\alpha}\cdot\ln\left|\frac{\alpha\vartheta_0+F_A-mg}{\alpha\vartheta+F_A-mg}\right|\right]=\frac{m}{\alpha}\cdot\left[\vartheta_0+\frac{mg-F_A}{\alpha}\cdot\ln\left|1+\frac{\alpha\vartheta_0}{F_A-mg}\right|\right]=$$

$$=\frac{0,314\,kg}{9,42\cdot 10^{-4}\,\frac{kg}{s}}\cdot\left[20\,\frac{m}{s}+\frac{0,314\,kg\cdot 9,8\,\frac{m}{s^2}-5,13\,N}{9,42\cdot 10^{-4}\,\frac{kg}{s}}\cdot\ln\left|1+\frac{9,42\cdot 10^{-4}\,\frac{kg}{s}\cdot 20\,\frac{m}{s}}{5,13\,N-0,314\,kg\cdot 9,8\,\frac{m}{s^2}}\right|\right]=$$

$$=333,333\ s\cdot\left[20\,\frac{m}{s}-2179,2\,\frac{m}{s}\cdot\ln\left(1,0091778\right)\right]=30,32\ m.$$

We divide the total movement time of the rubber ball thrown down by two. We determine the time it takes for the ball to slow down to stop and the time t_{up} it takes to rise up again.

$$t_{to'xt}=\frac{m}{\alpha}\cdot\ln\left|\frac{mg-(\alpha\vartheta_0+F_A)}{mg-(\alpha\vartheta+F_A)}\right|=\frac{m}{\alpha}\cdot\ln\left|1+\frac{\alpha\vartheta_0}{F_A-mg}\right|=\frac{0,314\,kg}{9,42\cdot 10^{-4}\,\frac{kg}{s}}\cdot$$

$$\cdot\ln\left|1+\frac{9,42\cdot 10^{-4}\,\frac{kg}{s}\cdot 20\,\frac{m}{s}}{5,13\,N-0,314\,kg\cdot 9,8\,\frac{m}{s^2}}\right|=333,333\ s\cdot\ln\left(1,0091778\right)=3,0453\ s.$$

The ball accelerates upward from its resting point. We determine the speed achieved when it jumps to the top

$$s=\frac{m}{\alpha}\cdot\left[\vartheta_0-\vartheta+\frac{F_A-mg}{\alpha}\cdot\ln\left|\frac{\alpha\vartheta_0+mg-F_A}{\alpha\vartheta+mg-F_A}\right|\right]=\frac{m}{\alpha}\cdot\left[\frac{F_A-mg}{\alpha}\cdot\ln\left|\frac{mg-F_A}{\alpha\vartheta+mg-F_A}\right|-\vartheta\right]$$

From the above equation, it is not possible to create a formula for determining speed ϑ. Because the speed comes both inside and outside the logarithm s=s(ϑ). To do this, we graph and then determine the value of speed $s=s_{to'xt}=30,32\ m$ that corresponds to a value equal ϑ. The graph σ=σ(θ) is depicted in Figure III.1.

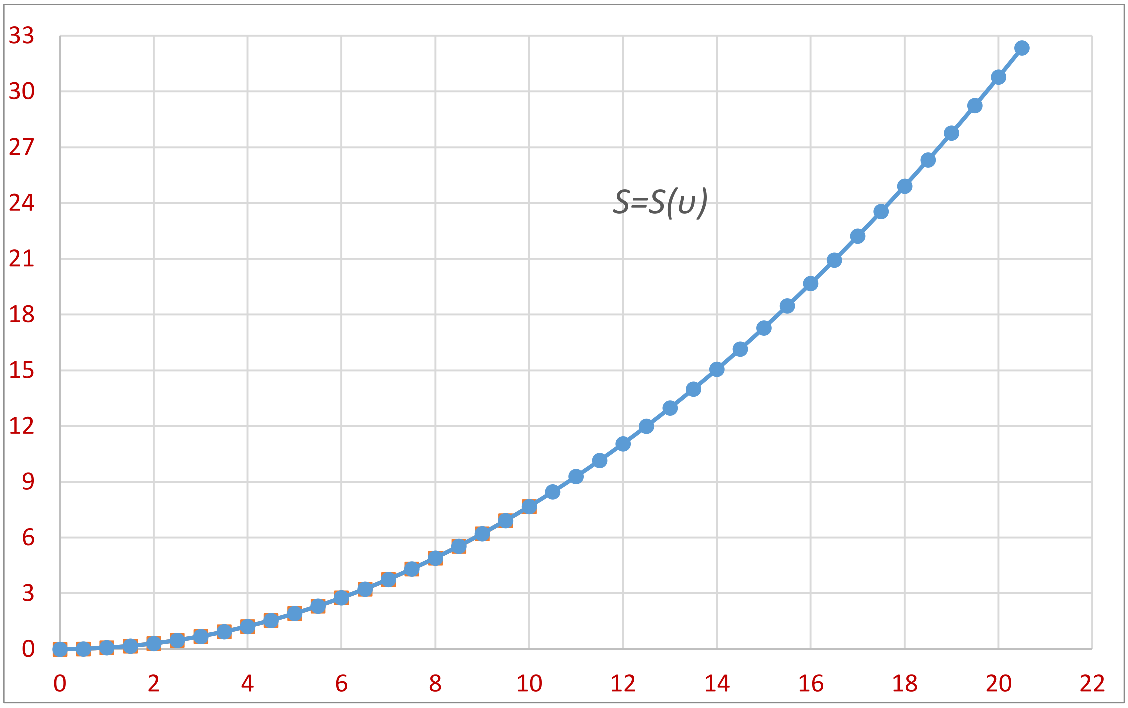
It can be seen from the picture that da will be equal to s=s(ϑ).

Now we determine the time it takes for the ball to reach the top. For this we use formula (II.1). Let's call it the initial speed.

$$t_{ko'tar}=\frac{m}{\alpha}\cdot\ln\left|\frac{\alpha\vartheta_0+mg-F_A}{\alpha\vartheta+mg-F_A}\right|=\frac{m}{\alpha}\cdot\ln\left|\frac{mg-F_A}{\alpha\vartheta+mg-F_A}\right|=\frac{0,314\,kg}{9,42\cdot 10^{-4}\,\frac{kg}{s}}\cdot$$

$$\cdot\ln\left|\frac{0,314\,kg\cdot 9,8\,\frac{m}{s^2}-5,13\,N}{9,42\cdot 10^{-4}\,\frac{kg}{s}\cdot 19,85\frac{m}{s}+0,314\,kg\cdot 9,8\,\frac{m}{s^2}-5,13\,N}\right|=$$

$$=333,333\ s\cdot\ln\left(1,00915061\right)=3,03633\ s$$



III.1-fig

Thus, we calculated all the requested sizes.

$$t_{umum} = t_{to'xt} + t_{ko'tar} = 3{,}0423 + 3{,}03633 = 6{,}07863\ s$$

The total time is the sum of the stop time and the rise time.

Problem solving 4: What is the law of acceleration during the rise of a ball of kerosene with a radius *r=1 cm*. What is the value of the maximum acceleration? After how much time t, the value of the acceleration decreases by a factor of e? The density of wax material is equal to, and that of kerosene is equal to $\rho_p = 200\ \dfrac{kg}{m^3}$, and the dynamic viscosity for water is equal to $\rho_k = 800\ \dfrac{kg}{m^3}$. Assume free fall acceleration $g = 9{,}8\ m/s^2$.

Solution: We use the formula (II.2) above to work out the acceleration. Let's call it the initial speed.

$$\vartheta = \left(\vartheta_0 - \frac{F_A - mg}{\alpha}\right)e^{-\frac{\alpha}{m}t} + \frac{F_A - mg}{\alpha} = \frac{F_A - mg}{\alpha}\left(1 - e^{-\frac{\alpha}{m}t}\right)$$

From the above formula, we get the 1st-order derivative with respect to time. In this case, the equation of change of acceleration over time is formed.

$$a = \frac{d\vartheta}{dt} = \frac{F_A - mg}{\alpha}\cdot\frac{d}{dt}\left(1 - e^{-\frac{\alpha}{m}t}\right) = \frac{mg - F_A}{\alpha}\cdot\frac{\alpha}{m}\cdot e^{-\frac{\alpha}{m}t} = \left(g - \frac{F_A}{m}\right)\cdot e^{-\frac{\alpha}{m}t}$$

So the acceleration is over time

$F_A = \rho_k V g = \dfrac{4}{3}\pi\rho_k g r^3$ for mass $m = \rho_p V = \dfrac{4}{3}\pi\rho_p r^3$ and for the drag coefficient $\alpha = 6\pi\eta r$

In the above equation, Archimedean force is for mass and for drag coefficient
if we use formulas, we will go to this view.

$$a=\left(g-\frac{F_A}{m}\right)\cdot e^{-\frac{\alpha}{m}t}=\left(g-\frac{\rho_k Vg}{\rho_p Vg}\right)\cdot e^{-\frac{6\pi\eta r}{\frac{4}{3}\pi\rho_p r^2}}=\frac{\rho_p-\rho_k}{\rho_p}g\cdot e^{-\frac{9\eta}{2\rho_p r^2}t}$$

So,

$$a(t)=\frac{\rho_p-\rho_k}{\rho_p}g\cdot e^{-\frac{9\eta}{2\rho_p r^2}t}$$

Maximun speed

$$a=\frac{\rho_p-\rho_k}{\rho_p}g\cdot e^{-\frac{9\eta}{2\rho_p r^2}\cdot 0}=\frac{\rho_p-\rho_k}{\rho_p}g=\frac{200\,\frac{kg}{m^3}-800\,\frac{kg}{m^3}}{200\,\frac{kg}{m^3}}\cdot 9{,}8\,\frac{m}{s^2}=-29{,}4\frac{m}{s^2}$$

In order for the acceleration to decrease by e times, the exponent in the equation *a=a(t)* must be equal to 1.

$$\frac{9\eta}{2\rho_p r^2}t=1,\ \rightarrow\ t=\frac{2\rho_p r^2}{9\eta}=\frac{2\cdot 200\frac{kg}{m^3}\cdot\left(10^{-2}m\right)^2}{9\cdot 1{,}5\cdot 10^{-3}\,Pa\cdot s}=2{,}963\ s$$

In this article, we have studied the effects of a resistive environment on a body sinking or rising in a liquid and the equations of motion and velocity in this article in order to give students a general understanding. We took the resistance of the liquid to be proportional to the 1st degree of speed.

When studying the motion of a body in a resistive medium, we consider the resistive force on the body and therefore the change in velocity of the body as a function of its velocity. Such drag forces are usually non-conservative and kinetic energy is usually converted into internal energy. Studying such a topic and solving related problems brings students closer to objective reality and develops the ability to imagine, as well as helps to develop skills in differential and integral calculations.

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