

## METHOD FOR MEASURING THE ELASTICITY OF MATERIALS

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<https://doi.org/10.5281/zenodo.8395090>

**Abstract.** *Currently, considerable attention is paid to improving the reliability of all categories of buildings and structures. Especially for the analysis of deformation and displacement field, the use of various methods is urgently required, one of which is digital holographic interferometry. For the practical application of this method, the authors have developed a technique for processing digital holographic interferograms. Digital holographic interferometers (DHI) used to measure the movements of diffuse-reflecting objects have two unique metrological properties: ultra-high sensitivity to movements (fractions of a nanometer) and the ability to perform measurements simultaneously over the entire surface of the object. Therefore, the main problem in assessing the accuracy of measurements is the lack of exemplary measuring instruments with higher or even similar characteristics.*

*To assess the accuracy of the measurement information in the laboratory of holographic research, the results of the analytical calculation of the elastic line (deflections) of a beam of constant rectangular cross section were used as a comparison base when it was loaded with pure bending. The test results of the technique showed the convergence of experimental and calculated data.*

**Keywords:** *digital holographic interferometry, modulus of elasticity, hologram, absolute value, breadboard beam.*

### Introduction

To date, new methods for obtaining interferometric data have been developed in the world - holographic interferometry. At the same time, much attention is paid to the study of transparent and reflective objects, including diffuse ones. In this area, one of the important tasks is to conduct targeted scientific research in the following areas: the development of ways to increase the sensitivity of holographic interferometry, the search for ways to compensate for distortions that occur at the stage of recording holograms, the development of digital methods of holographic interferometry, the development of optimal schemes for recording digital holograms that minimize technical and economic costs; application of digital holographic interferometry for measuring and analyzing fields of ultra-small displacements, deformations of materials and structures.

### Methods

On the basis of this method, a model beam design was made, the scheme of which is shown in Fig. 1. The beam was made of D-16 grade duralumin, which was attached to the massive steel plate of the optical table using steel corners. The photograph of the manufactured beam is shown in Fig.2. In Fig.3. the scheme of measurements is presented, using a model of a digital holographic interferometer, and in Fig.4. - layout beam loading scheme. As shown, holographic interferometry is one of the most promising non-contact methods for experimental study of the deformation properties of materials and analysis of the stress-strain state (SSS). Technology options based on the use of photographic materials have found a fairly wide application in experimental mechanics, their capabilities and limitations have been studied and described in detail [1,2]

The elastic line of the vertical part of the beam under pure bending loading is described by the well-known formula [3]:

$$Y = \frac{M}{2EJ} X^2 \quad (1)$$

Here  $X$  is the current coordinate of the beam surface point relative to the termination,  $Y$  is the deflection (normal displacement) of the surface of the vertical part of the beam at the point with the  $X$  coordinate (Fig. 1.),  $E$  is the modulus of elasticity of the beam material.

The moment of inertia of the beam section is determined based on the ratio:

$$J = \frac{bh^3}{12} \quad (2)$$

where  $b$  is the width of the beam;  $h$  is its thickness.

The bending moment is:

$$M = P \cdot L \quad (3)$$

where  $P$  is the magnitude of the force (load weight);  $L$  is the shoulder of force application.

Let us rewrite formula (1) by substituting the expressions of the input quantities.

$$Y = \frac{12PL}{2Ebh^3} X^2 \quad (4)$$

In order to compare the results of measurements performed using the CGI with the results of calculations for similar loading parameters of the beam, a series of experiments was carried out. At the same time, for the maximum approximation of the experimental conditions to the calculated ones, i.e. ideal, a precision exemplary beam was made. A technique was also developed and used, including statistical processing of measurement information and corrections.

The deflection value differential (the absolute error in calculating the displacement) depending on the measurement errors of the quantities included in expression (1) is calculated by the formula [4]:

$$\Delta Y = \frac{12PL}{2Ebh^3} X^2 \left( \frac{\Delta P}{P} + \frac{\Delta L}{L} + \frac{\Delta E}{E} + \frac{\Delta b}{b} + \frac{3\Delta h}{h} + \frac{2\Delta X}{X} \right) \quad (5)$$

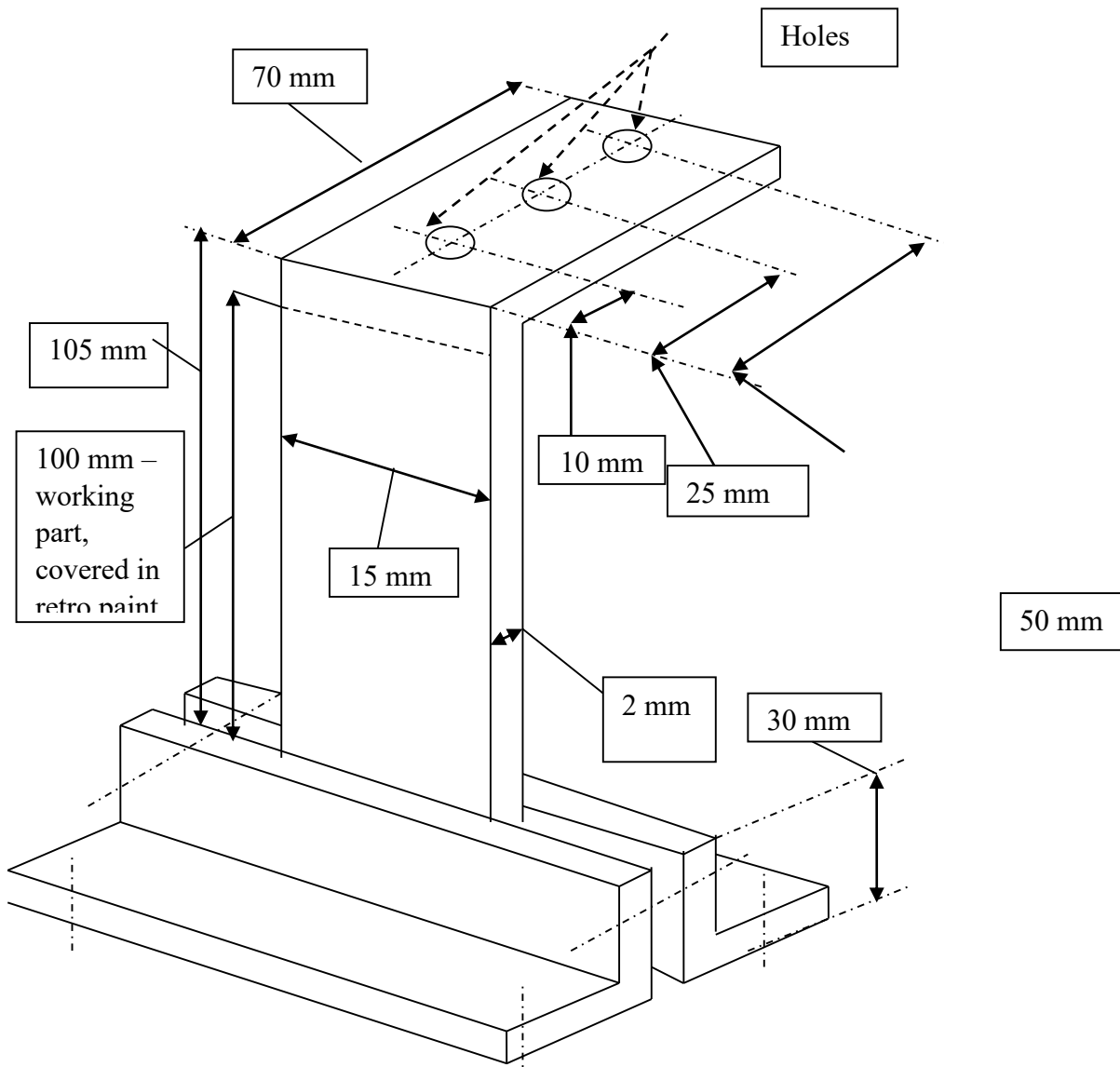
During measurements, the values of these parameters were:

$E = 0.695 \cdot 10^5$  MPa  $\sim 0.695 \cdot 10^{10}$  kg/m<sup>2</sup> - modulus of elasticity of a pressed strip of deformable aluminum alloy AMts (tabular data),  $b = 0,015$  m,  $h = 0,00214$  m,  $P = 0,001$  kg (standard weight),  $L = 0,06$  m,  $X_{\max} = 0,1$  m.

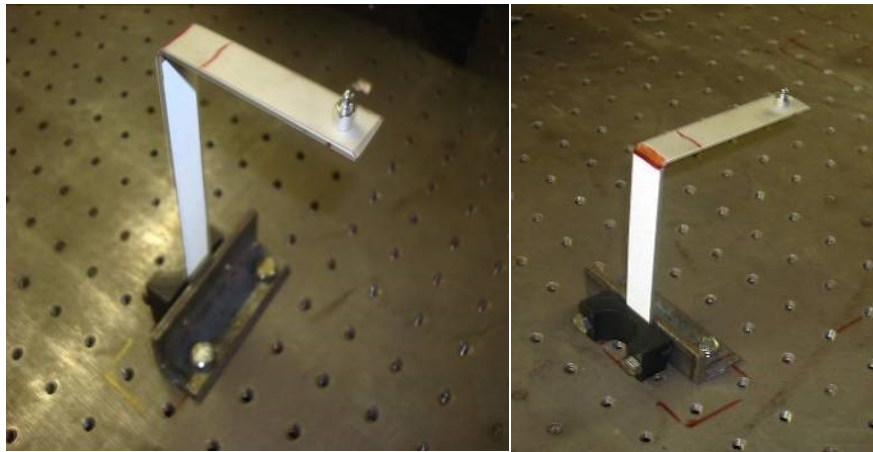
The value of the modulus of elasticity  $E$  is a reference value and is set with an accuracy of the second decimal place, thus we take  $\Delta E = 0.01 \cdot 10^{10}$  kg/m<sup>2</sup>.

$$\frac{\Delta E}{E} = 0.0144 \quad (6)$$

In the above estimate, the largest contribution to the total calculation error is made by the measurement error of the elastic modulus and beam thickness.



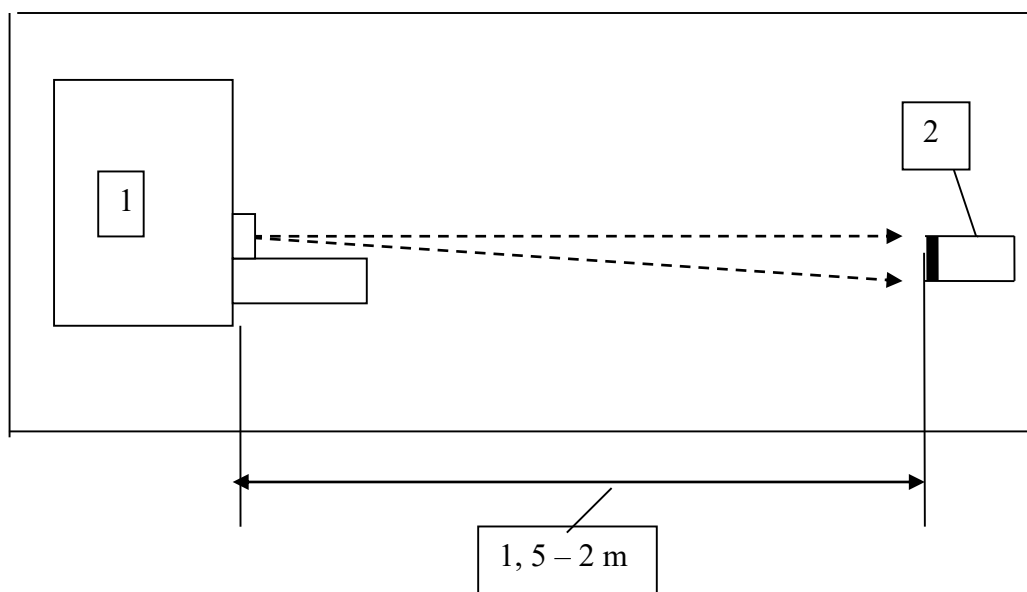
*Fig.1. Model beam diagram.*



a) Top view

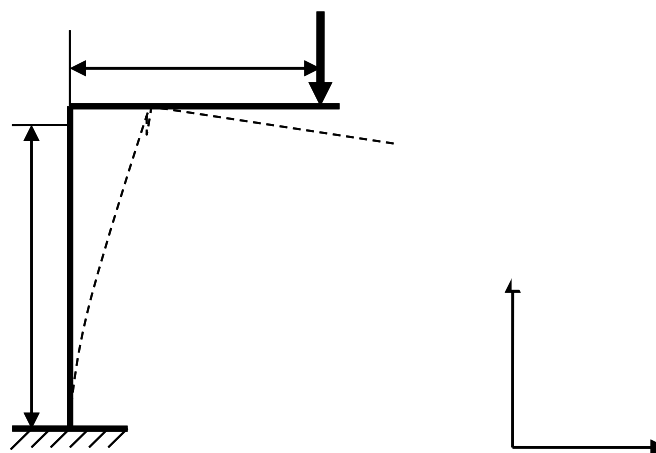
b) Rear view

**Fig.2. Photos of mock beam**



1 – scheme of DHI layout , 2 – model beam.

**Fig.3. Scheme of remote measurements of deformations model beam**



**Fig.4. Scheme of loading a model beam with a load P**

Experimental measurements of deformations were carried out according to the scheme shown in Fig.3. in the following way:

- 1 - a digital hologram of an unloaded model beam was taken.
- 2 - a digital hologram of the model beam was taken under load.
- 3 - a digital hologram of the model beam was taken after loading, i.e. hologram of a beam with residual deformation.
- 4 - with the help of the modified software, a 3D pattern of the displacement field was simulated under the action of a load in units of measurement "micron".

The holograms were taken using a 20mW He-Ne laser on a digital matrix with 1000x1000 pixels.

In Fig.5. digital holograms obtained as a result of one measurement cycle are shown. The arrows show which holograms were used to compare intermediate interferograms, and also shows the final result - a 3D displacement field of a loaded model beam.

To compare the experimental and calculated values, the displacement of the end of the beam under different loads was calculated and measured. The comparison results are shown in Fig.6. As can be seen from Fig.6., the measured and calculated data of the displacements of the model beam coincide up to the calculated measurement error.

Table 1 shows the measured values of displacements Y at the end of the working section of the beam (X=100 mm) in one series of measurements.

**Table 1**  
**Measured displacement values**

Measurement number	Measured value of beam end displacements, μm	Measurement number	Measured value of beam end displacements, μm
1	3,394	5	3, 397
2	3, 413	6	3,469
3	3, 462	7	3,456
4	3, 487	8	3,448

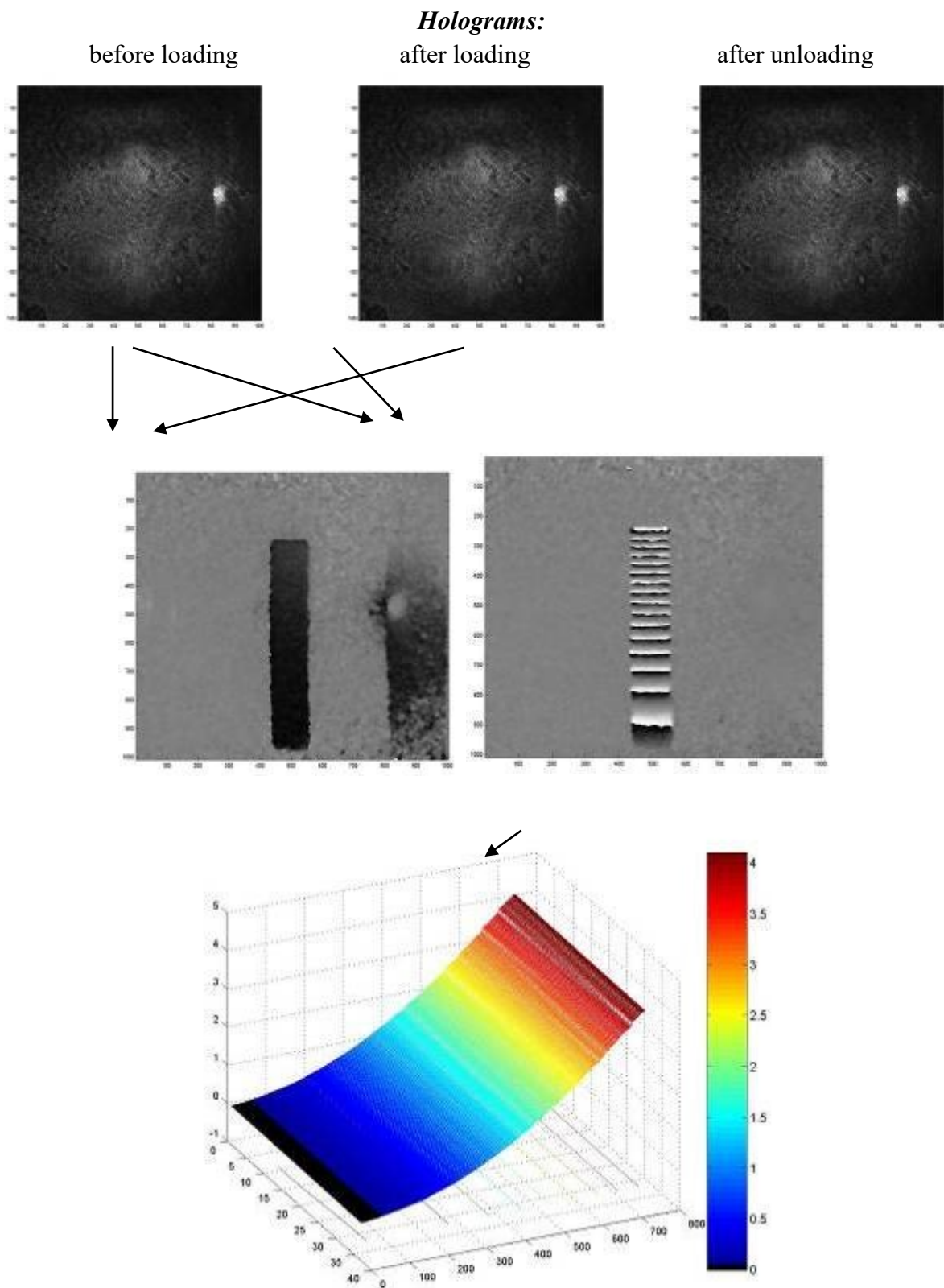
The average value of the measured displacements Y according to the formula:

$$Y_{cp} = \frac{\sum_{i=1}^8 Y_i}{8} = 3,44 \mu\text{m} \quad (7)$$

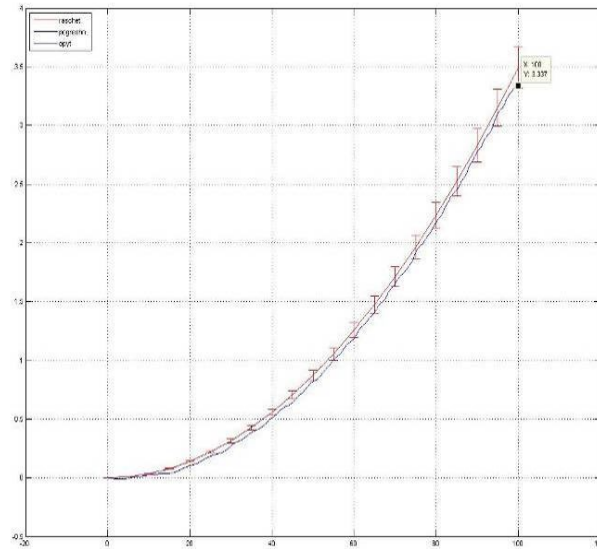
The absolute value of the maximum deviation of the measured values of displacements Y from the average value is calculated by the formula:

$$\Delta Y = \frac{Y_{\max} - Y_{\min}}{2} = (3,487 - 3,394) / 2 = 0,0465 \mu\text{m} \quad (8)$$

here  $Y_{\max}$ ,  $Y_{\min}$  - maximum and minimum displacement values from all measured.



*Fig.5. Model beam deflection measurement results*



**Fig.6. - Results of comparison of deflections measured with the CGI (lower curve) with the results of the calculation (upper curve - calculation with an error)**

### Results

The absolute value of the limiting relative deviation of the measured values from the average value of the measured displacements is calculated by the formula:

$$\Delta_{e\zeta i} = \frac{\Delta Y}{Y_{\bar{n}\delta}} = 0,0465 / 3,44 = 0,0135 = 1,35\% \quad (9)$$

Limit total relative error of displacement measurement

$$\Delta_{\max} = \pm(\Delta_{e\zeta i} + \Delta_{i\dot{a}\delta}) = \pm (1,35 + 5,21) = \pm 6,56\%$$

Here  $\pm 5,21\%$  - errors in the calculation of the elastic line of the beam.

### Conclusions

Thus, when carrying out real measurements of small displacements of points of a deformed object, a total relative error of 6.56% was achieved. It is shown that the developed technique can be used to measure the elastic constants, first of all, of "homogeneous" materials, such as metals and their alloys. For this, only a part from the material under study in the form of an "L" shaped beam is needed; also, based on a comparison with tabular data, you can find out from which "unknown" material the beam is made.

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