ADVANTAGES OF STUDYING MAXWELL'S EQUATIONS THROUGH THE LAWS OF ELECTROMAGNETISM

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Abstract. *Electromagnetism is one of the most difficult branches of physics. The studied phenomena in this section of physics are the basis of scientific and technological progress. Electric generators, electric motors, transformers and many other equipment operating on the basis of the laws of electromagnetism have long been firmly established in our everyday life. Consequently, the assimilation of this section by students is of paramount importance for understanding the principles of operation of these devices and equipment, as well as for expanding their horizons in general.*
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It is known that electromagnetism is one of the complex branches of physics. The studied phenomena in this section of physics form the basis of scientific and technological progress. Electric generators, electric motors, transformers and many other equipment based on the laws of electromagnetism have long been firmly established in our daily lives. Therefore, the assimilation of this section by students is of paramount importance for understanding the principles of operation of these devices and equipment, as well as for expanding their horizons in general.

It is known that there are textbooks that are good sources for students to study this section [1-8]. These sources, depending on the profession of future specialists, set out the basics of this section to the right and necessary extent for each profession.

At present, new pedagogical technologies and methods are the main tool of pedagogy and their use in teaching is considered as a guarantor of ensuring the quality of education. One of these approaches, in our opinion, is also testing students' knowledge and analyzing the fundamentals and laws of the Electromagnetism section from the point of view of Electrodynamics, more precisely, from the point of view of the Maxwell equations. From this point of view, in our opinion, the work [9] is a very good source. In this paper, the basics of the Electrodynamics section are presented in such a way that one can easily find a connection with the Electromagnetism section. Below we provide such an analysis of the foundations and laws of these sections.

As you know, Electrodynamics is the science of the electromagnetic field and its properties. And Maxwell's system of equations is the mathematical apparatus of Electrodynamics. The system of Maxwell's equations (or the system of equations of the electromagnetic field) for vacuum in differential form has the form:

$$\begin{cases} div\,\vec{E} = \frac{\rho}{\varepsilon_0} \\ rot\vec{E} = -\frac{\partial \vec{B}}{\partial t} \\ div\vec{B} = 0 \\ rot\vec{B} = \mu_0\vec{j} + \varepsilon_0\mu_0\frac{\partial \vec{E}}{\partial t}. \end{cases} \qquad (1)$$

Here, ε_0 - is the electrical constant, μ_0 - is the magnetic constant, is the volumetric charge density and is the current density. Here, the electric and magnetic constants, respectively, characterize the electrical and magnetic properties of vacuum and, therefore, this system of equations describes the electromagnetic field in vacuum, i.e. there are no parameters characterizing substances (environments). In this system of equations, each equation has its own physical meaning. To understand the essence of each equation in this system, it is necessary to reveal the meaning of such mathematical operators as gradient (grad), divergence (div) and rotor (rot). Note that these operators are also present in the Electromagnetism section $\vec{E} = -grad\varphi$. It is known that divergence from some field means the function of the source, and the rotor is the character of this field [10,11]. If the divergence from some field is not equal to zero, then this field has a source; the field lines of this field are closed, if the rotor is equal to zero, the field is not vortex, i.e. the lines of force of this field are not closed. Consequently, the system of Maxwell's equations (1) can be understood as follows: 1-equation read as follows: the source of the electric field is an electric charge; 2-equation: an alternating magnetic field generates an alternating electric field, which is also vortex; 3-equation: magnetic charges do not exist in nature; 4-equation: in addition to electric current, an alternating electric field also creates an alternating magnetic field, which is eddy. On the other hand, the right side of the 2-equation and the second term of the 4-equation also reflect the phenomenon of Faraday's electromagnetic induction, according to which alternating electric and magnetic fields induce each other. Thus, the explanation of the mathematical and physical meanings of the gradient, divergence and curl operators allows students to understand the essence of the equations in the Maxwell system of equations, which will help to comprehend the basics of this section as a whole, and also once again remember and consolidate the basics of the Electromagnetism section.

When presenting the system of Maxwell's equations, students, in our opinion, should pay attention to the following: First, the source of the electromagnetic field is a system of charged particles. And electric charge is the basic concept of Electrodynamics, i.e. charged particles or a system of charged particles create an electromagnetic field around them. And the nature of the electric charge itself is not yet fully understood. The electric charge is considered as an integral property of elementary particles - the electric charge exists together with the particle, but without the particle it does not exist by itself. And from the point of view of modern physics of elementary particles, the properties of elementary particles are described in 11-dimensional space. And the electric charge of hadrons is determined by the Gell-Mann-Nishijima relation [12,13]:

$$Q = T_3 + \frac{Y}{2} = T_3 + \frac{B+S}{2}, \quad (2)$$

here, T_3- the third isospin projection, Y – hypercharge, which is defined as the sum of the B-baryon charge, S -quantum number of strangeness. At present, the hypercharge has been expanded taking into account the newly discovered *c-, t-* and *b*-quarks $Y = B + S + C + B' + T$. Here C is the charm quantum number, $B' -$ beauty quantum number, T-quantum truth number. And these quantum numbers - $T_3 -$, $B -$, $S-, C-, B' -$ and $T -$ also called charge quantum numbers, which indicate the complex nature of the electric charge.

Secondly, magnetic charges, the existence of which was predicted by the outstanding English physicist P. Dirac in the 30s of the 20th century and named Dirac monopoles in his honor, have not yet been experimentally confirmed [14-16]. Dirac magnetic monopoles are assumed to have two signs, both positive and negative, similar to electric charges. If these monopoles are found experimentally, the third equation will change. In the system of Maxwell equations, if we use the stationarity condition. $\rho = const, \vec{j} = const$ и $\frac{\partial \vec{E}}{\partial t} = 0, \frac{\partial \vec{B}}{\partial t} = 0$, we obtain the system of equations of electrostatics (3) and magnetostatics (4) section Electromagnetism.

$$\begin{cases} div\vec{E} = \frac{\rho}{\varepsilon_0} \\ rot\vec{E} = 0 \end{cases} \quad (3)$$

$$\begin{cases} div\vec{B} = 0 \\ rot\vec{B} = \mu_0\vec{j} \end{cases} \quad (4)$$

It is known that the system of equations (3) describes an electrostatic field; field lines start at positive electric charges and end at negative ones, i.e.

$$rot\vec{E} = 0.$$

The system of equations (4) describes the magnetostatic field, this field remains vortex, i.e. the lines of the vector field are closed and have neither beginning nor end, therefore

$$rot\vec{B} \neq 0.$$

It is known that the system of Maxwell's equations is a system of 1-order differential equations, the solution of which is not easy from a practical point of view and often leads to cumbersome calculations. Therefore, it becomes necessary to give the system of Maxwell equations a different form in order to facilitate its solution. This is the reason for the need to introduce the potentials of the electromagnetic field $\vec{A}$ and φ. To introduce field potentials, one can use equation (1.3) of the system of Maxwell equations. From $div\vec{B} = 0$ (1.3) we can denote that $\vec{B} = rot\vec{A}$ (5). Because, from a mathematical point of view, for an arbitrary vector $\vec{a}$ $divrot\vec{a} = 0$ those. divergence from any curl of an arbitrary vector $\vec{a}$ is equal to zero and, therefore, relation (1.3) is satisfied. Here $\vec{A} = \vec{A}(\vec{r}, t)$ - is called the vector potential of the electromagnetic field. If we insert equation (5) into equation (1.2) of the Maxwell system of equations, we get the following expression

$$rot\vec{E} = -\frac{\partial}{\partial t} rot\vec{A} = -rot\frac{\partial \vec{A}}{\partial t}.$$

Or

$$rot\left(\vec{E} + \frac{\partial \vec{A}}{\partial t}\right) = 0.$$

The bracketed expression can be written as follows

$$\vec{E} + \frac{\partial \vec{A}}{\partial t} = -grad\varphi. \quad (6)$$

Because, the curl from any gradient of an arbitrary scalar function is equal to zero. Here $\boldsymbol{\varphi = \varphi(\vec{r}, t)}$ - is called the scalar potential of the electromagnetic field. From expression (6), the electric component of the electromagnetic field is expressed as follows

$$\vec{E} = -grad\varphi - \frac{\partial \vec{A}}{\partial t}. \quad (7)$$

Consequently, equations (1.3) and (1.2) of the system of Maxwell equations will take the following form

$$\vec{B} = rot\vec{A} \; \mathbf{(5)}$$

$$\vec{E} = -grad\varphi - \frac{\partial \vec{A}}{\partial t} (7)$$

The remaining equations of the system of Maxwell equations (equations (1.1) and (1.4)) with the help of equations (5) and (7) will take the following form

$$rot\vec{B} = rotrot\vec{A} = -\frac{1}{c^2}\frac{\partial^2 \vec{A}}{\partial t^2} - \frac{1}{c^2}grad\frac{\partial \varphi}{\partial t} + \mu_0 \vec{J} \qquad (8)$$

$$div\vec{E} = \Delta\varphi + \frac{\partial}{\partial t}div\vec{A} = -\frac{1}{\varepsilon_0}\rho \qquad (9)$$

Here, $\frac{1}{c^2} = \varepsilon_0\mu_0$. Using a well-known relation from mathematics $rotrot\vec{a} = graddiv\vec{a} - \Delta\vec{a}$ we get the following expression

$$rotrot\vec{A} = graddiv\vec{A} - \Delta\vec{A} = -\frac{1}{c^2}\frac{\partial^2 \vec{A}}{\partial t^2} - \frac{1}{c^2}grad\frac{\partial \varphi}{\partial t} + \mu_0 \vec{J}.$$

Or

$$\Delta\vec{A} - \frac{1}{c^2}\frac{\partial^2 \vec{A}}{\partial t^2} = -\mu_0\vec{J} + grad\left(div\vec{A} + \frac{1}{c^2}grad\frac{\partial \varphi}{\partial t}\right).$$

The last expression can be simplified if the expression in the parenthesis is set to zero

$$div\vec{A} + \frac{1}{c^2}grad\frac{\partial \varphi}{\partial t} = 0 \qquad (10)$$

Expression (10) is called the Lorentz gauge. From this expression we get

$$\frac{\partial}{\partial t}div\vec{A} = -\frac{1}{c^2}grad\frac{\partial^2 \varphi}{\partial t^2}.$$

Using this expression, equations (8) and (9) will take the following form

$$\begin{cases} \Delta\vec{A} - \frac{1}{c^2}\frac{\partial^2 \vec{A}}{\partial t^2} = -\mu_0\vec{J} \\ \Delta\varphi - \frac{1}{c^2}\frac{\partial^2 \varphi}{\partial t^2} = -\frac{1}{\varepsilon_0}\rho \end{cases} \qquad (11)$$

By solving the system of equations (11) it is possible to determine the potentials of the electromagnetic field $\vec{A}$ and φ. And with the help of equations (5) and (7) it is possible to determine the components of the electromagnetic field $\vec{E}$ and $\vec{B}$. The system of equations (11) is called the system of Maxwell's equations in the language of potentials or the system of electromagnetic field equations in the language of potentials for vacuum. This system of equations is a system of 2-order differential equations that are well studied in mathematics and are easily solved. This is precisely the meaning of introducing the potentials of the electromagnetic field and giving the system of Maxwell's equations the form of differential equations of the 2nd order. $\vec{A}$ and φ are auxiliary parameters, and the components of the electromagnetic field $\vec{E}$ and $\vec{B}$ are physical parameters. Here we should also reveal the meaning of the Lorentz gauge. The electromagnetic field must have four components, and only 2 components are observed, the electric and magnetic components - $\vec{E}$, $\vec{B}$. And the other two components are not observed. And according to the Lorentz calibration, two of the four components are excluded, and only the observed components remain - $\vec{E}$, $\vec{B}$. And the relationship between the potentials of the electromagnetic field, $\vec{A}$, φ and the components of the electromagnetic field $\vec{E}$, $\vec{B}$ are determined by relations (5) and (7).

And under the condition of stationarity, the system of equations (11) will take the following form

$$\begin{cases} \Delta\vec{A} = -\mu_0\vec{J} \\ \Delta\varphi = -\frac{1}{\varepsilon_0}\rho \end{cases} \qquad (12)$$

Equation (7) takes the following form

$$\vec{E} = -grad\varphi \quad (13)$$

It can be seen that equation (13) is a well-known relationship between the electric field strength $\vec{E}$ and the scalar φ potential of the Electromagnetism section.

If in the system of Maxwell equations we use the conditions $\rho = 0$, $\vec{j} = 0$, i.e. electromagnetic field in the absence of a source, then we obtain a system of equations of electromagnetic waves.

$$\begin{cases} div\,\vec{E} = 0 \\ rot\vec{E} = -\frac{\partial\vec{B}}{\partial t} \\ div\vec{B} = 0 \\ rot\vec{B} = \varepsilon_0\mu_0\frac{\partial\vec{E}}{\partial t} \end{cases} \quad (14)$$

This means that a system of charged particles generates an electromagnetic field and this field, leaving its source, can exist in the form of electromagnetic waves and propagate at the speed of light. According to (14.2) and (14.4), the electric and magnetic components of the electromagnetic field will create each other and propagate in the form of electromagnetic waves at the speed of light. Here you can also explain the difference between the concepts of "field" and "wave", which is of no small importance for understanding the essence of these concepts and the nature of waves and fields by students and expanding their horizons in general.

If in the system of Maxwell equations for vacuum we make the following substitutions

$$\varepsilon_0 \to \varepsilon_0\varepsilon, \ \mu_0 \to \mu_0\mu$$

then we get the system of Maxwell equations for the substance (environment)

$$\begin{cases} div\,\vec{D} = \rho \\ rot\vec{E} = -\frac{\partial\vec{B}}{\partial t} \\ div\vec{B} = 0 \\ rot\vec{H} = \vec{j} + \frac{\partial\vec{D}}{\partial t} \end{cases} \quad (15)$$

and the following known relations between the quantities $\vec{E}$ and $\vec{D}$, $\vec{B}$ and $\vec{H}$

$$\vec{D} = \varepsilon_0\varepsilon\vec{E},$$
$$\vec{H} = \frac{1}{\mu_0\mu}\vec{B}.$$

where, respectively, ε is the permittivity and μ is the magnetic susceptibility of the substance (medium). From these relations, we can conclude that $\vec{E}$ and $\vec{B}$ characterize, respectively, the electric and magnetic fields in vacuum, $\vec{D}$ and $\vec{H}$characterize, respectively, the electric and magnetic fields in matter (medium).

From the system of Maxwell's equations, you can also get the relationship between c- the speed of light and the electrodynamic parameters - ε_0, μ_0 (i.e. connection with the Optics section)

$$\frac{1}{c^2} = \varepsilon_0\,\mu_0\ .$$

As well as an expression for the speed of light in a substance (medium), which has the form

$$c' = \frac{c}{\sqrt{\varepsilon \mu}} ,$$

Where, $\sqrt{\varepsilon\mu} = n -$ refractive index of a substance (medium).

Thus, the use of the mathematical apparatus of Electrodynamics - the system of Maxwell's equations is a good tool for consolidating knowledge not only in electromagnetism, but also for ensuring the qualitative development of the very section of Electrodynamics by students. According to the curriculum, students study the Electrodynamics section after studying the Electromagnetism section. And this circumstance allows students to once again recall the basics of the course and analyze the processes of electromagnetism from the point of view of the section of Electrodynamics, from the point of view of the system of Maxwell's equations. On the other hand, the topic “The phenomenon of electromagnetic induction” is the most complex and difficult to master for students. Therefore, it is desirable to repeat this theme again in different manifestations of this phenomenon, i.e. including also mutual- and self-induction. Here we have given only some points of such an analysis, but in fact, in practice, it is possible to bring the correspondence between these sections much wider according to their curricula.

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