SPATIAL PROBLEM OF INTERACTION P (LONGITUDINAL) - WAVES ON A CYLINDRICAL CAVITY IN AN ELASTIC MEDIUM

[1]N.U. Kuldashov, [2]M. Choriev, [3]U.A.Urolov
[1,2,3] Tashkent Institute of Chemical Technology
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Abstract. *The spatial problem of diffraction of elastic waves, the angle of incidence relative to the longitudinal axis of the cavity, is considered. The aim of the study is the diffraction of elastic harmonic waves in a cavity depending on the angle of incidence. An analytical solution of the problem of diffraction of elastic waves in a cavity is obtained, which makes it possible to determine the force factors of the stress-strain state of the cavity from the angle of incidence. The environment is described by the Helmholtz equation and the solution of which is described by the Bessel and Neumann functions with complex arguments. The stress and permeability of the cavity point take on a maximum value in the region of long waves. A technique and algorithm for solving the spatial problem of diffraction of hormone waves in a cavity has been developed.*
Keywords: *cavity, wave, wave equation, wave incidence angle, Bessel and Neumann function.*

1. Introduction. Many scientists dealt with the problems of diffraction of plane harmonic waves in homogeneous isotropic elastic bodies [1,2,3]. When elastic waves interact with inhomogeneities of the medium or with obstacles, the wave field changes to some extent. Due to the interaction of the wave with the interface, reflected and transmitted plane waves appear, depending on the angle of incidence. The incident elastic wave is considered to be flat, the angle of incidence of the wave front relative to the longitudinal axis is considered in [5,7]. The diffraction of elastic waves in inhomogeneous cylindrical elastic bodies placed in a deformable medium is the subject of works [4, 8].

The papers [8, 9] consider the problem of elastic wave diffraction in a viscoelastic cylindrical body. At the contact of the cylindrical body with the medium, the conditions of rigid contact are set. At infinity, the Somerfeld radiation conditions are set.

Scattering of elastic waves in an ideal infinite circular cavity is the simplest diffraction problem with an exact solution. In addition, problems that have exact solutions are the problems of scattering of acoustic waves in a sphere, an ellipsoid, and in other bodies whose surfaces are the coordinate surfaces of the corresponding curvilinear coordinate systems.

In this paper, we consider the diffraction of elastic harmonic waves in a cavity, which depends on the angle of incidence. Numerical results are obtained

2. Method

2.1. Problem statement and solution technique

In a cylindrical coordinate r, θ, z system, the propagation of longitudinal P (incident) and PP (reflected), transverse PS (reflected along r, θ) and PZ (reflected along z) waves is considered. The design scheme is shown in (ras1)

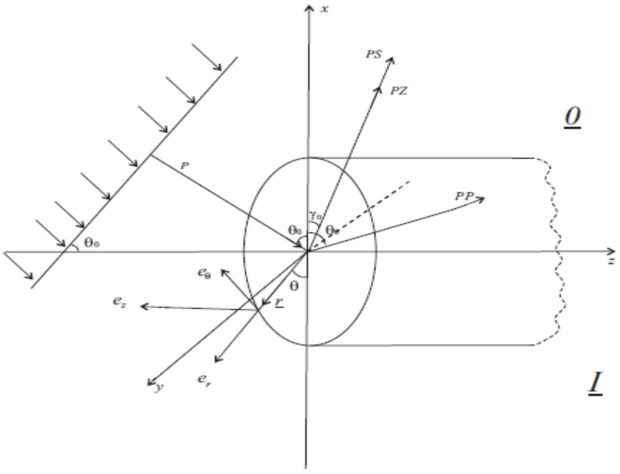

Fig.1.

When the longitudinal P waves fall. The linear equation of motion of the medium is determined by a system of differential equations in partial derivatives, expressed in terms of displacements, by equations (Lame equations) [10]:

$$\mu\nabla^2\vec{u} + (\lambda + 2\mu)\, graddi\vartheta\vec{u} = \rho\frac{\partial^2\vec{u}}{\partial t^2} \; ; \quad (1)$$

$$\lambda = \frac{vE}{(1+v)(1-2v\ )}; \quad \mu = \frac{vE}{2(1+v\ )}; \quad (2)$$

Here $\vec{u}(u_x, u_y, u_z)$ is the displacement vector of the medium points; ρ- medium density; v - Poisson's ratio; , λ , μ - Lame coefficient.

We represent the medium displacement vector as a potential

$$u = grad\varphi + rot\vec{\psi}, \vec{\psi}(\psi_r, \psi_\theta, \psi_z) \quad (3)$$

(3) substituting (1) we obtained the following differential of the medium equation by the Helmholtz equations [9]:

$$\nabla^2\varphi - \frac{1}{c_p^2}\frac{\partial^2\varphi}{\partial t^2} = 0;$$

$$\nabla^2\psi_Z - \frac{1}{c_S^2}\frac{\partial^2\psi_Z}{\partial t^2} = 0;$$

$$\nabla^2\psi_\theta - \frac{\psi_\theta}{r^2} + \frac{2}{r^2}\frac{\partial\psi_r}{\partial\theta} - \frac{1}{c_s^2}\frac{\partial^2\psi_\theta}{\partial t^2} = 0; \quad (4)$$

$$\nabla^2\psi_r - \frac{\psi_r}{r^2} + \frac{2}{r^2}\frac{\partial\psi_\theta}{\partial\theta} - \frac{1}{c_s^2}\frac{\partial^2\psi_r}{\partial t^2} = 0;$$

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2} + \frac{\partial^2}{\partial z^2}$$

C_p, C_s - longitudinal and transverse velocity of wave propagation in the medium.

The boundary condition is set on the free surface of the cavity

$$G_{rr}^{(p)}+G_{rr}^{(pp)}=G_{r\theta}^{(p)}+G_{r\theta}^{(pp)}=G_{rz}^{(p)}+G_{rz}^{(pp)}=0\ ; \qquad (5)$$

Solutions to equation (4) are sought in the form

$$\varphi_r=e^{i(n\theta-\frac{w\cos\theta_0}{C_{P_0}}z-wt)}\varphi(r)$$
$$\psi_z=e^{i(n\theta+\frac{w\cos\theta_0}{C_{P_0}}z-wt)}\psi(r) \qquad (6)$$
$$\begin{pmatrix}\psi_\theta\\ \psi_r\end{pmatrix}=e^{i(n\theta+\frac{w\cos\theta_0}{C_{P_0}}z-wt)}\psi(r)$$

(6) we substitute into (4), as a result we obtain the equations of the medium in the form of the Bessel equation [6]:

$$\frac{\partial^2\varphi(r)}{\partial r^2}+\frac{1}{r}\frac{\partial\varphi(r)}{\partial r}+\left(\frac{\omega^2}{c_p^2}(1-\cos^2\theta_0)-\frac{n^2}{r^2}\right)\varphi(r)=0$$
$$\frac{\partial^2\psi_z(r)}{\partial r^2}+\frac{1}{r}\frac{\partial\psi_z(r)}{\partial r}+\left(\frac{\omega^2}{c_s^2}(1-\cos^2\gamma_0)-\frac{n^2}{r^2}\right)\psi_z(r)=0 \qquad (7)$$
$$\frac{\partial^2\psi_{r,\theta}(r)}{\partial r^2}+\frac{1}{r}\frac{\partial\psi_{r,\theta}(r)}{\partial r}+\left(\frac{\omega^2}{c_s^2}(1-\cos^2\gamma_0)-\frac{1}{r^2}(n^2+1)\right)\psi_{r,\theta}(r)=0$$

Where

$$\alpha=\left(\frac{\omega^2}{c_p^2}(1-\cos^2\theta_0)-\frac{n^2}{r^2}\right)$$
$$\beta=\left(\frac{\omega^2}{c_s^2}(1-\cos^2\gamma_0)-\frac{n^2}{r^2}\right) \qquad (8)$$
$$\Omega=\left(\frac{\omega^2}{c_s^2}(1-\cos^2\gamma_0)-\frac{1}{r^2}(n^2+1)\right)$$

Corresponding wave numbers of longitudinal and transverse waves.

The general solution (7) in potentials has the following form.

$$\varphi^{(p)}=\varphi_0\sin\theta_0\cdot\sum_{n=0}^{\infty}\ell^{i(n\theta-\frac{\omega\cos\theta_0}{c_{p0}}z-\omega t)}J_n(\alpha r)\ ;$$
$$\varphi^{(pp)}=A\sin\theta_0\cdot\sum_{n=0}^{\infty}\ell^{i(n\theta+\frac{\omega\cos\theta_0}{c_{p0}}z-\omega t)}H_n^{(2)}(\alpha r)\quad ; \qquad (9)$$
$$\begin{pmatrix}\psi^{(ps)}\\ \psi^{(pz)}\end{pmatrix}=\begin{pmatrix}B\\ C\end{pmatrix}\cos\gamma_0\sum_{n=0}^{\infty}\ell^{i(n\theta+\frac{\omega\cos\gamma_0}{c_{s0}}z-\omega t)}\begin{pmatrix}H_n^{(2)}(\Omega r)\\ H_n^{(2)}(\beta r)\end{pmatrix}\ ;$$

$\varphi^{(p)}$- incident expansion wave, $\varphi^{(pp)}$- reflected expansion wave, $\psi^{(ps)},\psi^{(pz)}$- reflected shear wave along the Z, θ, r axis;

φ_0 - amplitude of longitudinal incident waves; θ_0, γ_0 - angle of incident and reflected waves; $J_n(\alpha r), H_n^{(2)}(\alpha r), H_n^{(2)}(\Omega r), H_n^{(2)}(\beta r)$ - Bessel and Hankel function of the second kind of the n-th order; A, B, C - unknown coefficients which is determined from the boundary conditions (5). When the value of the argument (7) increases more than Pi, then the asymptotic value is used to calculate the value of the Bessel and Hankel function. At infinity, the Sommerfeld radiation conditions for the potentials of reflected waves must be satisfied [8].

Voltage vector.

$$\frac{1}{2\mu}G_{rr} = \frac{1-\nu}{1-2\nu}\frac{\partial^2\varphi}{\partial r^2} + \frac{\nu}{r^2(1-2\nu)}\frac{\partial^2\varphi}{\partial\theta^2} + \frac{\nu}{r(1-2\nu)}\frac{\partial\varphi}{\partial r} + \\ + \frac{\nu}{1-2\nu}\frac{\partial^2\varphi}{\partial z^2} + \frac{1}{r}\frac{\partial^2\psi_z}{\partial r\partial\theta} - \frac{1}{r^2}\frac{\partial\psi_z}{\partial\theta} - \frac{\partial^2\psi_\theta}{\partial r\partial z}$$

$$\frac{1}{2\mu}G_{r\theta} = \frac{2}{r}\frac{\partial^2\varphi}{\partial r\,\partial\theta} - \frac{2}{r^2}\frac{\partial\varphi}{\partial\theta} + \frac{1}{r^2}\frac{\partial^2\psi_z}{\partial\theta^2} - \frac{\partial^2\psi_z}{\partial r^2} + \frac{1}{r}\frac{\partial\psi_z}{\partial r} + \frac{1}{r}\frac{\partial^2\psi_\theta}{\partial\theta\,\partial z} \\ \frac{\partial^2\psi_r}{\partial r\,\partial z} - \frac{1}{r}\frac{\partial\psi_r}{\partial r}; \qquad (10)$$

$$\frac{1}{2\mu}G_{rz} = \frac{\nu}{1-2\nu}\frac{\partial^2\varphi}{\partial r^2} + \frac{\nu}{r^2(1-2\nu)}\frac{\partial^2\varphi}{\partial\theta^2} + \frac{\nu}{r(1-2\nu)}\frac{\partial\varphi}{\partial r} + \frac{\partial^2\psi_\theta}{\partial r\partial z} \\ + \frac{1}{r}\frac{\partial\psi_\theta}{\partial z} - \frac{1}{r}\frac{\partial^2\psi_r}{\partial\theta\partial z};$$

$$\frac{1}{2\mu}G_{\theta\theta} = \frac{\nu}{1-2\nu}\frac{\partial^2\varphi}{\partial r^2} + \frac{1-\nu}{r^2(1-2\nu)}\frac{\partial^2\varphi}{\partial\theta^2} + \frac{1-\nu}{r(1-2\nu)}\frac{\partial\varphi}{\partial r} + \\ + \frac{\nu}{1-2\nu}\frac{\partial^2\varphi}{\partial z^2} - \frac{1}{r}\frac{\partial^2\psi_z}{\partial r\partial\theta} + \frac{1}{r^2}\frac{\partial\psi_z}{\partial\theta} - \frac{1}{r}\frac{\partial\psi_\theta}{\partial z} - \frac{1}{r}\frac{\partial^2\psi_r}{\partial z\partial\theta};$$

Since the spatial problem is considered, the angle of incidence, reflection of waves according to the Fermat, Hugenz and Snell's laws is the law of geolatric seismology. Can be generalized to rays vertically inhomogeneous media [5,7]

$$\frac{\sin\theta_0}{Cp_0} = \frac{\sin\gamma_0}{Cs_0}; \qquad \gamma_0 = \arcsin\left(\frac{Cs_0}{Cp_0}\sin\theta_0\right);$$

Substituting (9) into (10) we obtain the value

$$\frac{1}{2\mu}G_{rr}^{(p)} = \varphi_0\sin\theta_0\left(\frac{1-\nu}{1-2\nu}\alpha^2 J_n^{//}(\alpha\, r)\right) + \frac{\nu(-n^2)}{r^2(1-2\nu)}J_n(\alpha\, r) + \\ + \frac{\nu\alpha}{r(1-2\nu)}J_n^{/}(\alpha r) - \frac{\nu}{1-2\nu}\frac{\omega^2\cos^2\theta_0}{c_{p_0}^2}J_n(\alpha r)$$

$$\frac{1}{2\mu}G_{rr}^{(pp)} = A\cdot\sin\theta_0\left(\begin{array}{l}\frac{1-\nu}{1-2\nu}\alpha^2\cdot H_n^{(2)//}(\alpha r)+\frac{\nu(-n^2)}{r^2(1-2\nu)}H_n^{(2)}(\alpha r)+\\ \frac{\nu\alpha}{r(1-2\nu)}H_n^{(2)/}(\alpha r)-\frac{\nu}{(1-2\nu)}\frac{\omega^2\cos^2\theta_0}{c_{p_0}^2}H_n^{(2)}(\alpha r)\end{array}\right)-$$

$$-B\cos^2 r_0\cdot\frac{i\omega}{c_{so}}\cdot\Omega\cdot H_n^{(2)}(\Omega r)+c\cdot\cos\gamma_0\left(\frac{1}{r}\cdot i\cdot n\cdot\beta\cdot H_n^{(2)}(\beta r)-\frac{1}{r}\cdot i\cdot n\cdot H_n^{(2)}(\beta r)\right)$$

$$\frac{1}{2\mu}G_{r\theta}^{(p)} = \alpha_0\sin\theta_0\left(\frac{1}{r}\cdot i\cdot n\cdot\alpha\cdot J_n^{/}(\alpha r)-\frac{1}{r^2}\cdot i\cdot n\cdot\cdot J_n(\alpha r)\right) \quad (11)$$

$$\frac{1}{2\mu}G_{r\theta}^{(pp)} = A\cdot\sin\theta_0\left(\frac{2}{r}\cdot i\cdot n\cdot\alpha\cdot H_n^{(2)/}(\alpha r)-\frac{2}{r^2}\cdot i\cdot n\cdot H_n^{(2)}(\alpha r)\right)-$$

$$-B\cdot\cos^2 r_0\cdot n\cdot\frac{\omega}{c_{so}}\cdot H_n^{(2)}(\Omega r)+C\cdot\cos r_0\cdot$$

$$\cdot\left(\frac{1}{r^2}\cdot(-n)\cdot H_n^{(2)}(\beta r)-\rho^2\cdot H_n^{(2)}(\beta r)+\frac{1}{r}\cdot\beta\cdot H_n^{(2)/}(\beta r)\right)$$

$$\frac{1}{2\mu}G_{rz}^{(p)} = \varphi_0\sin\theta_0\left(\frac{\nu}{1-2\nu}\cdot\alpha^2\cdot J_n^{//}(\alpha r)+\right.$$

$$\left.+\frac{\nu}{r^2(1-2\nu)}\cdot(-n^2)\cdot J_n(\alpha r)+\frac{\nu}{r(1-2\nu)}\cdot\alpha\cdot J_n^{/}(\alpha r)\right)$$

$$\frac{1}{2\mu}G_{rz}^{(pp)} = A\cdot\sin\theta_0\left(\frac{1-\nu}{1-2\nu}\alpha^2\cdot H_n^{(2)//}(\alpha r)+\frac{\nu}{r^2(1-2\nu)}\cdot(-n^2)\cdot\right.$$

$$\left.\cdot H_n^{(2)}(\alpha r)+\frac{\nu}{r(1-2\nu)}\cdot\alpha\cdot H_n^{(2)//}(\alpha r)\right)+$$

$$+B\cdot\cos\gamma_0\cdot\left(\frac{i\omega\cos\gamma_0}{c_{so}}\cdot\Omega\cdot H_n^{(2)/}(\Omega r)+\frac{1}{r}\cdot\frac{i\omega\cos\gamma_0}{c_{so}}\cdot H_n^{(2)}(\Omega r)\right)$$

$$\frac{1}{2\mu}G_{\theta\theta}^{(P)} = \varphi_0\sin\theta_0\left(\frac{\nu}{1-2\nu}\cdot\alpha^2\cdot J_n^{''}(\alpha r)+\frac{1-\nu}{r^2(1-2\nu)}\cdot(-n^2)\cdot J_n(\alpha r)+\right.$$

$$\left.+\frac{\nu}{r(1-2\nu)}\cdot\alpha\cdot J_n^{/}(\alpha r)+\frac{1-\nu}{(1-2\nu)}\cdot\left(\frac{-\omega^2\cdot\cos^2\theta_0}{c_{p_0}^2}\right)\cdot J_n(\alpha r)\right) \quad (12)$$

$$\frac{1}{2\mu}G_{\theta\theta}^{(pp)} = A\cdot\sin\theta_0\left(\begin{array}{l}\frac{1-\nu}{1-2\nu}\alpha^2\cdot H_n^{(2)//}(\alpha r)+\frac{1-\nu}{r^2(1-2\nu)}\cdot(-n^2)\cdot H_n^{(2)}(\alpha r)+\\ +\frac{1-\nu}{r^2(1-2\nu)}\cdot\alpha\cdot H_n^{(2)/}(\alpha r)+\frac{\nu}{1-2\nu}\cdot\left(-\frac{\omega^2\cos^2\gamma_0}{c_{so}^2}\right)\cdot H_n^{(2)}(\Omega r)\end{array}\right)-$$

$$-B\cdot\cos^2\gamma_0\cdot\frac{1}{r}\cdot\frac{i\omega}{c_{so}}\cdot H_n^{(2)}(\Omega r)+C\cdot\cos\gamma_0\cdot\left(-\frac{1}{r}\cdot i\cdot n\cdot\beta\cdot H_n^{(2)/}(\beta r)\right)$$

Substituting stress (11) into boundary conditions (5), we obtain a system of complex algebraic equations [3x3] in the form

$$[C](ABC)^{\mathrm{T}} = [P]\ ; \qquad (13)$$

[C] is a quadratic complex matrix whose elements have the following form:

$$C_{11} = \sin\theta_0\left(\begin{array}{l}\frac{1-\nu}{1-2\nu}\alpha^2\cdot H_n^{(2)//}(\alpha r)+\frac{\nu(-n^2)}{r^2(1-2\nu)}H_n^{(2)}(\alpha r)+\frac{\nu\alpha}{r(1-2\nu)}H_n^{(2)/}(\alpha r)-\\ -\frac{\nu}{(1-2\nu)}\frac{\omega^2\cos^2\theta_0}{c_{p_0}^2}H_n^{(2)}(\alpha r)\end{array}\right)$$

$$C_{12} = \cos^2 r_0\cdot\frac{i\omega}{c_{so}}\cdot\Omega\cdot H_n^{(2)/}(\Omega r);$$

$$C_{13} = \cos\gamma_0\left(\frac{1}{r}\cdot i\cdot n\cdot\beta\cdot H_n^{(2)/}(\beta r)-\frac{1}{r}\cdot i\cdot n\cdot H_n^{(2)}(\beta r)\right);$$

$$C_{21} = \sin\theta_0\left(\frac{2}{r}\cdot i\cdot n\cdot\alpha\cdot H_n^{(2)/}(\alpha r)-\frac{2}{r^2}\cdot i\cdot n\cdot H_n^{(2)}(\alpha r)\right);$$

$$C_{22} = \cos^2 r_0\cdot n\cdot\frac{\omega}{c_{so}}\cdot H_n^{(2)}(\Omega r);$$

$$C_{23} = \cos\gamma_0\cdot\left(\frac{1}{r^2}\cdot(-n)\cdot H_n^{(2)}(\beta r)-\rho^2\cdot H_n^{(2)//}(\beta r)+\frac{1}{r}\cdot\beta\cdot H_n^{(2)/}(\beta r)\right);$$

$$C_{31} = \sin\theta_0\left(\begin{array}{l}\frac{1-\nu}{1-2\nu}\alpha^2\cdot H_n^{(2)//}(\alpha r)+\frac{\nu}{r^2(1-2\nu)}\cdot(-n^2)\cdot H_n^{(2)}(\alpha r)+\\ +\frac{\nu}{r(1-2\nu)}\cdot\alpha\cdot H_n^{(2)/}(\alpha r)\end{array}\right);$$

$$C_{32} = \cos\gamma_0\cdot\left(\frac{i\omega\cos\gamma_0}{c_{so}}\cdot\Omega\cdot H_n^{(2)/}(\Omega r)+\frac{1}{r}\cdot\frac{i\omega\cos\gamma_0}{c_{so}}\cdot H_n^{(2)}(\Omega r)\right);\ C_{33} = 0;$$

[P] – Vector column of external loads

$$P_1 = \varphi_0 \sin\theta_0 \left(\frac{1-\nu}{1-2\nu} \alpha J_n^{//}(\alpha r) \right) + \frac{\nu(-n^2)}{r^2(1-2\nu)} J_n(\alpha r) +$$

$$+ \frac{\nu\alpha}{r(1-2\nu)} J_n^{/}(\alpha r) - \frac{\nu}{1-2\nu} \frac{\omega^2 \cos^2\theta_0}{c_{p_0}^2} J_n(\alpha r);$$

$$P_2 = \varphi_0 \sin\theta_0 \left(\frac{1}{r} \cdot i \cdot n \cdot \alpha \cdot J_n^{/}(\alpha r) - \frac{1}{r^2} \cdot i \cdot n \cdot \cdot J_n(\alpha r) \right);$$

$$P_3 = \varphi_0 \sin\theta_0 \left(\frac{\nu}{1-2\nu} \cdot \alpha^2 \cdot J_n^{//}(\alpha r) + \frac{\nu}{r^2(1-2\nu)} \cdot (-n^2) \cdot J_n(\alpha r) + \frac{\nu}{r(1-2\nu)} \cdot \alpha \cdot J_n^{/}(\alpha r) \right);$$

Solving the complex algebraic equations (13) we find the unknown complex coefficients A, B, C, substituting these coefficients into (11.12) we find the corresponding stresses.

3. Results and analysis

Numerical results and graphs obtained using C++ software

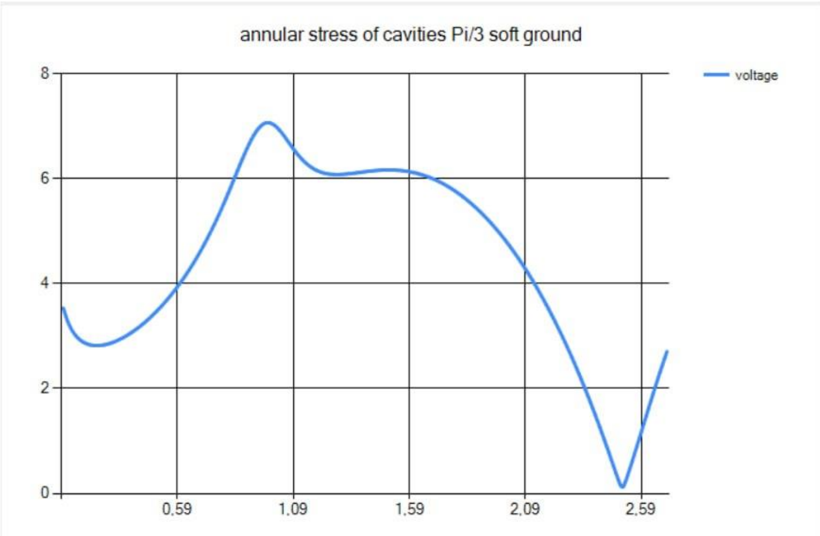

Fig 2.

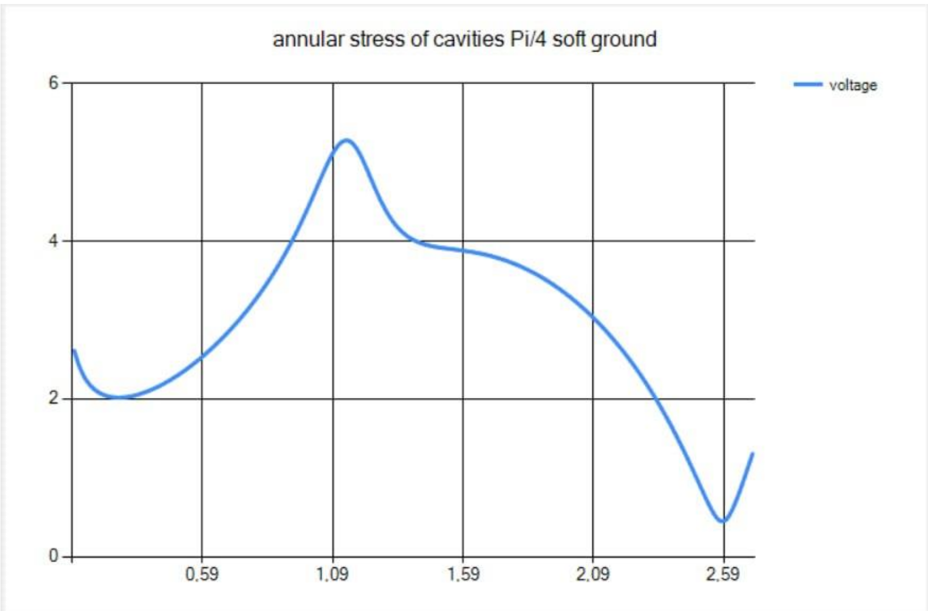

Fig 3.

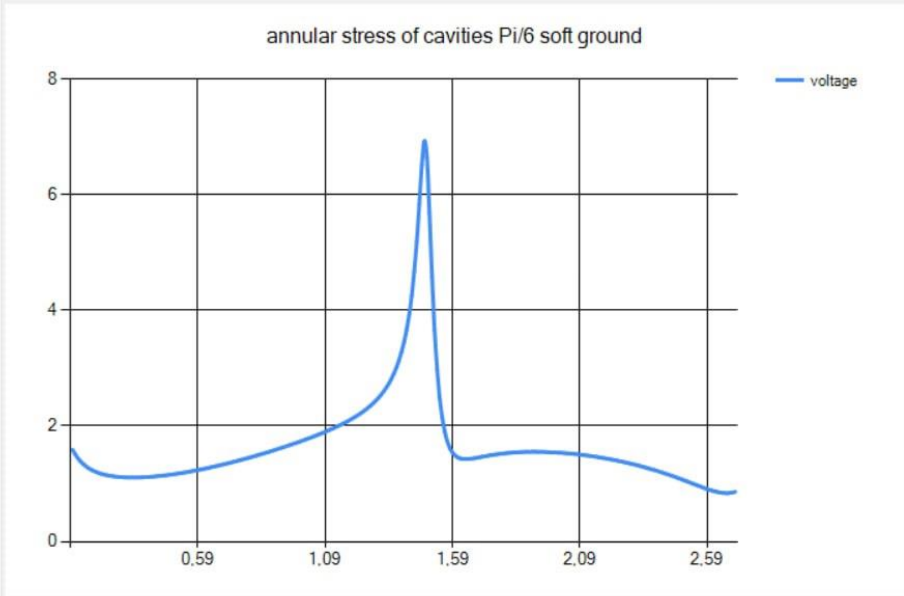

Fig 4.

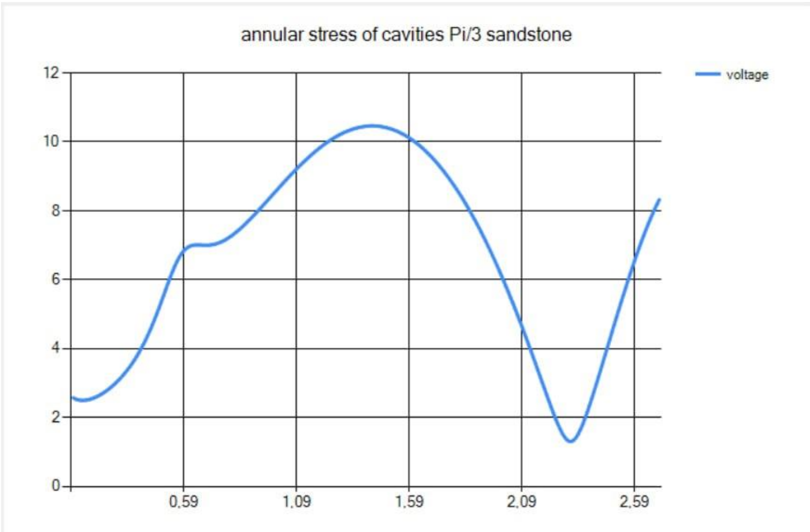

Fig 5.

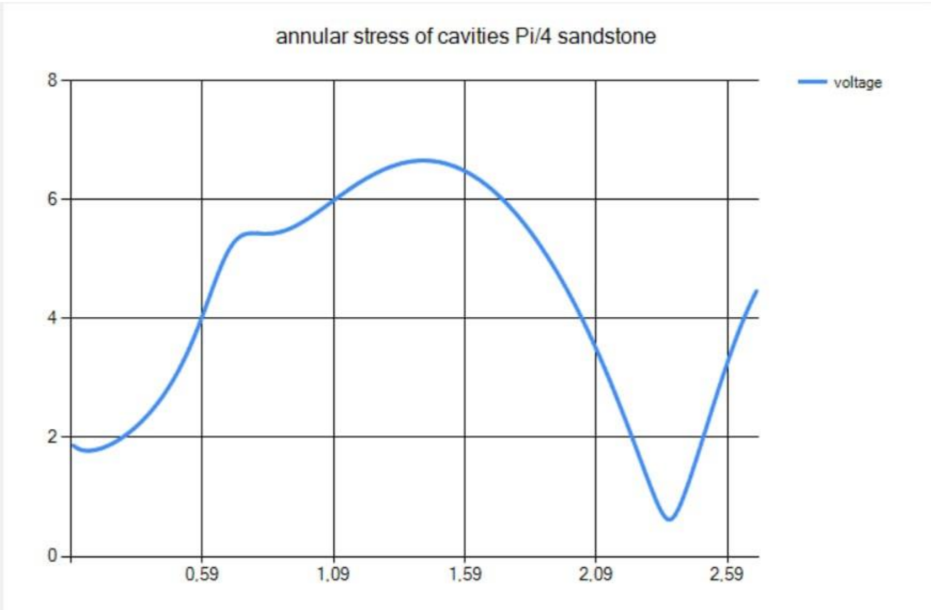

Fig 6.

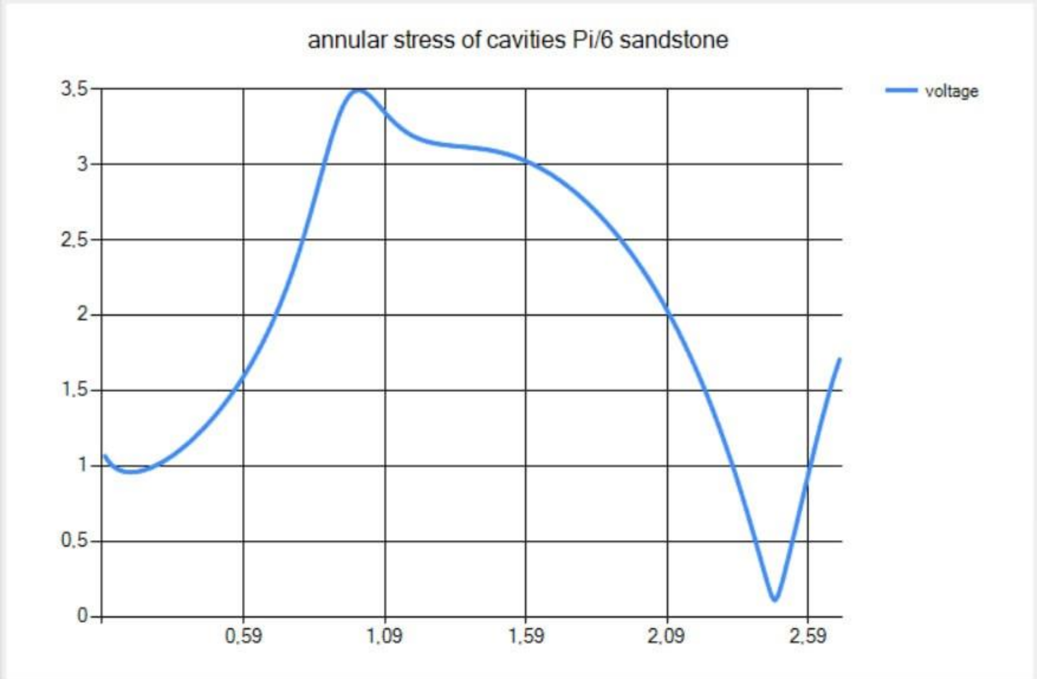

Fig 7.

In fig 1, 2, 3 Figure 3 shows the hoop stresses of the medium cavity when soft soil is considered; the maximum hoop stresses at pi/3 reaches 6.54 dimensionless units, at pi/4 it reaches 5.53 dimensionless units, at pi/6 it reaches 6.26 dimensionless units. When sandstone is considered, the maximum hoop stresses at pi/3 reaches 10.27 dimensionless units, at pi/4 it reaches 6.39 dimensionless units, at pi/6 it reaches 3.49 dimensionless units. In three cases, it can be seen that when a longitudinal wave falls with an increase in the angle of incidence, the hoop stress of the cavity also increases

4. Conclusions

1. A technique and algorithm have been developed for solving the spatial problem of diffraction of harmonic waves in a cavity, an elastic medium.

2. when a longitudinal wave falls with an increase in the angle of incidence relative to the longitudinal axis of the cavity, the hoop stress increases.

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