MECHANICAL CALCULATIONS OF THE SHELL BREAKAGE OF APRICOT PITS

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Abstract. *This article is devoted to the development of a mathematical model of the process of crack formation on the shell of apricot kernels under mechanical action.*
Keywords: *pit shell, condition of fluidity, thickness of the pit shell, elliptical paraboloid, apricot pit, arrow of convexity, plastic state, Gaussian curvature, node number, moment setting.*

Static and kinematic formulations of the tasks of limiting equilibrium of pits shows that the condition of fluidity is important in determining the upper and lower estimates of the ultimate fracture and breakage of pits.

In the general case, when the impact force is acted, internal forces arise at each point of the pit shell: normal forces N_x and N_y, shear forces N_{xy}, transverse forces Q_x and Q_y, bending moments M_x and M_y, as well as torque moments M_{xy}. Usually, the effect of transverse forces on the transition of the material to the plastic state is small, so often the plasticity conditions can be associated with only six of the eight internal forces.

Let us denote the following curvatures

$$K_x = \frac{d^2z}{dx^2}; \qquad K_y = \frac{d^2y}{dy^2}; \qquad K_{xy} = \frac{d^2z}{dxdy};$$

and passing to without momentary force

$$n_x = N_x N_0^{-1};\ n_y = N_y N_0^{-1};\ N_{xy} = N_{xy} N_0^{-1};\ N_0 = \mathfrak{S}\text{kh}, \qquad (1)$$

Where h is thickness of the shell of the stone, then we obtain a system of linear equations for the grid node with the number

$$\begin{cases} (k_x n_x)_{i,j} - 2(k_{xy} n_{xy})_{ij} + (k_y n_y)_{i,j} + q_{i,j} = 0, \\ (n_x)_{i-1,j} - (n_x)_{i,j} + (n_{xy})_{i,j-1} - (n_{xy})_{i,j} = 0, \\ (n_y)_{i,j-1} - (n_y)_{ij} + (n_{xy})_{i-1,j} - (n_{xy})_{i,j} = 0 \end{cases} \qquad (2)$$

Equation (2) can be written for each node of the grid area.

As can be seen from the description of mathematical models of limit equilibrium problems (1) and (2), an important role is played by the equilibrium conditions. They are usually written in differential form.

$$\begin{cases} \frac{dn_x}{d_x} + \frac{dx_{xy}}{d_y} = 0; \qquad \frac{dn}{d_y} + \frac{dn_{xy}}{d_x} = 0; \\ \frac{d^2 2}{d_{x^2}} n_x - 2\frac{d^2 2}{d_x d_y} n_{xy} + \frac{d^2 z}{d_{y^2}} n_y + \frac{d^2 m_x}{dx^2} - 2\frac{d^2 m_{xy}}{d_x d_{xy}} + \frac{d^2 m_y}{dy^2} - q = 0 \end{cases} \qquad (3)$$

This is the partial differential equation.

In general case, that is, in the moment theory of the shell of pits, in order to represent the second derivatives of the moments in the equation, we use the second differences. For i, j point we get.

$$\begin{cases} \dfrac{d^2 m_x}{dx^2} = \Delta x^{-2}\left((m_x)_{i-1,j} - 2(m_x)_{i,j} + (m_x)_{i+1,j}\right), \\ \dfrac{d^2 m_y}{dy^2} = \Delta y^{-2}\left((m_y)_{i,j-1} - 2(m_y)_{i,j} + (m_y)_{i,j+1}\right), \\ \dfrac{d^2 m_x m_y}{dxdy} = (h\Delta x\Delta y)^{-1}\left((m_{xy})_{i-1,j-i} + (m_{xy})_{i,j} - (m_{xy})_{i-1,j} - (m_{xy})_{i,j-1}\right) \end{cases} \quad (4)$$

where $\Delta x, \Delta y$ are grid steps in each of the directions, the values $m_{x,}$ $m_{y,}$ $m_{xy,}$ are bending moments, introduced by the formulas

$$m_x m_0^{-1};\quad m_y m_y^{-1};\quad m_{xy} m_{xy} m_0^{-1};\quad m_0 = 4^{-1}\mathfrak{S}\mathrm{h}^2$$

Substituting relations (10) into (5) allows the third equation to be replaced by the corresponding algebraic expression. It is important to note that its algebraic equations obtained by grid discretization are linear with respect to $n_x, n_y, \dots, m_{xy.}$

Let us consider ¼ of the shell of an apricot pit in a square plan.

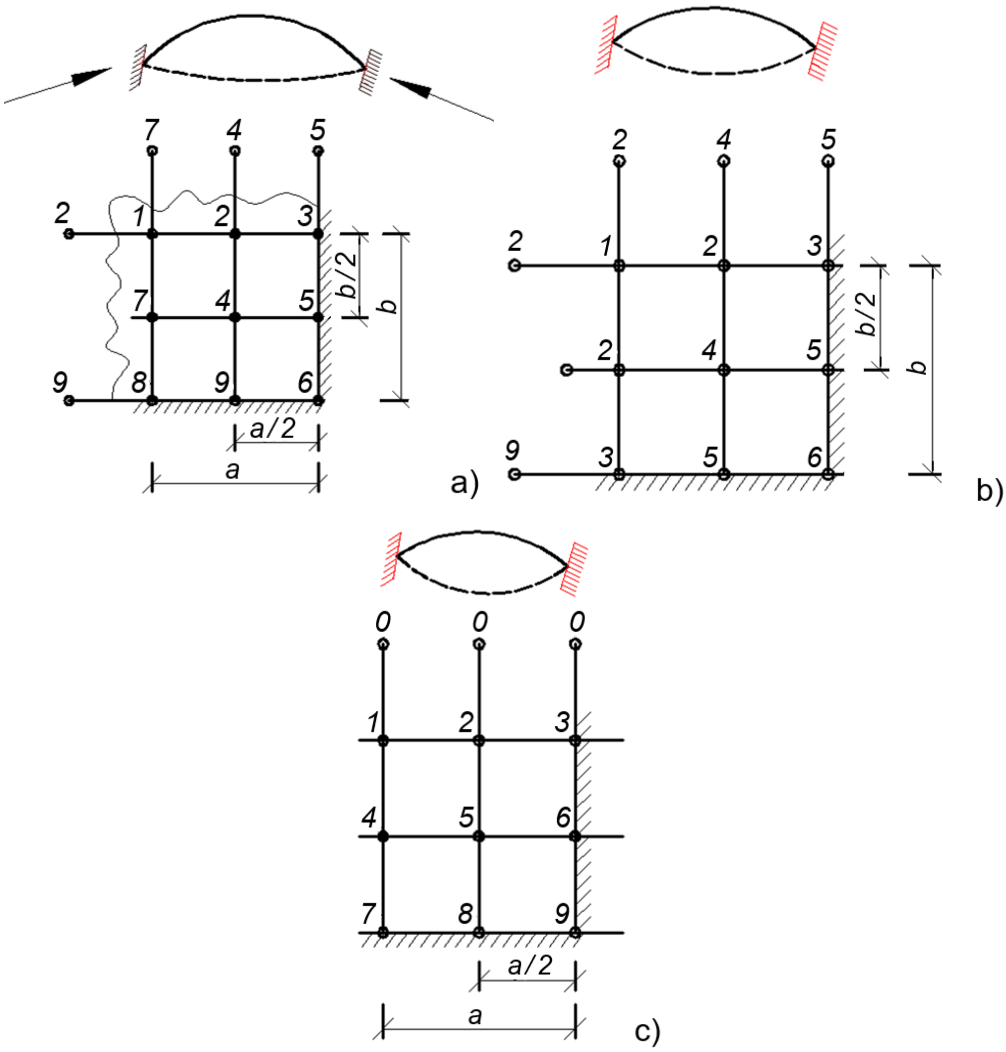

Fig. 1. Finite-difference grid and numbering of nodes.

Fig. 1 shows the finite-difference grid and the numbering of the nodes. Compiling the equilibrium equations (5) in algebraic form, we consider that from equilibrium equations (9), it is necessary to compose only loose knots of the pit shell.

If we assume that the shells of apricot pit are supported along two and special points of the sides, then we will write equilibrium equations only for internal nodes 1, 2 and 4. Since we compose three equilibrium equations for each node, we get nine algebraic equations in total:

- symmetry conditions of the problem allow identifying nodes (Fig. 1b) 2 and 7, 3 and 8, 9 and 5 as well as 7, 10. 4 and 11, 2 and 13, 4 and 14 when compiling the equilibrium equations, and due to this, we can noticeably simplify the equations;

- the conditions for fixing the edges allow immediately specifying the values of internal forces at grid points coinciding with the contour, for the free bearing of the edges when

$$x =\neq a,\ N_x = 0;\ M_x = 0;\ N_y \neq 0 \text{ etc.}$$

$$y =\neq a,\ N_y = 0;\ M_y = 0;\ N_x \neq 0 \text{ etc.}$$

For definiteness, the middle surface of the shell of an apricot pit be an elliptical paraboloid.

$$Z = \frac{f}{2a^2}(x^2 + y^2) \tag{5}$$

where 2a is the size of the side of the shell of the stone in plan; f is arrow of convexity.

Calculating the curvature of the middle shell of an apricot pit, we find

$$\begin{cases} K_x = \frac{d^2z}{dx^2} = fa^{-2}, \\ K_y = \frac{d^2z}{dy^2} fa^{-2}, \\ K_{xy} = \frac{d^2z}{d_x d_y} = 0 \end{cases} \tag{6}$$

Additionally, denoting $\Upsilon = fa^{-1}$ the flatness of the shell of the stone, $e = kf^{-1}$ the relative thickness, $p = q\mathfrak{S}_0^{-1}$ the dimensionless intensity of the uniform force, we will take for a specific example $\Upsilon = 0.2y$, $e = 0.055$. In order to control the error in approximating hypersurfaces by polyhedron, it is necessary to construct strongly both inscribed and hewn polyhedrons.

2. The size of the problem, the volume of calculations, the time consumption significantly depends on the number of faces of polyhedron and increases sharply with an increase in their number. With a small number of hyperplanes of the polyhedron, the volume of the problem is small, but the accuracy of the results is also low. Therefore, the problem of choosing the optimal number of faces with the least amount of time arises. Equilibrium equations under the action of a uniform transverse impact force are compiled for nodes 1, 2 and 4 of the grids, which are given in Table. 1. In the moment statement of the problem, Table. 2 (Fig. 1-c).

Table 1

Number of points																			
№ / **н/п**	①			②			③			④			⑤			⑥			**Force load**
	n_x	n_y	n_{xy}	n_x	n_y	n_{xy}	n_x	n_y	n_{xy}	n_x	n_y	n_{xy}	n_x	n_y	n_{xy}	n_x	n_y	n_{xy}	0 0 0
1	-1		-1	1		1													0
2	1			-1		-1						1							0
3				1			-1		-1										0
4				1		1				1		-1							0
5									1	1			-1		-1				0
6													1		1	-1		-1	0

7		-1	-1		1	1													0
8			1		-1	-1					1								0
9						1		-1	-1					1					0
10					1	1					-1	-1							0
11								1				1		-1	-1				0
12														1	1		-1	-1	0
13	-1	-1																	0.5
14				-1	-1														0.5
15										-1	-1								0.5

The linear algebraic equations obtained by the described method include the value P-intensity of the distributed impact force, as well as internal forces $n_x, n_y, n_{xy}, m_x, m_y, m_{xy}$.

In accordance with the theorem of the lower limit of the limiting impact force, we obtain a static formulation of the problem: it is necessary, by varying the fields of internal forces $n_x, n_y, n_{xy}, m_x, m_y, m_{xy}$, to find min p.

Table 2

①						②						③						④						⑤					
n_x			n_x			n_x			n_x			n_x			n_x			n_x			n_x			n_x			n_x		
	n_y			n_y			n_y			n_y			n_y			n_y			n_y			n_y			n_y			n_y	
		n_{xy}			n_{xy}			n_{xy}			n_{xy}			n_{xy}			n_{xy}			n_{xy}			n_{xy}			n_{xy}			n_{xy}
1		1				-1														-1									
						1						-1		1												-1			
																		1		1				-1					
																								1		1			
	1	1						-1											-1										
							1	1						-1											-1				
																			1	1						-1			
																									1	1			
					-0.					0.05							0.05							-1	-1		-0.1	-0.1	

①						②						③							④					⑤				
n_x			n_x			n_x			n_x			n_x			n_x			n_x			n_x		n_x			n_x		
	n_y			n_y			n_y			n_y			n_y			n_y			n_y			n_y		n_y			n_y	
		n_{xy}			n_{xy}			n_{xy}			n_{xy}			n_{xy}			n_{xy}			n_{xy}		n_{xy}			n_{xy}			n_{xy}
					05																							
-1	-1		-0.1	-0.1					0.1													0.1						
			0.1			-1	-1		0.2	0.2					-0.1												0.2	
				0.05														-1	-1									

Table 2 (cont.)

⑥						⑦						⑧						⑨						P
n_x			n_x			n_x			n_x			n_x			n_x			n_x			n_x			
	n_y			n_y			n_y			n_y			n_y			n_y			n_y			n_y		
		n_{xy}			n_{xy}			n_{xy}			n_{xy}			n_{xy}			n_{xy}			n_{xy}			n_{xy}	
								-1																
-1																								
														-1										
							-1																	
		-1											-1											

			0.05								1105				-0.1	-0.1								0.5
																								0.5
			-0.1																					0.5
										0.05					-0.1	-0.1								0.5

Since these quantities are included in the equilibrium equations in the first confluence, an effective linear programming method could be used to find the solution, which guarantees a solution in a finite number of steps. An obstacle to the application of this method is the non-linear form of the conditions of fluidity, so the formulation of the problem under consideration in terms of linear programming requires the linearization of the conditions of fluidity.

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