ON THE NONSMOOTH OPTIMIZATION PROBLEM FOR CONTROL SYSTEM UNDER CONDITIONS OF UNCERTAINTY

[1]Otakulov Salim, [2]Murotboev Mirjalol Baxtiyor ogli
[1]Doctor of Physical and Mathematical Sciences, Professor, Jizzakh Polytechnic Institute
[2]Jizzakh Branch of the National University of Uzbekistan, master's student
https://doi.org/10.5281/zenodo.7855384

Abstract. *In the paper we considere a model of linear dynamic control system under conditions of uncertainty. For this model minimaks optimal control problem of ensemble of trajectories on terminal funksional is researched. In the problem the necessary and sufficient conditions of optimality are obtained.*

Keywords: *linear system, conditions of uncertainty, nonsmooth terminal funksional, minimaks problem, ensemble of trajectories, optimal control.*

1. INTRODUCTION

To research mathematical models of management problems, advanced methods of modern mathematics such as differential equations and mathematical physics, theory of dynamic systems, functional and convex analysis, non-smooth analysis are widely used. This is the main factor in the comprehensive development of the mathematical theory of optimal processes [1-8]. The application of this theory to various practical issues in the management of complex technological processes, technology, economy, energy and other fields is expanding.

In the field of practice, the importance of finding solutions suitable for the real situation in solving existing management issues is increasing. Therefore, it is important to build mathematical models and research them, taking into account the uncertainty of information about various parameters affecting the management process [7-10]. Such models have a wide field of application in a number of actual practical issues, especially in long-term forecasting problems in the economy, seismic strength of engineering and construction facilities, dynamics of infectious diseases.

In the study of control issues, models with unique characteristics are formed when information about the parameters of the control system is incomplete. In conditions of uncertainty, in contrast to deterministic models of optimal control, it is necessary to look for such control effects, which are not intended to control individual trajectories, but to control an ensemble of trajectories. Extensive research is being conducted on models and issues related to trajectory ensemble management [8–14]. The model considered in this research work and the obtained results are inextricably linked with the works of [13,15,16].

2. RESEARCH OBJECT AND METHODS

The following

$$\dot{x} = A(t)x + B(t)u(t) + C(t)v(t)\ ,\ t \in T = [t_0, t_1] \qquad (1)$$

for the linear control system $x(t_0) = x^0 \in M_0$ is the initial state and $v(\cdot) \in V(T)$ under conditions of uncertainty of external influences $x = x(t; x^0, u(\cdot), v(\cdot))$ are defined on the right edge of the trajectories

$$g(x(t_1; x^0, u(\cdot), v(\cdot))) = \sum_{i=1}^{s} \min_{l_i \in L_i} (l_i, x(t_1; x^0, u(\cdot), v(\cdot))) \qquad (2)$$

we consider the problem of optimal control according to the functional in appearance [16]. This issue, according to the principle of obtaining a guaranteed result in management,

$$J(u(\cdot)) = \max_{x^0 \in M_0, v(\cdot) \in V(T)} g(x(t_1; x^0, u(\cdot), v(\cdot))) \qquad (3)$$

consists of the problem of minimizing the terminal functional in the form of

$$\max_{x^0 \in M_0, v(\cdot) \in V(T)} g(x(t_1; x^0, u(\cdot), v(\cdot))) \to min, u(\cdot) \in U(T) \qquad (4)$$

given a minimax optimal control problem.

In system (1), matrix elements $A(t)$, $B(t)$ are continuous in the interval $T = [t_0, t_1]$ components of vector functions $u(t)$ and $v(t)$ are dimensional and bounded in this interval. The set of valid controls $U(T)$ consists of dimensional functions $U \subset R^m$ taking values from the given convex and compact set $u = u(t),\ t \in T = [t_0, t_1]$. The non-local set of the uncertain external driving force parameter $V(T)$ is a convex and compact all-dimensional $V \subset R^k$ taking values from the set $v = v(t), t \in T = [t_0, t_1]$ consists of functions.

Determining the necessary and sufficient conditions for optimality is an important step in solving the problem (4). In the study of these spheres (2) and (3) functionals and (1) system

$$X(t; u(\cdot)) = \cup\{x(t; x^0, u(\cdot), v(\cdot)) | x^0 \in M_0, v(\cdot) \in V(T)\}, t \in T$$

we use the properties of the ensemble of trajectories. These properties are studied in [ ]. In the study of optimality conditions, we also use convex and functional analysis methods.

[16] according to the results of work (4).

$$J(u(\cdot)) = \min_{l \in coL} \rho(u(\cdot), l) \qquad (5)$$

A concave functional can be thought of as a minimization problem over a set of misplaced controls, where $U(T)$

$$\rho(u(\cdot),l) = \max_{\xi \in M_0}\left(F(t_1,t)\xi, l\right) + \int_{t_0}^{t_1}\left(F(t_1,t)B(t)u(t), l\right)dt + \int_{t_0}^{t_1}\max_{v \in V}\left(F(t_1,t)C(t)v, l\right)dt \quad (6)$$

Lemma 1. The functional defined by equality (6) is convex in $u(\cdot) \in U(T)$ with respect to for$l$ every R^n assigned $\rho(u(\cdot),l)$ Indeed, the concept of the basis function and the properties of the multivalued reflection integral are functional $\rho(u(\cdot),l)$.

$$\rho(u(\cdot),l) = \sigma\left(F(t_1,t)M_0, l\right) + \int_{t_0}^{t_1}\left(F(t_1,t)B(t)u(t), l\right)dt + \sigma\left(\int_{t_0}^{t_1} F(t_1,t)C(t)v dt, l\right)$$

can be written in the form. According to the properties of the basis function, each of the three components on the right side of this equation consists of convex functions in l in R^n. As a result, their sum functional is also convex in $\rho(u(\cdot),l)$ with respect to l in R^n.

We also consider the following function:

$$\mu(l) = \max_{\xi \in M_0}(F(t_1,t_0)\xi, l) + \int_{t_0}^{t_1}\min_{u \in U}(F(t_1,t)B(t)u, l)dt + \int_{t_0}^{t_1}\max_{v \in V}(F(t_1,t)C(t)v, l)dt$$

Lemma 2. $\mu(l)$ function continuous at R^n.

The correctness of this assertion depends on the $\mu_2(t,l)=\min_{u\in U}(F(t_1,t)B(t)u,l)$, $\mu_1(l)=\max_{v\in W}(F(t_1,t_0)\xi,l)$ $\mu_3(t,l)=\max_{v\in V}(F(t_1,t)C(t)v,l)$ continuity of these functions and $l\in R^n$, follows from equality. $\mu(l)=\mu_1(t)+\int_{t_0}^{t_1}\mu_2(t,l)dt+\int_{t_0}^{t_1}\mu_3(t,l)dt$

3. RESEARCH RESULTS

In problem solving (4), we study the necessary and sufficient conditions of optimality.

Theorem 1. Let us assume that problem $u^*(t),t\in T,$ (4) is a piecewise-continuous optimal control. Then the function $l\to\rho(u^*(\cdot),l)$ is for an arbitrary global minimum point l^* in the set

$$(F(t_1,t)B(t)u^*(t),l^*)=\min_{u\in U}(F(t_1,t)B(t)u,l^*),\forall t\in T \quad (7)$$

relationship is fulfilled.

Proof. If - $u^*(t),t\in T,$ is an optimal control, that is, if the functional (3) has a minimum point in the set $U(T)$, then by definition

$$J(u^*(\cdot))\le J(u(\cdot)),\forall u(\cdot)\in U(T)$$

relationship is fulfilled. Considering formulas (5)-(6), we write this last relation as follows:

$$\begin{gathered}\min_{l\in coL}\left[\max_{\xi\in M_0}(F(t_1,t)\xi,l)+\int_{t_0}^{t_1}(F(t_1,t)B(t)u^*(t),l)dt+\int_{t_0}^{t_1}\max_{v\in V}(F(t_1,t)C(t)v,l)dt\right]\le\\ \min_{l\in coL}\left[\max_{\xi\in M_0}(F(t_1,t)\xi,l)+\int_{t_0}^{t_1}(F(t_1,t)B(t)u(t),l)dt+\int_{t_0}^{t_1}\max_{v\in V}(F(t_1,t)C(t)v,l)dt\right],\forall u(\cdot)\in U(T)\end{gathered} \quad (8)$$

According to Lemma 1 $l\to\rho(u^*(\cdot),l)$, the function consists of a convex function in R^n. According to the properties of convex functions, it is also a continuous function. So this function has a global minimum in a convex compact set coL. Let - be an arbitrary global minimum point of the vector function in the set. taking into account the definition of the vector $l^*\in coL-$ vektor $l\to\rho(u^*(\cdot),l)$, we get the following relation from l^* (8):

$$\begin{gathered}\max_{\xi\in M_0}(F(t_1,t)\xi,l^*)+\int_{t_0}^{t_1}(F(t_1,t)B(t)u^*(t),l^*)dt+\int_{t_0}^{t_1}\max_{v\in V}(F(t_1,t)C(t)v,l^*)dt\le\\ \le\max_{\xi\in M_0}(F(t_1,t)\xi,l^*)+\int_{t_0}^{t_1}(F(t_1,t)B(t)u(t),l)dt+\int_{t_0}^{t_1}\max_{v\in V}(F(t_1,t)C(t)v,l^*)dt.\end{gathered}$$

for from here:

$$\forall u(\cdot)\in U(T)$$

$$\int_{t_0}^{t_1}(F(t_1,t)B(t)u^*(t),l^*)dt\le\int_{t_0}^{t_1}(F(t_1,t)B(t)u(t),l^*)dt \quad (9)$$

the fulfillment of the inequality follows.

Let us assume that, $u^*(t),t\in T,-$ the optimal control is piecewise-continuous. For clarity, $u^*(t),t\in T,$ we consider the function to be continuous from the left. Now, taking an arbitrarily small enough number $\tau\in[t_0,t_1),$ and $\varepsilon>0$ number and $\forall\bar{u}\in U$,

$$u(t)=u(t,\tau,\varepsilon,\bar{u})=\begin{cases} u^*(t), t\notin(\tau,\tau+\varepsilon), \\ \bar{u},\ t\in(\tau,\tau+\varepsilon) \end{cases}$$

we look at the permissible management in the form. This $u(t)=u(t,\tau,\varepsilon,\bar{u})$ management (9) we get the inequality. And that time

$$\int_{\tau}^{\tau+\varepsilon}\left[(F(t_1,t)B(t)\bar{u},l^*)-(F(t_1,t)B(t)u^*(t),l^*)\right]dt\geq 0$$

we get the relationship. So,

$$\varepsilon\left[(F(t_1,\tau)B(\tau)\bar{u},l^*)-(F(t_1,t)B(\tau)u^*(\tau),l^*)\right]+0(\varepsilon)\geq 0$$

get the inequality. The last equality $\varepsilon>0$ devided and $\varepsilon\to 0$ go to limit,

$$(F(t_1,\tau)B(\tau)\bar{u},l^*)\geq(F(t_1,\tau)B(\tau)u^*(\tau)),\forall\bar{u}\in U$$

we get the relationship. So,

$$(F(t_1,\tau)B(\tau)u^*(\tau),l^*)=\min_{u\in U}(F(t_1,\tau)B(\tau)u,l^*), \tag{10}$$

that is, equality (7) $\tau\in[t_0,t_1)$ holds for arbitrary. (10) using the continuity of the left and right sides of the relation, taking into account the transition to the limit and

, $\tau\to t_1$ go to limit and $F(t_1,t_1)=E$ taking into account the transition to the limit and

$$(B(t_1)u^*(t_1),l^*)=\min_{u\in U}(B(t_1)u,l^*)$$

we form the equality, that is, (7) is also fulfilled $t=t_1$ for the equality. Thus, it was shown that relation (7) holds for all $t\in T\in[t_0,t_1]$. The theorem is proved.

Theorem 2. $u^*=u^*(t),t\in T$, the global minimum point of the function should be found, and equality (6) should be $\mu(l)$ fulfilled in order for the $l^*\in coL$ piecewise-continuous admissible control to be the optimal control in problem (7)

Theorem 3. If $u^*=u^*(t),t\in T$ such a $l^*\in coL$ global minimum point of the function for the piecewise-continuous $\mu(l)$ local control is found, and equality (7) is satisfied, then such control will be the optimal control in problem (4).

Proof. Suppose that the vector $l^*\in coL$ function $\mu(l)$ is the global minimum point coL in the set and equality (7) is fulfilled. Then we will have the following for optional $u(\cdot)\in U(T)$:

$$\min_{l\in coL}\rho(u(\cdot),l)=\min_{l\in coL}\left[\max_{\xi\in M_0}(F(t_1,t_0)\xi,l)dt+\int_{t_0}^{t_1}(F(t_1,t)B(t)u(t),l)dt+\int_{t_0}^{t_1}\max_{v\in V}(F(t_1,t)C(t)v,l)dt\right]\geq$$

$$\geq\min_{l\in coL}\left[\max_{\xi\in M_0}(F(t_1,t_0)\xi,l)+\int_{t_0}^{t_1}\min_{u\in U}(F(t_1,t)B(t)u,l)dt+\int_{t_0}^{t_1}\max_{v\in W}(F(t_1,t)C(t)v,l)dt\right]=\min_{l\in coL}\mu(l)=$$

$$=\mu(l^*)=\max_{\xi\in M_0}(F(t_1,t_0)\xi,l^*)dt+\int_{t_0}^{t_1}\min_{u\in U}(F(t_1,t)B(t)u,l^*)dt+\int_{t_0}^{t_1}\max_{v\in V}(F(t_1,t)C(t)v,l^*)dt=$$

$$=\max_{\xi\in M_0}(F(t_1,t_0)\xi,l^*)dt+\int_{t_0}^{t_1}(F(t_1,t)B(t)u^*(t),l^*)dt+\int_{t_0}^{t_1}\max_{v\in V}(F(t_1,t)C(t)v,l^*)dt=$$

$= \rho(u^*(\cdot),l^*) \geq \min_{l \in coL} \rho(u^*(\cdot),l).$

It follows from here that $\forall u \in U(T)$, that is $J(u(\cdot)) \geq J(u^*(\cdot))$, there will be an optimal control in problem $u^* = u^*(t),\ t \in T$ (4). The theorem was proved.

4. DISCUSSION

In the theorems presented above, equality (7) was obtained as a necessary and sufficient condition for optimality. This condition can be use $\mu(l)$ function coL $l^* \neq 0$ has a global minimum in the set.

The obtained result can be written in the form of Pontryagin's maximum principle in the theory of optimal control. Let's look at the function for this. $\psi(t,l) = F'(t_1,t)l, t \in [t_0,t_1]$. We calculate $t \in T$ the special derivative of this function. It is known that the fundamental matrix

$F(t_1,t)$

$$\frac{\partial F(t_1,t)}{\partial t} = -F(t_1,t)A(t),\ \ t \in [t_0,t_1],\ F(t_1,t_1) = E$$

satisfies the conditions. Given that,

$$\frac{\partial \psi(t,l)}{\partial t} = \frac{\partial F'(t_1,t)l}{\partial t} = -(F(t_1,t)A(t))'l = -A'(t)F'(t_1,t)l = -A'(t)\psi(t,l), \psi(t_1,l) = l$$

we form equalities..So, $\psi(t,l) = F'(t_1,t)l, t \in [t_0,t_1]$ fuction (1) corresponds to the system

$$\dot{\psi} = -A'(t)\psi,\ \ t \in [t_0,t_1] \tag{11}$$

will be a solution of the joint system satisfying the condition $\psi(t_1) = l$.

(7) equality

$$(B(t)u^*(t), F'(t_1,t)(-l^*)) = \max_{u \in U}(B(t)u, F'(t_1,t)(-l^*)), \forall t \in T$$

can be written in the form. This equality $\psi(t,l) = F'(t_1,t)l, t \in [t_0,t_1]$ with help of function

$$(B(t)u^*(t), \psi(t,-l^*)) = \max_{u \in U}(B(t)u, \psi(t,-l^*)), \forall t \in T$$

can be written in the form. Thus, the necessary and sufficient conditions of the obtained optimality can be expressed as follows:

In order for the piecewise continuous control to be optimal in problem (4), $u^* = u^*(t),\ t \in T = [t_0,t_1]$ $\psi^*(t_1) = -l^*$ $\psi = \psi^*(t)$

$$(B(t)u^*(t), \psi^*(t)) = \max_{u \in U}(B(t)u, \psi^*(t)), \forall t \in T \tag{12}$$

the fulfillment of the maximum condition is necessary and sufficient, where the vector is the global $l^* \in coL$ vector $\mu(l)$ funksiyaning global minimum point of the function.

5. CONCLUSION.

Based on the obtained results, it can be said that in order to find the optimal control in problem (4), it is necessary to first find the global minimum point of the $\mu(l)$ function $l^* \in coL$, $l^* \neq 0$ then determine the solution of the joint system that satisfies the condition (11) $\psi^*(t_1) = -l^*$, and determine the optimal control from the maximum $u^* = u^*(t),\ t \in T = [t_0,t_1]$ condition (12).

REFERENCES

1. Понтрягин Л.С., Болтянский В.Г., Гамкрелидзе Р.В., Мищенко Е.Ф. Математическая теория оптимальных процессов. -М.: Наука,1961.
2. Габасов Р.Ф., Кириллова Ф.М. Качественная теория оптимальных процессов. М.: Наука, 1971.
3. Ли Э.Б., Маркус Л. Основы теории оптимального управления. М.: Наука, 1972.
4. Габасов Р.Ф., Кириллова Ф.М. Оптимизация линейных систем. Методы функционального анализа. Минск: Изд-во БГУ, 1973.
5. Благодатских В.И., Филиппов А.Ф. Дифференциальные включения и оптимальное управление. Труды математического института АН СССР, Т.169, 1985. - с. 194-252.
6. Кларк Ф. Оптимизация и негладкий анализ. -М.: Наука, 1988.
7. Куржанский А.Б. Управление и наблюдение в условиях неопределенности. -М.: Наука, 1977.
8. Овсянников Д.А. Математические методы управления пучками траекторий. Ленинград: Изд-во ЛГУ, 1981.
9. Куржанская Н.А. Оптимальное управление пучком траекторий системы с недоопределенными параметрами. Дифференциальные уравнения, Т.21, №3, 1985. - с. 404-410.
10. Otakulov S. The control problems of ensemble of trajectories for differential inclusions. LAP Lambert Academic Publishing, 2019.
11. Otakulov S., Kholiyarova F.Kh. About the conditions of optimality in the minimax problem for controlling differential inclusion with delay. Academica: An International Multidisciplinary Research Jounal,Vol.10, Issue 4 (April 2020).- p. 685–694.
12. Otakulov S.,Haydarov T.T. The nonsmooth control problem for dynamic system with parameter under conditions of incomplete initial date. International Conference On Innovation Perspectives, Psychology and Social Studies (ICIPPCS-2020), may 11-12 2020. International Engineering Journal for Research & Development(IEJRD). pp.211-214.
13. Otakulov S., Rahimov B. Sh., Haydarov T.T. The nonsmooth optimal control problem for ensemble of trajectories of dynamic system under conditions of indeterminacy. Middle European Scientific Bulletin, vol. 5, October 2020. -p. 38-42.
14. Отакулов С., Рахимов Б.Ш. Об условиях управляемости ансамбля траекторий дифференциальных включений. Physical and mathematical scinces. Vol. 3, Issue 1,2020.- p.45-50.
15. Otakulov S., Murotboyev M.B. Noanilik sharoitidag boshqaruv tizimi uchun silliqmas terminal funksionalning xossalari. “Zamonaviy innovatsion tadqiqotlarning dolzarb muammolari va rivojlanish tendensiyalari: yechimlar va istiqbollar”. Respublika miqyosidagi ilmiy-texnik anjuman materiallari toʻplami, 2022 yil 13-14 may, Oʻ zMU Jizzax filiali. Jizzax, 2022. 349–352 b.
16. Otakulov S., Murotboyev M.B. Noaniqlik sharoitidagi chiziqli sistema uchun optimal boshqarish masalasi modeli haqida. International scientific journal “Science and innovation”, 2022, №7 , ser. A. p.544-553.