THE BASIC RULES OF COMBINATORIAL THEORY IN THE SCHOOL COURSE OF MATHEMATICS

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Abstract. *In the thesis psychological and pedagogical aspects of teaching the elements of probability theory, statistic and combinatory at high school. Offers a course of methodical recommendations of teaching stochastic line in high school on the following topics: the probability of random events, discrete spaces of elementary events, classical and statistical definition of probability, the algebra of events, introduction to mathematical statistics, the application of the formulas of combinatoric and binomial theorem of Newton to calculate the probability of events.*

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At the heart of combinatorial actions, in particular, the enumeration of all possible options, are actions with finite sets. An objective analysis of the situation described in the combinatorial problem and the correct execution of operations with sets, which are discussed in the problem, presuppose:

• possession at a sufficiently high level of a number of logical and theoretical - multiple concepts (some, each, all, individual, set, part, whole);

• understanding of the meaning of unions-links and, or;

• the ability to establish predetermined relationships between elements of sets and between sets.

Targeted propaedeutic work allows you to prepare children to get acquainted with combinatorial tasks. First, such problems are solved on the basis of practical actions by enumeration. The enumeration may involve the detection of both all possible combinations with objects, and only their part that satisfies the conditions of the problem.

The theory of combinatorics is based on two basic principles - the rule of addition and the rule of multiplication. Let's consider them in more detail.

Addition rule: Let the object A we can choose from the set in m ways, and the object B can be chosen in nn ways, then the object "A+B" can be chosen in m+nm+n ways.

Perhaps this rule will seem like abracadabra to an uninitiated person, but there is nothing complicated. Consider an example - let there be mm balls in one box, and nn balls in the second box. In how many ways can a ball be drawn from one of these boxes? Obviously, there are m+nm+n ways to get ONE ball.

Multiplication rule: Let object A be chosen in m ways, object B is chosen in nn ways, then both objects can be chosen in mnmn ways.

It's very simple - each of the mm ways of choosing object A is combined with each of the nn ways of choosing object B, that is, the number of ways is simply multiplied by each other.

Consider a simple example: how many numbers can be made from the digits 0,1,2,3,4,5,6,7,8,9 if the number must be two digits?

You can make 90 numbers - the first digit of the number (object A) can be chosen in 9 ways, since the number cannot start from zero. The second digit of the number (object B) can be chosen in 10 ways, since we have 10 digits. The result is 9∗10=909∗10=90 numbers.

These were the main rules on which all methods of solving problems in combinatorics are based. You can find even more theory about the beginnings of combinatorics in the online tutorial: Elements of combinatorics online.

Two basic rules of combinatorial theory

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Elements of combinatorics that will be required for further study of probability theory. It should be noted that combinatorics is an independent section of higher mathematics (and not a part of terver) and weighty textbooks have been written in this discipline, the content of which, at times, is no easier than abstract algebra. However, a small share of theoretical knowledge will be enough for us, and in this article I will try to analyze the basics of the topic with typical combinatorial problems in an accessible form. What are we going to do? In a narrow sense, combinatorics is the calculation of various combinations that can be made up of a certain set of discrete objects. Objects are understood as any isolated objects or living beings - people, animals, mushrooms, plants, insects, etc. At the same time, combinatorics does not care at all that the set consists of a plate of semolina, a soldering iron and a marsh frog. It is fundamentally important that these objects can be enumerated - there are three of them (discreteness) and it is essential that there are no identical ones among them.

With a lot sorted out, now about the combinations. The most common types of combinations are permutations of objects, their selection from a set (combination) and distribution (placement). Let's see how this happens right now:

Permutations, combinations and placements without repetition

Do not be afraid of obscure terms, especially since some of them are really not very successful. Let's start with the headline tail - what does "no repetition" mean? This means that in this section we will consider sets that consist of different objects. For example, ... no, I won’t offer porridge with a soldering iron and a frog, something tastier is better =) Imagine that an apple, a pear and a banana materialized on the table in front of you (if there are any, the situation can be simulated in real). We lay out the fruits from left to right in the following order:

apple / pear / banana

Question one: in how many ways can they be rearranged?

One combination has already been written above and there are no problems with the rest:

apple / banana / pear

pear / apple / banana

pear / banana / apple

banana / apple / pear

banana / pear / apple

Total: 6 combinations or 6 permutations.

Well, it was not difficult to list all possible cases here, but what if there are more items? Already with four different fruits, the number of combinations will increase significantly!

No torment - 3 objects can be rearranged in ways.

Question two: how many ways can you choose a) one fruit, b) two fruits, c) three fruits, d) at least one fruit?

a) One fruit can be chosen, obviously, in three ways - take either an apple, or a pear, or a banana. The entry in this case should be understood as follows: “in how many ways can you choose 1 fruit out of three?”

b) We list all possible combinations of two fruits:

apple and pear;

apple and banana;

pear and banana.

The number of combinations is easy to check using the same formula:

The entry is understood similarly: “in how many ways can you choose 2 fruits out of three?”.

c) And finally, three fruits can be chosen in a unique way: $C_3^3 = \frac{3!}{0!\cdot 3!} = 1$

By the way, the formula for the number of combinations also makes sense for an empty sample: $C_3^0 = \frac{3!}{3!\cdot 0!} = 1$

In this way, you can choose not a single fruit - in fact, take nothing and that's it.

d) How many ways can you take at least one piece of fruit? The “at least one” condition implies that we are satisfied with 1 fruit (any) or any 2 fruits or all 3 fruits:

$C_3^1 + C_3^2 + C_3^3 = 3 + 3 + 1 = 7$ ways you can choose at least one fruit.

Readers who have carefully studied the introductory lesson on probability theory have already guessed something. But about the meaning of the plus sign later.

To answer the next question, I need two volunteers ... ... Well, since no one wants, then I will call to the board =)

Question three: in how many ways can one fruit be distributed to Dasha and Natasha?

In order to distribute two fruits, you must first select them. According to point "b" of the previous question,

you can do it $C_3^2 = 3$ ways, I will rewrite them:

apple and pear;

apple and banana;

pear and banana.

But now there will be twice as many combinations. Consider, for example, the first pair of fruits:

you can treat Dasha with an apple, and Natasha with a pear;

or vice versa - Dasha will get the pear, and Natasha will get the apple.

And such a permutation is possible for every pair of fruits.

In this case, the formula for the number of placements works: $A_3^2 = 2 \cdot 3 = 6$

It differs from a formula in that it takes into account not only the number of ways in which multiple objects can be selected, but also all permutations of objects in each possible selection. So, in the considered example, it is important not only that you can simply choose, for example, a pear and a banana, but also how they will be distributed (placed) between Dasha and Natasha.

The basic formulas of combinatorics and try to understand well the difference between permutations, combinations and placements. In the simplest cases, you can recalculate all possible combinations manually, but most often it becomes an overwhelming task, which is why you need to understand the meaning of the formulas.

The rule of addition and the rule of multiplication of combinations

These rules are very reminiscent of the algebra of events, and many readers have already familiarized themselves with paragraph No. 4 of the reference material Basic formulas of combinatorics, where they are stated in a general form. I will try to repeat the principles as briefly as possible:

The plus sign should be understood and read as a union OR. Recall the demo problem with an apple, pear, and banana:

$C_3^1 + C_3^2 + C_3^3 = 3 + 3 + 1 = 7$ ways you can choose at least one fruit.

That is, you can take 1 fruit (any of the three) OR some combination of two fruits OR all three fruits. Note that adding combinations implies choice indifference (whether one, two, or 3 fruits are selected).

Let's look at a more thorough example:

Task.

The student group consists of 23 people, including 10 boys and 13 girls. In how many ways can two people of the same gender be chosen?

Solution: in this case, the calculation is not suitable, since the total number of combinations includes opposite-sex couples.

The condition "choose two people of the same sex" implies that it is necessary to choose two boys or two girls, and the verbal formulation itself indicates the right way to solve:

$C_{10}^2 = \frac{10!}{8! \cdot 2!} = \frac{8! \cdot 9 \cdot 10}{8! \cdot 2!} = \frac{9 \cdot 10}{2} = 45$ ways you can choose 2 young men;

$C_{13}^2 = \frac{13!}{11! \cdot 2!} = \frac{11! \cdot 12 \cdot 13}{11! \cdot 2!} = \frac{12 \cdot 13}{2} = 78$ ways you can choose 2 girls.

Thus, two people of the same sex (no difference - boys or girls) can be selected: $C_{10}^2 + C_{13}^2 = 45 + 78 = 123$ ways.

Answer: 123

Rule for multiplying combinations:

The sign "multiply" should be understood and read as the union I.

Consider the same student group that went to the dance. In how many ways can a boy and a girl be paired?

$C_{10}^{1}=10$ in this ways you can choose 1 young man;

$C_{13}^{1}=13$ in this ways you can choose 1 young girl.

Thus, one boy and one girl can be chosen in this way: $C_{10}^{1}\cdot C_{13}^{1}=10\cdot 13=130$

When 1 object is selected from each set, then the following principle of counting combinations is valid: "each object from one set can pair with each object of another set."

That is, Akmal can invite any of the 13 girls to dance, Kamol - also any of the thirteen, and other young people have a similar choice. Overall: $10\cdot 13=130$ possible pairs.

It should be noted that in this example, the "history" of pair formation does not matter; however, if initiative is taken into account, then the number of combinations must be doubled, since each of the 13 girls can also invite any boy to dance. It all depends on the conditions of a particular task!

A similar principle is also valid for more complex combinations, for example: in how many ways can two boys and two girls be chosen to participate in a FRC (fun and resourceful club)?

Union And unambiguously hints that combinations must be multiplied: $C_{10}^{2}\cdot C_{13}^{2}=45\cdot 78=3510$ possible group of artists.

In other words, each pair of boys (45 unique pairs) can compete with any pair of girls (78 unique pairs). And if we consider the distribution of roles between the participants, then there will be even more combinations. ... I really want to, but still I will refrain from continuing, so as not to instill in you an aversion to student life =).

I also remind you that now we are talking about a set with different objects, and if an apple / pear / banana is replaced by 3 apples or even 3 very similar apples, then in the context of the considered problem they will still be considered different.

Let's dwell on each type of combination in more detail:

Permutations

Permutations are combinations consisting of the same n **different objects and differ only in the order of their location. The number of all possible permutations is expressed by the formula** $P_n=n!$

A distinctive feature of permutations is that each of them involves the ENTIRE set, that is, all objects. For example, a friendly family:

<u>Task 1</u>

<u>In how many ways can 5 people be seated at a table?</u>

<u>Solution: use the formula for the number of permutations:</u>

$P_5=5!=120$

Answer: 120 ways

Unbelievable but true. Please note that it does not matter here whether the table is round, square, or in general all the people sat down, got up, lay down on a bench along one wall - only the number of objects and their relative position are important. In addition to

permuting people, the problem of permuting different books on a shelf is often encountered, but this would be too simple even for a teapot:

Task 2

How many four-digit numbers can be made from four cards with numbers 0, 5, 7, 9?

In order to make a four-digit number, you need to use all four cards (the numbers on which are different!), And this is a very important prerequisite for applying the formula $P_n = n!$ Obviously, by rearranging the cards, we will get different four-digit numbers, ... wait, is everything all right here? ;-)

Think carefully about the problem! In general, this is a characteristic feature of combinatorial and probabilistic problems - they NEED TO THINK. And often think in a worldly way, as, for example, in the analysis of the introductory example with fruits. No, of course, I do not call for stupidly working out other sections of mathematics, but I must note that the same integrals can be learned to solve purely mechanically.

Solution and answer at the end of the lesson.

Increasing turnover:

Combinations

Textbooks usually give a concise and not very clear definition of combinations, therefore, in my mouth, the wording will not be particularly rational, but, I hope, intelligible:

Combinations are various combinations of objects that are selected from a variety of different objects and that differ from each other by at least one object. In other words, a single combination is a unique selection of elements in which their order (location) is not important. The total number of such unique combinations is calculated by the formula $C_n^m = \frac{n!}{(n-m)! \cdot m!}$.

Task 3

There are 15 parts in a box. In how many ways can 4 parts be taken?

Solution: first of all, I again draw attention to the fact that, according to the logic of the condition, the parts are considered different - even if they are actually of the same type and visually the same (in this case, they can, for example, be numbered).

In the task we are talking about a selection of 4 parts, in which their "further fate" does not matter - roughly speaking, "just chose 4 pieces and that's it." Thus, we have a combination of parts. We count their number:

$$C_{15}^4 = \frac{15!}{(15-4)! \cdot 4!} = \frac{15!}{11! \cdot 4!} = (*)$$

Here, of course, you do not need to move huge numbers. $11! = 39916800, \quad 15! = 1307674368000$.

In a similar situation, I advise you to use the following technique: in the denominator, choose the largest factorial (in this case $11!$) and reduce the fraction by it. To do this, the numerator should be represented as $15! = 11! \cdot 12 \cdot 13 \cdot 14 \cdot 15$. I will write in great detail:

$$(*) = \frac{11! \cdot 12 \cdot 13 \cdot 14 \cdot 15}{11! \cdot 4!} = \frac{12 \cdot 13 \cdot 14 \cdot 15}{4!} = \frac{12 \cdot 13 \cdot 14 \cdot 15}{24} = 1365$$

ways you can take 4 parts from the box.

Once again, what does this mean? This means that from a set of 15 different parts, one thousand three hundred and sixty-five unique combinations of 4 parts can be made. That is, each such combination of four parts will differ from other combinations by at least one detail.

Answer: 1365 ways

In formula $C_n^m = \frac{n!}{(n-m)!\cdot m!}$ it is necessary to pay close attention, since it is a "hit" of combinatorics. At the same time, it is useful to UNDERSTAND and write down the “extreme” values without any calculations: $C_n^0 = 1, \quad C_n^1 = n, \quad C_n^{n-1} = n, \quad C_n^n = 1$. As applied to the analyzed problem:

$C_{15}^0 = 1$ – the only way you can not select a single detail;

$C_{15}^1 = 15$ the way you can select a single detail from 15;

$C_{15}^{14} = 15$ ways you can take 14 parts (in this case, one of the 15 will remain in the box);

$C_{15}^{15} = 1$ – the only way is to take all fifteen parts.

I recommend that you carefully familiarize yourself with Newton's binomial and Pascal's triangle, according to which, by the way, it is very convenient to check calculations C_n^m for small values of "n".

Thus, the following principles underlie the training system for solving such problems:

• the psychological content of education is a strategy for developing the flexibility of children's thinking;

• taking into account the process of interpretation (initial performance of tasks in practical activities, then transferring practical actions through speech into a plan of mental actions);

• consistent use of the enumeration method in order to teach rational methods of systematic enumeration as a basis for introducing combinatorial rules and formulas in the future.

The complexity of combinatorial problems lies in the fact that when solving them, such a system of constructed enumeration must be chosen that would give full confidence that all possible cases have been considered (without repeating combinations).

The enumeration is always carried out according to some attribute of objects and is directly related to the operation of classifying objects. Therefore, an important element of the child's readiness to master the methods of solving combinatorial problems is his ability to single out various features of objects, to classify sets of the same objects on various grounds.

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