UDC 530.12

WHICH FORMULA OF SPECIAL THEORY OF RELATIVITY IS TRUE?

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https://doi.org/10.5281/zenodo.7786468

Abstract. *The paper considers three formulas for calculating the observed time interval according to the readings of a clock moving towards the observer. The first formula follows from the Lorentz transformations of the special theory of relativity (STR), the second - from the STR relation for the Doppler effect, the third formula is given in an article published in the scientific journal "Uspekhi Fizicheskikh Nauk". It is shown that under the same conditions, with the speed of movement towards the observer equal to 0.99 of the speed of light, the formulas give different results, differing from each other by 100 and 709 times. The question is raised about the choice of the true formula. The relevance of the work is due to the fact that SRT is used in applied sciences, in particular in the development of global satellite navigation systems, rocket and space technology. It is shown that the use of SRT formulas can lead to errors in determining the observed time interval of clocks of satellite navigation systems of the order* $1\cdot10^{-05}$. *Such an error in determining the time can lead to an error in determining the distance to the navigation satellite equal to 3000 m. The error in determining the location of GLONASS navigation receivers in the horizontal plane should not exceed 12 m, therefore, the errors in calculating the time intervals that SRT formulas lead to are certainly unacceptable. The use of erroneous formulas can lead not only to the erroneous operation of aviation and space technology in which these formulas are used, but also to catastrophic consequences.*

Keywords: *BeiDou, GLONASS, GALILEO, GPS, global navigation satellite system, special relativity, satellite navigation system, STR, observer, observed clock readings, clock, Doppler effect.*

Acknowledgment

We express our sincere gratitude to Dr. Yakovlev Viktor Timofeevich for discussion of the issues considered in the work, support and recommendations on the presentation of the material.

1.Introduction

This discussion article raises the question of which of the formulas of the special theory of relativity (STR) is true. The importance of this issue is due to the fact that SRT formulas are used in the development of space and rocket technology, as well as global satellite navigation systems GPS, GLONASS, GALILEO, BeiDou [1-7].

The use of erroneous formulas in navigation and surveillance systems can lead to errors in the operation of these systems and, as a result, to disastrous consequences.

Introduction to ADS-B Automatic Dependent Surveillance Broadcast

Automatic Dependent Surveillance Broadcast (ADS-B) is a broadcast transmission from an aircraft (aircraft) of its position (latitude and longitude), altitude, speed, as well as identification and other information received from on-board systems. The ADS-B message may include information about the time when the message was broadcast (Figure 1).

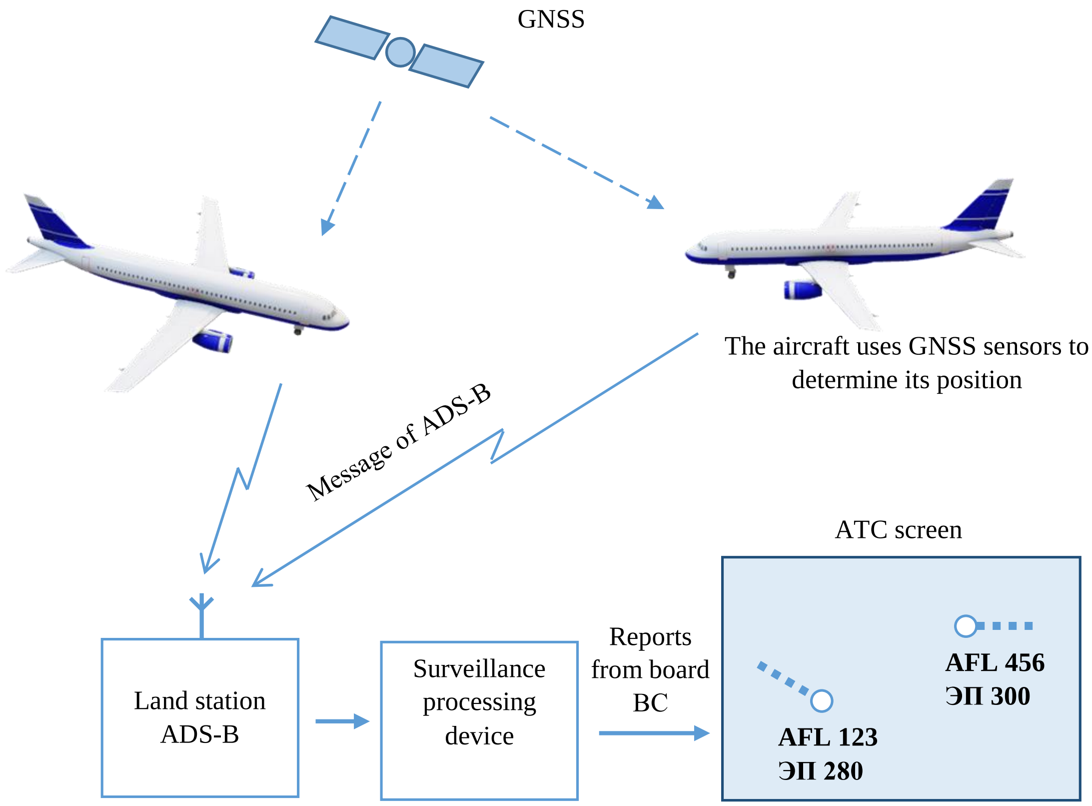

Fig. 1. Broadcast Automatic Dependent Surveillance ADS-B

Aircraft position, speed data is usually obtained from the onboard GNSS (Global Navigation Satellite System). Altitude data is usually obtained from a barometric altimeter encoder.

2. Materials andmethods

SRT is presented in numerous publications, including the works of A. Einstein himself, which should be studied first of all [8-13].

Let us consider a platform moving towards a motionless observer located at the origin of the coordinates of the inertial calculation system (ISS) (Fig. 2).

Platform 1 contains:

1. Clock 2 with an arrow or digital dial, which discretely, at the beginning of the next second, change their readings.

2. Lamp 3 flashing with frequency f_0^l, equal to 1 Hz. The flashes of the lamp occur synchronously with the change in the readings of the second hand of the watch (or the readings of the seconds on the digital display of the watch), that is, at the moments of the beginning of the next second.

The radio transmitter 4 of the exact time signals, which broadcasts the exact time signals synchronously with the change in the clock readings and the flashes of the lamp, at the moments of the beginning of the next second. The frequency of broadcasting messages of the exact time f_0^t equal to1 Hz.

1. ADS-B automatic dependent surveillance broadcast transmitter 5, which broadcasts messages with information about the time points of the message transmission. ADS-B messages are transmitted on the air synchronously with the change in the indications of the second hand of

the clock (seconds on the digital display of the watch) at the moments of the beginning of the next second. Broadcast frequency of messages ADS-B f_0^{ADS} equal to1 Hz.

2. Transmitter 6 e / m (electromagnetic) wave frequency f_0^{v}equal to1 Hz. One wave period per second is transmitted on the air, and the beginning of each next wave period occurs at the moments of change in the indications of the clock seconds (at the moments of the beginning of the next second).

Near the observer at the origin of coordinates, there is a measuring unit 7, which makes it possible to observe (register) signals sent from devices located on the moving platform and measure their physical quantities.

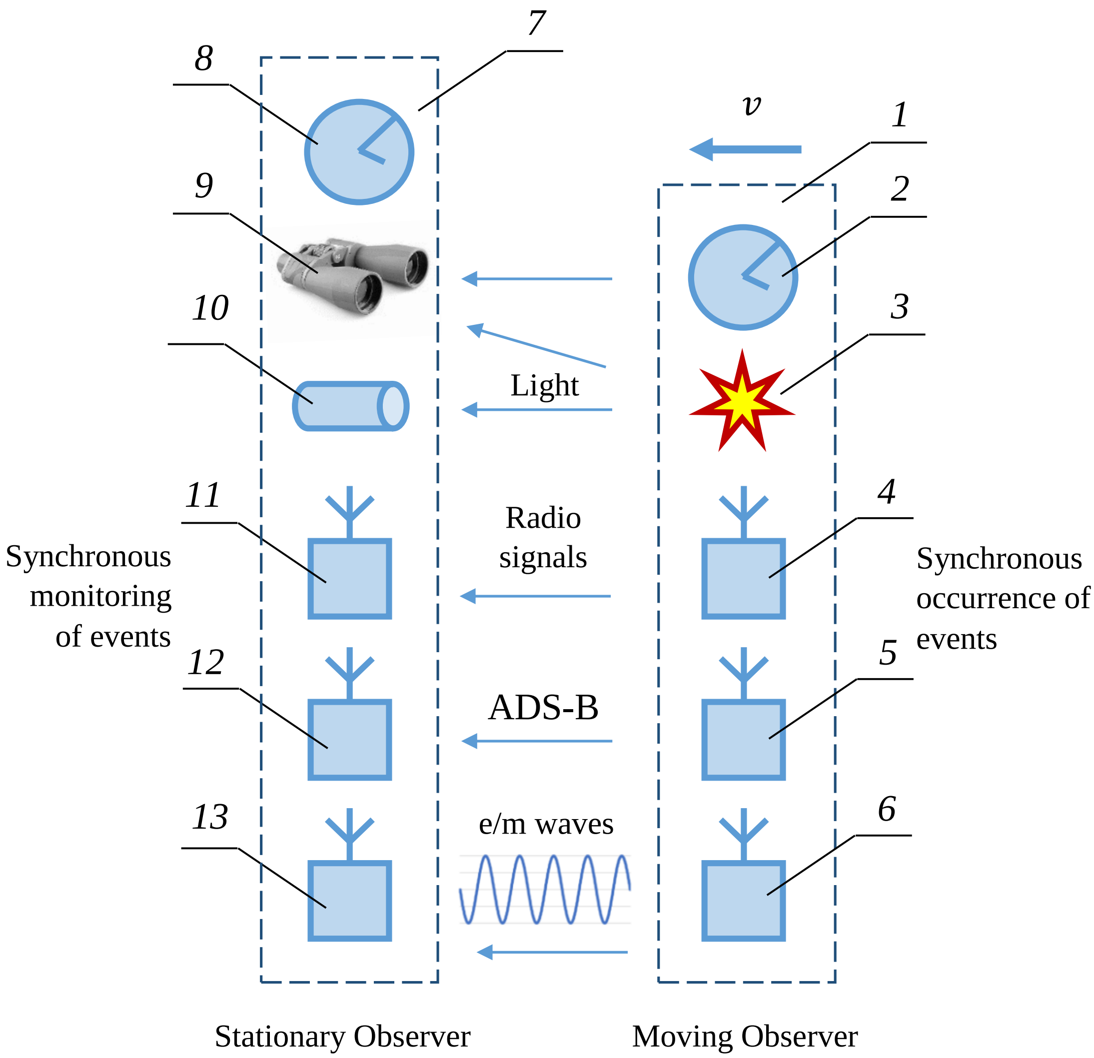

Fig. 2. Observation of the moving clock: 1 - moving platform; 2 – observed hours; 3 - lamp; 4 – radio transmitter of exact time signals; 5 – ADS-B transmitter; 6 - transmitter of e / m waves; 7 – measuring setup; 8 - stationary clock; 9 - video recorder; 10 - photodetector; 11 - Radio receiver of signals of exact time; 12 – ADS-B receiver; 13 - receiver of e / m waves

Measuring setup 7 includes:

1. Video recorder 9, which provides observation of the readings of watches moving on the platform in the optical range, for example, using a telescope, as well as video recording of the observed readings of hours 8 for the purpose of subsequent viewing and analysis of their course. The video recorder provides a record of the time of the observer's fixed clock located in the immediate vicinity of him (frame within frame). Thus, the recording of the video recorder makes

it possible to compare with high accuracy the readings of the observer's stationary clock with the observed readings of clocks moving on the platform.

The video recorder also provides registration and recording of the observed flashes of the lamp located on the platform. DVR recording makes it possible to measure the frequency of observed lamp flashes f^l.

1. Photodetector of 10 flashes of the lamp, also allowing to measure the frequency of the observed flashes of the lamp f^l.
2. Radio receiver 11 signals of exact time, providing the ability to measure the frequency of the moments of reception of these signals f^t.
3. Receiver 12 broadcasting automatic dependent surveillance ADS-B, providing the ability to record on the objective control system information about the time of transmission of ADS-B messages, as well as the time of receipt of these messages. The ADS-B receiver provides the ability to measure the observed frequency at which messages are received f^{ADS}, and also to compare the observed readings of clocks moving on the platform with the readings of a stationary clock (stationary) of the observer located at the origin of coordinates.
4. Receiver 13 e / m waves, providing the ability to measure the frequency f^v of the received e/m wave, as well as the possibility of registering the moments of time (by the clock of a stationary observer) of the beginning of the period of the received e/m wave.

All devices located on the platform moving towards the stationary observer are synchronized. From the point of view of another observer who is on a moving platform, the following events occur simultaneously (synchronously):

1. Changes in the indications of the clock located on the platform, which count the time by the second;

2. Flash lamp;

3. Broadcasting radio signals of exact time;

4. Broadcast of ADS-B messages;

5. The beginning of the next period of the e / m wave transmitted on the air.

Lamp flash frequency f_0^l, frequency of transmission of radio signals of exact time f_0^t, message rate ADS-B $f_0^{\text{ADS}}$and the frequency of the transmitted e / m wave f_0^v, as well as the frequency of change of seconds on the clock face on the platform f_0^{cl}, equal to 1 Hz,

$$f_0^l = f_0^t = f_0^{\text{ADS}} = f_0^v = f_0^{cl} = 1 \text{ Гц}. \quad (1)$$

Information about all the listed events moves to a stationary observer in the air through all observation channels at the same speed - the speed of light c. The light from the dial located on the clock platform goes to the video recorder at the origin at a speed *c*, with the same speed moving: lamp flashes, radio time signals, ADS-B messages, as well as the transmitted electromagnetic wave.

From here, firstly, it follows that the following events at the origin will be simultaneous (synchronous):

1. Observation of changes in the readings of a clock located on a moving platform, which counts time by second;

2. Observation of lamp flashes;

3. Reception of radio signals of exact time;

4. Reception of ADS-B messages;

5. Reception of the beginning of the period of the e / m wave.

In addition, from the fact of synchronous transmission of information about the listed events on the platform and the movement of information about these events to a stationary observer (measuring installation) at the origin of coordinates with the same speed of light, it inevitably follows that the observed (at the origin) lamp flash frequency f^l, frequency of receiving radio signals of exact time f^t, message reception frequency ADS-B f^{ADS}, frequency of received e/m wave f^v, as well as the observed frequency of change of seconds on the clock face on the platform f^{cl}, equals.

With other words,

$$f^l = f^t = f^{\mathrm{ADS}} = f^v = f^{cl}. \quad (2)$$

In accordance with (2), when observing from the origin of devices on a moving platform, the seconds according to the observed clock readings go synchronously with the observed flashes of the lamp, the received signals of the exact time, the received ADS-B messages, and also with the periods of the received e / m wave.

This is a very important conclusion.

From (2) follows the following. Since the frequency of the received e / m wave is equal to the observed frequency of the seconds of the moving clock, that is $f^v = f^{cl}$, then the time interval Δt_{obs}, counted from the observed readings of the moving clock for the corresponding time interval of the stationary clock Δt, is numerically equal to the average number of periods of the received e/m wave $n^v(\Delta t)$ (as well as the average number of lamp flashes observed, the average number of time signals received, the average number of ADS-B messages received) during this stationary clock interval Δt.

In other words

$$\Delta t_{obs} = n^l(\Delta t) = n^t(\Delta t) = n^{\mathrm{ADS}}(\Delta t) = n^v(\Delta t) = n^{cl}(\Delta t), \quad (3)$$

where

$$n^l(\Delta t) = f^l \Delta t,$$
$$n^t(\Delta t) = f^t \Delta t,$$
$$n^{ADS}(\Delta t) = f^{ADS} \Delta t,$$
$$n^v(\Delta t) = f^v \Delta t,$$
$$n^{cl}(\Delta t) = f^{cl} \Delta t,$$

$n^l(\Delta t), n^t(\Delta t), n^{\mathrm{ADS}}(\Delta t), n^v(\Delta t), n^{cl}(\Delta t)$ – respectively, the average number of observed lamp flashes, the average number of received accurate time signals, the average number of received ADS-B messages, the average number of periods of the received e / m wave, and the average number of seconds according to the observed readings of the clock located on the moving platform, fixed for the time interval Δt by the fixed clock of the (stationary) observer.

From formulas (3) it follows that

$$\Delta t_{obs} = f^l \Delta t = f^t \Delta t = f^{ADS} \Delta t = f^v \Delta t = f^{cl} \Delta t. \quad (4a)$$

We see that the observed time interval according to the readings of the moving clock Δt_{obs} can be determined in many ways, using a telescope and video recorder for observing moving clocks in the optical range, a photodetector for registering lamp flashes, a transmitter and receiver of radio signals of exact time, an ADS-B message transmitter and receiver, an E/M wave transmitter and receiver.

Equalities (3) and (4) indicate that no matter what method of observing the moving clock is chosen, in all cases of observation the observed rate of the moving clock will be the same.

In other words: regardless of how the moving clock is observed (using a telescope in the optical range, a lamp flash photodetector, radio time signals, ADS-B or EM waves), a stationary observer will record the same moving clock readings, the same the same observed rate of progress of the readings of a moving clock.

So, for example, if on a platform moving towards a stationary observer there is only a clock and a transmitter of an e / m wave with a frequency of 1 Hz (one wave per second), then the value of the time interval observed from the readings of the moving clock Δt_{obs} according to *(4a)* can be determined by the formula

$$\Delta t_{obs} = f^v \Delta t, \qquad (4b)$$

where f^v – the frequency of the e/m wave received at the origin, which can be measured;
Δt – the interval of time that has passed according to the fixed clock of a (stationary) observer.

The value of the time interval observed from the indications of a moving clock

It was proved above that regardless of the method of observing the readings of a moving clock (including using e / m waves), in all cases the result of observation will be the same.

Let us now consider what speaks about the observed time interval according to the readings of the moving STR clock.

The first formula for calculating the time interval observed from the readings of a moving clock follows from the following SRT statement: "If you observe the clock from a system in relation to which they move uniformly at a speed v, then it turns out that they go to $1/\sqrt{1-(v/c)^2}$ times slower than the same clock that is stationary with respect to this system" [11, p. 156].

In [13 p. 78] the following formula is given, confirming this position:

$$\Delta t = \frac{\Delta t_{obs}}{\sqrt{1-\left(\frac{v}{c}\right)^2}}.$$

As well:

$$\Delta t_{obs} = \Delta t\sqrt{1-\left(\frac{v}{c}\right)^2} \qquad (5)$$

where Δt – the time interval passed by the fixed clock of the (stationary) observer;
Δt_{obs} – the interval of time elapsed according to the observed readings of a moving clock.
The second formula follows from the Doppler effect. In STR, this effect is determined by the known relations:
when the transmitter approaches the receiver:

$$f^v = f_0^v \frac{\sqrt{1-\left(\frac{v}{c}\right)^2}}{1-\frac{v}{c}},$$

when moving the transmitter away from the receiver:

$$f^v = f_0^v \frac{\sqrt{1-\left(\frac{v}{c}\right)^2}}{1+\frac{v}{c}},$$

where f_0^v – transmitter frequency;
f^v – the frequency of the received e / m wave.

Considering that in our example the transmitted e / m wave has a frequency of 1 Hz, that is $f_0^v = 1$, then when the platform with the clock approaches the stationary observer

$$f^v = \frac{\sqrt{1-\left(\frac{v}{c}\right)^2}}{1-\frac{v}{c}}. \qquad (6)$$

Since the value of the time interval observed from the readings of the moving clock Δt_{obs} according to *(4b)* is directly proportional to the frequency f^v of the received e/m wave, taking into account (6), we can obtain:

$$\Delta t_{obs} = \Delta t \cdot f^v = \Delta t \frac{\sqrt{1-\left(\frac{v}{c}\right)^2}}{1-\frac{v}{c}}. \quad (7)$$

This formula gives the second answer to the question about the magnitude of the observed time interval according to the readings of a clock approaching a stationary observer Δt_{obs}, if the time interval has passed by the fixed clock Δt.

The third formula can be obtained from the work, which was published in the authoritative journal "Uspekhi Fizicheskikh Nauk", recommended by the Higher Attestation Commission and included in the Scopus and WoS databases [14].

In this work, the formula

$$\Delta t = \Delta t_{obs}\left(1 - \frac{v}{c}\right),$$

where Δt_{obs} – the duration of an event occurring on a body moving towards a stationary observer; Δt – the duration of the same event, measured by a stationary observer.

In accordance with this, the value of the observed time interval is equal to

$$\Delta t_{obs} = \Delta t \frac{1}{1-\frac{v}{c}}, \quad (8)$$

Formula (8) was obtained by the author in [15-21]. In [15], the results of experimental confirmation of the validity of formula (8) are presented.

3. Results and discussion

An important result obtained is the following statement: the average number of seconds observed from the indications of a moving clock $n^{cl}(\Delta t)$ per time interval Δt for a fixed clock is equal to the average number of periods of the received e / m wave $n^v(\Delta t)$ for the same time interval Δt at a transmitter frequency of e / m waves equal to 1 Hz:

$$n^{cl}(\Delta t) = n^v(\Delta t) = f^v \Delta t.$$

Since the value of the time interval observed from the readings of the moving clock Δt_{obs} is the number of seconds observed by the moving clock $n^{cl}(\Delta t)$, passed during the time interval Δt, then the relation ($4b$)

$$\Delta t_{obs} = f^v \Delta t.$$

Main conclusion: the value of the time interval observed from the indications of a moving clock Δt_{obs} per time interval Δt about a stationary clock, numerically equal to the frequency of received electromagnetic waves (at a transmitter frequency of 1 Hz) multiplied by the value of this time interval Δt.

The main conclusion *(4b)* is a consequence of the synchronous observation of the progress of seconds on a moving clock, lamp flashes, receipt of radio time signals and ADS-B messages, as well as periods of the received e/m wave.

Three different formulas for calculating the same physical quantity have been obtained: the time interval that has elapsed according to the observed indications of a clock moving towards a stationary observer Δt_{obs} (5), (7) и (8). These formulas are summarized in Table 1.

Table 1

Three formulas for calculating the time interval elapsed according to the observed indications of a clock moving towards a stationary observer Δt_{obs}

Formula source	Lorentz transformations	Doppler effect	Publications [14-21]
Formula	$\Delta t_{obs} = \Delta t\sqrt{1-\left(\frac{v}{c}\right)^2}$	$\Delta t_{obs} = \Delta t\frac{\sqrt{1-\left(\frac{v}{c}\right)^2}}{1-\frac{v}{c}}$	$\Delta t_{obs} = \Delta t\frac{1}{1-\frac{v}{c}}$

So, answering the question: how much time will pass according to the observed readings of those moving at a speed v to a stationary observer of the clock, if the time interval Δt passes by the clock of this observer, we have obtained three different formulas.

Table 2 shows the numerical values of the observed time interval, expressed in seconds, that has passed in one second according to the stationary clock of the (stationary) observer at a speed of movement (towards the observer) equal to 0.99 of the speed light. The values calculated by different formulas differ significantly from each other. Values in table 2 marked with superscripts $^{(1)}$ and $^{(2)}$, differ from each other by 100 times, and the values marked with indices $^{(1)}$ and $^{(3)}$, differ from each other by 708.9 times!

Table 2

Results of calculations of the observed time interval according to formulas (5), (7) and (8), sec

Speed, v	Lorentz transformations, (5)	Doppler Effect, (7)	Publications [14-21], (8)
0,99 c	0,1410$^{(1)}$	14,1067$^{(2)}$	100$^{(3)}$
3500 м/с	0,999 999 999 932$^{(4)}$	1,000 011 666 735$^{(5)}$	1,000 011 666 803$^{(6)}$

At a movement speed comparable to the linear speed of movement of satellites of global navigation systems (GNSS) equal to 3500 m/s, formula (5) in comparison with formula (8) gives a relative error in the value of the observed time interval $1{,}167\cdot10^{-05}$, formula (7) in comparison with formula (8) leads to a relative error $6{,}805\cdot10^{-11}$. Since the accuracy of clocks on GNSS satellites is of the order $5\cdot10^{-11}$, then the errors to which the STR formulas lead are unacceptably high in terms of modern requirements for the accuracy of determining time in GNSS.

Note that the use of formula (5) in comparison with formula (8) entails an error in the calculation of the observed time interval, equal to $1{,}167\cdot10^{-05}$. During this time, electromagnetic waves propagate over a distance of approximately 3.5 km. Since the positioning error in GLONASS (global navigation satellite system) according to modern requirements should not exceed 12 m in the horizontal plane [22], it follows from this that the use of erroneous SRT formulas in satellite navigation systems can lead to unacceptable errors in positioning.

So, in the case of using erroneous STR formulas, errors in estimating time and place in satellite navigation systems that are unacceptable according to modern requirements may occur.

In other words, STR cannot be used in modern satellite navigation systems.

Answering one question, we received three different answers. Obviously, there can be only one answer to this question. Otherwise, according to the apt expression of one of the scientists - Doctor of Technical Sciences, "an observer who, watching one clock, simultaneously sees three different values of time, will need to be taken to a psychiatric hospital."

The conclusion inevitably follows from this: at least one of the SRT formulas (5) or (7) is erroneous. It is possible that both of these formulas are erroneous.

The question about the observed indications of a clock approaching the observer led to three radically (100 and 709 times) different answers. The physical meaning of the result obtained is as follows: if a stationary observer follows the readings of a clock moving towards him, then, in accordance with SRT, using its various formulas, as well as using the formula given in [14-21], he will come to the conclusion: observable watches, according to their observed indications, simultaneously run at three different speeds (with different rates of movement). These speeds of the same clock, at the same time, differ hundreds of times. Obviously, this cannot be (otherwise, the observer who discovered this amazing phenomenon is waiting for a psychiatric hospital).

4. The key issue of this work

Thus, an important question arises: which of the three formulas (5), (7) and (8) is true?

Formula (5) follows from the Lorentz transformations. Formula (7) is from the SRT relation for the Doppler effect.

5. Summary

In this problematic article, three different formulas for calculating the same physical quantity in accordance with SRT are given: the observed time interval according to the readings of a clock moving towards a stationary observer.

With the same initial data, these formulas give values that at a speed of 0.99 the speed of light differ from each other by 100 and 709 times.

Using different SRT formulas, the time interval of the clocks of GNSS navigation satellites observed from the Earth was calculated. It turned out that at a navigation satellite speed of 3500 m/s, STR formulas lead to an error of the order $1 \cdot 10^{-05}$. This time estimation error can lead to an error in determining the distance traveled by the e/m waves, on the order of 3000 m.

With the limit of permissible error in determining the location of GLONASS consumers in the horizontal plane equal to 12 m, the errors that SRT formulas lead to are, of course, unacceptable.

The question arises: which of the three formulas (5), (7) or (8) is true?

The answer to this question is important due to the fact that STR is used in applied sciences, in particular, in the development of global navigation satellite systems, rocket and space technology.

The use of erroneous formulas can lead not only to the erroneous operation of aviation and space technology in which these formulas are used, but also to catastrophic consequences. That is why determining which of the STR formulas is true is an important and relevant scientific task.

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