CREATIVE APPROACHES TO THE SOLUTION OF TASKS IN SCHOOL TEXTBOOKS

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Abstract. *In this thesis, the concept of the distance between two intersecting straight lines, which is introduced as a problematic issue in general education school textbooks and has wide applications in practice, is described and the methods of finding this distance are described in detail.*

Keywords: *intersecting straight lines, parallel straight lines, parallel planes, distance between parallel straight lines, distance between parallel planes, volume of the pyramid.*

In modern mathematics textbooks, great emphasis is placed on dividing straight lines and finding the distance between them. The reason for this can be shown by the fact that finding the distance between straight lines is widely encountered in the problems of mechanics and differential geometry and has wide applications in life.

Below are some basic definitions of bisecting straight lines and finding the distance between them, with plenty of examples. First, let's define straight lines:

Definition 1. The intersection of two straight lines that do not lie in the same plane in space are called straight lines (Fig. 1).

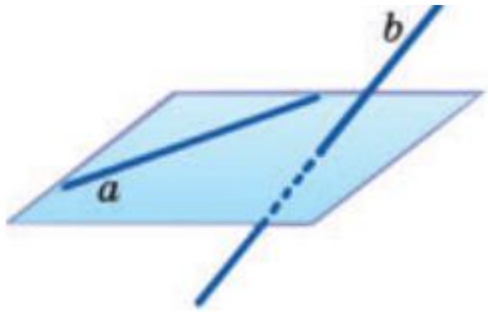

Figure 1

Before introducing the concept of distance between two intersecting straight lines, we define the concept of distance between two shapes.

Definition 2. For points F_1 and F_2 and arbitrary $A_1 \in F_1$, $A_2 \in F_2$of the given forms $B_1 \in F_1$, $B_2 \in F_2$ if inequality is appropriate. Points $|A_1A_2| \leq |B_1B_2|$ are called the closest points of the given shapes. The distance between A_1 va A_2 forms means the distance between their nearest points (if it exists).

Now we present several definitions that are convenient for finding the distance between two intersecting straight lines in practice:

Definition 3. The distance between the nearest points of two intersecting straight lines is called the distance between the straight lines.

Definition 4. The distance between parallel planes where two intersecting straight lines are located is called the distance between straight lines.

Definition 5. The distance from one of the two intersecting straight lines to the parallel plane on which the second intersecting straight line lies is called the distance between these lines.

Below we show the process of solving the problem in several ways:

Problem solving. A right prism $ABCDA_1B_1C_1D_1$whose base is a square is given. If the side of the base of the prism is 4 and the height is $2\sqrt{2}$, find the distance between the diagonals DA_1 and CD_1 (Fig. 2).

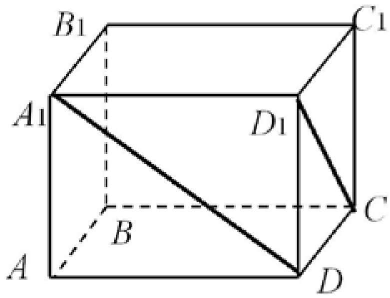

Figure 2

Note: Below (ABC) - the inscription in the form ABC indicates the plane.

1-way. It is known that CD_1 lies on the diagonal (CB_1D_1). Also, from the fact that $DA_1||CB_1$follows that the straight line DA_1 (CB_1D_1) is parallel to the plane. So, the distance between the planes DA_1 and (CB_1D_1) is equal to the distance between the diagonals DA_1 and CD_1 according to definition 5. In particular, to find this distance, it is enough to find the distance from the point A_1 of the straight line DA_1 (CB_1D_1) to the plane (Fig. 3).

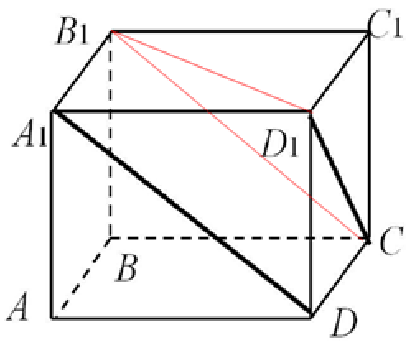

Figure 3

We designate the points of intersection of the diagonals of the bases of the given right prism by $O\ and\ O_1$, respectively.

It is known that the planes (ACC_1) and (CB_1D_1)intersect in the straight line O_1C.

So, to find the distance from A_1point (CB_1D_1) to the plane, it is enough to find the distance A_1 from point O_1C to the straight line A_1H (Fig. 4).

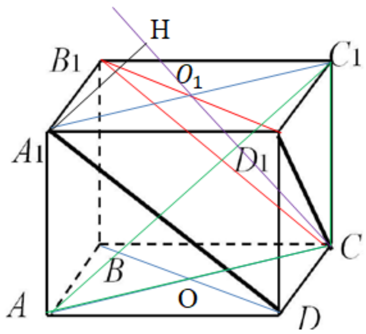

Figure 4

According to the condition of the problem, $C_1C = 2\sqrt{2}$, $C_1D_1 = 4$. So, ΔCO_1C_1 for $C_1O_1 = C_1C = 2\sqrt{2}$ right triangle. From this comes the equality $\measuredangle CO_1C_1 = 45^0$ ^0. Also, $\measuredangle CO_1C_1 = \measuredangle HO_1A_1$

ΔA_1HO_1- in a right triangle, $A_1O_1 = 2\sqrt{2}\, va \sin \measuredangle HO_1A_1 = \frac{\sqrt{2}}{2}$. Therefore, $HA_1 = A_1O_1 \cdot \sin \measuredangle HO_1A_1 = 2\sqrt{2} \cdot \frac{\sqrt{2}}{2} = 2$.

2-way. (BDA_1) and (CB_1D_1) planes are parallel to each other, because $DA_1||CB_1, CD_1||BA_1$. Also, the straight lines A_1D and CD_1 lie on (BDA_1) and (CB_1D_1) planes, respectively. So, to find the distance between these straight lines, it is enough to find the distance between(BDA_1) and (CB_1D_1) planes. In turn, to find the distance between these planes, it is necessary to find the distance from the point O on the plane (BDA_1) to the plane (CB_1D_1).

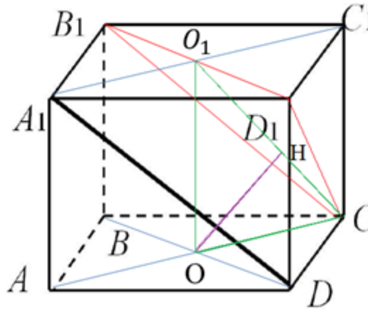

Figure 5

(OCO_1) and (CB_1D_1) planes are perpendicular to each other and intersect in a straight line CO_1. We pass OH perpendicular from point O to straight line CO_1. The length of this section OH is equal to the distance from the point O (CB_1D_1) to the plane, so it is also equal to the distance between the straight lines A_1D and CD_1 (Fig. 5). We find the length of this section.

It is known that $AA_1 = OO_1 = 2\sqrt{2}$, $AO = CO = 2\sqrt{2}$. . Also from ΔOAA_1: $A_1O = CO_1 = \sqrt{AA_1^2 + AO^2} = 4$.

$\Delta OHO_1 \sim \Delta COO_1$. Because the $OO_1 -$ side is general and $\measuredangle CO_1O = \measuredangle OO_1H$. So, $\frac{OH}{CO} = \frac{OO_1}{CO_1}$. From this, $OH = \frac{CO \cdot OO_1}{CO_1} = \frac{2\sqrt{2} \cdot 2\sqrt{2}}{4} = 2$.

3-way. Method of volumes

It is known that $DA_1||CB_1$ and $CD_1||BA_1$ relations are relevant. From these conditions, the relation $(BDA_1)||(CB_1D_1)$ is derived (Fig. 6).

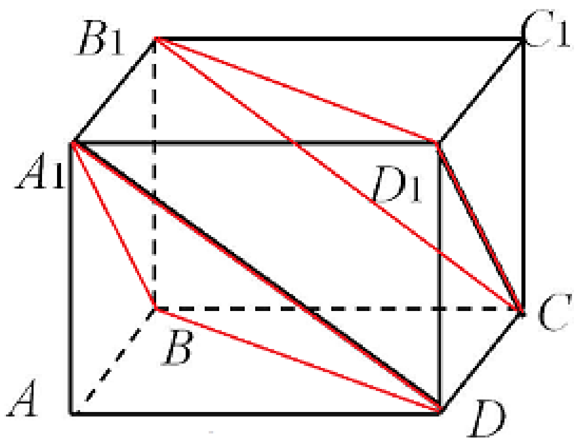

Figure 6

$BCDA_1$ − Let's look at the pyramid (Fig. 7).

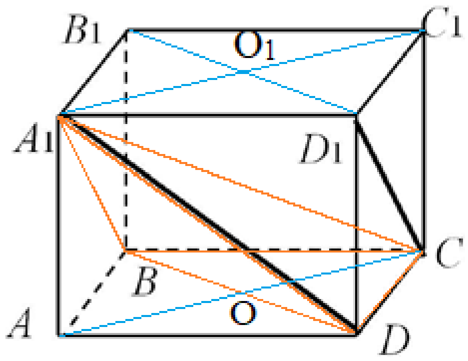

Figure 7

We denote by h the height dropped from the end of this pyramid C to the base BDA_1 The length of this height is equal to the distance between the diagonals DA_1 and CD_1. Because this length is equal to the distance from the end C of the diagonal CD_1 to the plane BDA_1 lying on the diagonal DA_1. We find the length of this height.

From the conditions of the problem, it can be directly found that $BD = AC = 4\sqrt{2},\ AO = 2\sqrt{2}$. From right angle $\Delta OAA_1\ O_1$: $A_1O = CO_1 = \sqrt{AA_1^2 + AO^2} = 4$. .

We find the volume of the pyramid $BCDA_1$ − - through the base BDA_1- and h- height:

$$V_{BCDA_1} = \frac{1}{3}S_{BDA_1} \cdot h = \frac{1}{3} \cdot \frac{A_1O \cdot BD}{2} \cdot h = \frac{8\sqrt{2}}{3} \cdot h.$$

We find the volume of the pyramid $BCDA_1$- by the base BCD − and the height AA_1:

$$V_{BCDA_1} = \frac{1}{3}S_{BCD} \cdot AA_1 = \frac{1}{3} \cdot \frac{BC \cdot CD}{2} \cdot AA_1 = \frac{16\sqrt{2}}{3}.$$

So, $V_{BCDA_1} = \frac{8\sqrt{2}}{3} \cdot h = \frac{16\sqrt{2}}{3} \Rightarrow h = 2.$

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