PROPERTIES OF THE SET OF SOLUTIONS AND AN ENSEMBLE OF TRAJECTORIES OF A DIFFERENTIAL INCLUSION WITH A DELAY ARGUMENT

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Abstract. *In this paper, a mathematical model of the control object in the form of a differential inclusion with a delay argument. For such models of dynamical systems, the Cauchy problem is studied. Some properties of the set of solutions and the ensemble of trajectories of the considered class of differential inclusions are studied. Conditions for continuous dependence, closedness, and convexity of multivalued mappings related to the studied properties of differential inclusions are obtained.*
Keywords: *differential inclusion, delay argument, Cauchy problem, sets of solutions, ensemble of trajectories, continuous dependence, closedness, convexity.*

INTRODUCTION

Great interest in differential inclusions led to their effective applications to optimal control problems and other problems in applied mathematics . Since the 60 s of the last century, questions of the theory of differential inclusions and their applications have been studied in the works of A.F. Filippov, T Wazewski, J . P. Aubin, C.Castaing, N. Kikuchi, A. Cellina , V.I. Blagodatskikx, A.B. Kurzhansky , S.M. Aseev, V.A. Plotnikov, E.S. Polovinkin, A.A. Tolstonogov and others [1,2,4,6,8,11] It should be noted that differential inclusions have important applications in the theory of differential games.

Differential inclusions have various generalizations. Research is being developed on differential-functional and integro-differential inclusions, differential inclusions with delays, differential inclusions in partial derivatives, differential inclusions in Banach spaces, differential inclusions with a fuzzy right hand side, controlled differential inclusions [5,7,8,10,17,18].

Along with other developing areas in the theory of differential inclusions, research on differential inclusions with a delay argument is being actively conducted. Differential inclusions with a delay argument have been studied in the works of A.D. Myshkis , A.B. Kurzhansky , L.I. Minchenko, E.V. Duda, V.N. Teslyuk and others [5, 7] .

Controlled differential inclusions, i.e. differential inclusions with control parameter, begin to study since the 80 s of the last century. Studies devoted to the properties of a family of solutions and the reachability set of controlled differential inclusions, questions of control and optimization of the ensemble of trajectories of such systems according to various criteria are developed in the works of N. Kikuchi, N. S. Papageorgio, G.N. Konstantinov, T.F. Filippova, A.V. Plotnikov and others [17–20]. Problems of control of an ensemble of trajectories of differential inclusions with a delay argument are one of the developing directions in the theory of controlled differential inclusions. Separate aspects of issues related to the properties of trajectories of some particular classes of controlled differential inclusions with delays are considered in [12–16].

OBJECT OF STUDY AND METHODS

In what follows, we will use the following notation: $\Omega(G)$ is the set of all compact sets from the space G; $co\Omega(G)$ is the set of all convex compact sets from G; $\rho^*(X,Y)=\sup\{\rho(x,Y): x\in X\}$; $h(X,Y)=\max\{\rho^*(X,Y),\ \rho^*(Y,X)\}$– Hausdorff metric of sets $X\subset G$ and $Y\subset G$; R^n– n-dimensional Euclidean space of vectors $x=(x_1,..., x_n)$ with scalar product $(x,y)=\sum_{i=1}^{n} x_i y_i$ and norm $\|x\|_{R^n}=\sqrt{(x,x)}$; $C(X,\psi)=\sup\{(x,\psi):\ \ x\in X\}$ is the support function of the set $X\subset R^n$; $C^n(T)$- the space of continuous on the interval $T\subset R^1$ n-dimensional vector functions; $L_1(T)$ is the space of T Lebesgue integrable functions.

Continuity and measurability of multivalued mappings will be understood as in [2,3,11].

Consider a control object whose state at each moment of time $t\in T=[t_0,t_1]$ is expressed by *an* n-dimensional vector $x=x(t)$ satisfying the differential inclusion with delays

$$\dot{x}\in F(t,x(t),x(t-h_1(t)),...,x(t-h_k(t)),u), \tag{1}$$

where $F(t,x,y,u)\subset R^n$, $x\in R^n$, $y=(y_1,..., y_k)$, $y_i\in R^n$, $i=\overline{1,k}$, $u\in V\subset R^m$. Here, the parameter $u=u(t)$ plays the role of control actions, and $h_i(t)\geq 0$, $i=\overline{1,k}$, are given functions. The model of a dynamic system of the form (1) is called a controlled differential inclusion with delays.

For the control system model (1), for the class of admissible controls, we choose measurable bounded m- vector functions $u=u(t),\ \ t\in T$. Denote by U the set of all measurable bounded controls $u=u(t),\ \ t\in T$ that take almost everywhere on T values from a convex compact set $V\subset R^m$.

We assume that the delays $h_i(t),\ \ i=\overline{1,k}$, are non-negative and continuous on T functions. Let $t_*=\min_{i-\overline{1,x}}\min_{t\in T}[t-h_i(t)]$, $T_0=[t_*,t_0]$, $u(\cdot)\in U$, $D^0\subset C^n(T_0)$. Consider the Cauchy problem for system (1):

$$\dot{x}\in F(t,x(t),x(t-h_1(t)),..,x(t-h_k(t)),u(t)),\ x_{T_0}(\cdot)\in D^0, \tag{2}$$

where $x_{T_0}(\cdot)$ is the restriction of the function $x(t), t\in T_1=[t_*,t_1]$, to the segment T_0, i.e. $x_{T_0}(t)=x(t)$ at $t\in T_0$. By the solution of this problem we mean continuous on T_0 and absolutely continuous on T n-vector function $x=x(t)$ satisfying the differential inclusion (1) almost everywhere on T and the initial condition $x_{T_0}(\cdot)\in D^0$.

Let $H(u,D^0)$ the set of all solutions of the Cauchy problem (2). Let's put $X(\tau,u,D^0)=\{\xi\in R^n:\xi=x(\tau), x(\cdot)\in H(u,D^0)\}$, $\tau\in T_1$. We will use the following well-known conditions for the compactness and convexity of the sets $H(u,D^0)$ and $X(\tau,u,D^0)$.

Lemma 1. [9] Let the following conditions be met:

a1) for any $(t,x,y,u)\in T\times R^n\times R^{nk}\times V$ the set $F(t,x,y,u)$ is convex compact from R^n;

b1) the set-valued mapping $(t,x,y,u)\to F(t,x,y,u)$ is measurable with t respect to $\forall(x,y,u)\in R^n\times R^{nk}\times V$ and continuous with (x,y,u) respect to almost all $t\in T$;

c1) there are functions $g_i(t,u),\ i=\overline{1,2}$ such that functions $g_i(t,u(t)),\ i=\overline{1,2}$ are summable on T for any $u(\cdot)\in U$ and

$$\|f\|\le g_1(t,u)\left(\|x\|+\sum_{i=1}^{k}\|y_i\|\right)+g_2(t,u),$$
$$\forall f\in F(t,x,y,u),\ (t,x,y,u)\in T\times R^n\times R^{nk}\times V. \tag{3}$$

Then for any $u(\cdot)\in U$ and $D^0\in\Omega(C^n(T_0))$ the sets $H(u,D^0)$ and $X(t,u,D^0),\ t\in T_1$, are non-empty compact sets from, $C^n(T_1)$ and R^n, respectively.

Corollary 1. Let Ω^0 be a set of compact sets D ^{0} from $C^n(T_0)$, and there exists $M_0>0$, such that $\|w\|=\max_{t\in T_0}\|w(t)\|\le M_0,\ \ \forall w\in D^0, D^0\in\Omega^0$. Let us assume that in condition (3) the functions $g_i(t,u)\ i=\overline{1,2}$, such that $\bar g_i(t)=\sup_{u\in V} g_i(t,u),\ \ i=\overline{1,2}$ are summable on T. Then, the set $H(u,D^0)$ for $(u,D^0)\in U\times\Omega^0$, are uniformly bounded.

Lemma 2 . [12] Let the conditions a1) – c1) of Lemma 1 be satisfied. We also assume that :

d1) the support function $c(F(t,x,y,u),\psi)=\max_{f\in F(t,x,y,u)}(f,\psi)$ of the set $F(t,x,y,u)$ is concave in (x,y) for almost all $t\in T$ and all $(u,\psi)\in V\times R^n$.

Then for any $u(\cdot)\in U$ and $D^0\in co\Omega(C^n(T_0))$ the sets $H(u,D^0)$ and $X(t,u,D^0),\ t\in T_1$, are non-empty convex compact sets from, $C^n(T_1)$ and R^n, respectively.

Corollary 2. Let

$$F(t,x,y,u)=A(t)x+\sum_{i=1}^{k}A_i(t)y_i+b(t,u), \tag{4}$$

where $A(t),\ A_i(t),\ i=\overline{1,k},-\ n\times n$ matrices whose elements are summable on T, $b(t,u)\in co\Omega(R^n)$, the multivalued mapping $(t,u)\to b(t,u)$ is measurable with respect to $t\in T$ and continuous with respect to $u\in V$, and $\sup_{\beta\in b(t,u)}\|\beta\|\le\beta_1(t)\|u\|+\beta_2(t),\ \ \beta_i(\cdot)\in L_1(T),\ \ i=\overline{1,2}.$

Then all the statements of Lemma 2 remain valid.

In this paper, as an object of study, we will consider controlled differential inclusions with a delay argument of the form (1) and their linear models (4).

RESULTS OF THE RESEARCH

We will study some properties of multivalued mappings $(u,D^0)\to H(u,D^0),\ (t,u,D^0)\to X(t,u,D^0)$, where $H(u,D^0)$ is the set of absolutely continuous solutions to problem (2), and $X(t,u,D^0)=\{\xi\in R^n:\xi=x(t), x(\cdot)\in H(u,D^0)\}$ the corresponding reachable set at time $t \in T$. The multivalued mapping $t \to X(t,u,D^0)$, $t \in T$, is called the ensemble of trajectories of system (2).

We assume that the right side of the differential inclusion (1) following:

a_2) for any $(t,x,y,u) \in T \times R^n \times R^{nk} \times V$ the set $F(t,x,y,u)$ is a convex compact set from R^n;

b_2) the multivalued mapping $(t,x,y,u) \to F(t,x,y,u)$ is measurable with $t \in T$ respect to $\forall\,(x,y,u) \in R^n \times R^{nk} \times V$ and satisfies the Lipschitz condition with (x,y,u) respect to almost all $t \in T$, i.e.

$$h(F(t,x,y,u),F(t,\xi,\eta,v)) \le l(t)[\|x-\xi\| + \sum_{i=1}^{k}\|y_i-\eta_i\| + \|u-v\|],$$

$$\forall\,(x,y,u),(\xi,\eta,v) \in R^n \times R^{nk} \times V,$$

where is $l(t)$ – the square summable function on T ($l(\cdot) \in L_2(T)$);

c_2) there is an element $(\xi^0,\eta^0,v^0) \in R^n \times R^{nk} \times V$ and a function summable on T $l_1(t)$ such that $\|F(t,\xi^0,\eta^0,v^0)\| \le l_1(t), t \in T$;

d_2) support function $c(F(t,x,y,u),\psi) = \max\{\ (f,\psi) : f \in F(t,x,y,u)\ \}$ is concave in (x, y, u) for almost all $t \in T$.

Under conditions a_2) – c_2) all conditions a_1) – c_1) of Lemma 1 are satisfied. Therefore, we have

Lemma 3. For any $u \in U$, $D^0 \in \Omega(C^n(T_0))$ and $t \in T$ the following relations are true:

$$H(u,D^0) \in \Omega(C^n(T_1)),\ X(t,u,D^0) \in \Omega(R^n).$$

Lemma 4. Let (u_1,D_1), (u_2,D_2) be arbitrary points from $U \times \Omega(C^n(T_0))$. Then for any $x_1(\cdot) \in H(u_1,D_1)$ there exists $x_2(\cdot) \in H(u_2,D_2)$, such that

$$\|x_1(\cdot)-x_2(\cdot)\|_{C^n(T_1)} \le \left[h(D_1,D_2) + \|l(\cdot)\|_{L_2(T)}\|u_1(\cdot)-u_2(\cdot)\|_{L_2^m(T)}\right]M_2,$$

where $M_3 = \exp\left[(k+1)\int_{t_0}^{t_1} l(s)ds\right]$.

Theorem 1. Let conditions a_2) – c_2) be satisfied. Then the multivalued mappings $(u,D) \to H(u,D)$ and $(t,u,D) \to X(t,u,D)$ are continuous on $U \times \Omega(C^n(T_0))$ and, $T \times U \times \Omega(C^n(T_0))$ respectively.

Proof. Let $u_k(\cdot) \in U,\ D_k \in \Omega(C^n(T_0)),\ u_k(\cdot) \to u^*(\cdot)$ (in the metric $L_2(T)$), $D_k \to D$ (in the Hausdorff metric) for $k \to \infty$, $u^*(\cdot) \in U$, $D^* \in \Omega(C^n(T_0))$.

Take an arbitrary element $x_k(\cdot) \in H(u_k,D_k)$. Then, by virtue of Lemma 4 $x^*(\cdot) \in H(u^*,D^*)$, there exists such that

$$\rho(x_k(\cdot),H(u^*,D^*)) \le \|x_k(\cdot)-x^*(\cdot)\|_{C^n(T_1)} \le \Delta_k, \tag{5}$$

$$\Delta_k = \left[h(D_k, D^*) + \|l(\cdot)\|_{l_2(T)} \|u_k(\cdot) - u^*(\cdot)\|_{l_2^m(T)} \right] M_2. \tag{6}$$

From (5) we get

$$\sup_{\xi(\cdot)\in H(u_k, D_k)} \rho(\xi(\cdot), H(u^*, D^*)) \le \Delta_k. \tag{7}$$

Similarly, we have

$$\sup_{\xi(\cdot)\in H(u^*, D^*)} \rho(\xi(\cdot), H(u_k, D_k)) \le \Delta_k. \tag{8}$$

From (7) and (8) follows the estimate

$$h(H(u_k, D_k), H(u^*, D^*)) \le \Delta_k. \tag{9}$$

Since according to (6) $\Delta_k \to 0,\ k \to \infty$, then from (9) we obtain the relation

$$\lim_{k\to\infty} h(H(u_k, D_k), H(u^*, D^*)) = 0,$$

which shows the continuity of the multivalued mapping $(u, D) \to H(u, D)$ on $U \times \Omega(C_n(T_0))$.

Consider now an arbitrary sequence of points $(t_k, u_k, D_k) \in T \times U \times \Omega(C^n(T_0))$ converging to $(t^*, u^*, D^*) \in T \times U \times \Omega(C^n(T_0))$. Let's take an arbitrary trajectory $x^*(\cdot) \in H(u^*, D^*)$. Using the formula

$$x^*(t) = x^*(t_0) + \int_{t_0}^{t} \dot{x}^*(s)ds,\ \ t \in T,$$

it is easy to show that

$$\|x^*(t_k) - x^*(t^*)\| \le \delta_k, \tag{10}$$

where

$$\delta_k = \left| \int_{t_k}^{t^*} [g_1(t)(k+1)M_3 + g_2(t)]dt \right|.$$

It is clear that $\rho(x^*(t_k), X(t^*, u^*, D^*) \le \|x^*(t_k) - x^*(t^*)\| \le \delta_k$. Therefore, given the arbitrariness of $x^*(\cdot) \in H(u^*, D^*)$, it is easy to verify that

$$h(X(t_k, u^*, D^*), X(t^*, u^*, D^*) \le \delta_k. \tag{11}$$

Due to the properties of the Hausdorff metric, the following inequalities are valid:

$$\begin{aligned} h(X(t_k, u_k, D_k), X(t^*, u^*, D^*)) \le h(X(t_k, u_k, D_k), X(t_k, u^*, D^*)) + \\ + h(X(t_k, u^*, D^*), X(t^*, u^*, D^*)), \end{aligned} \tag{12}$$

$$h(X(t_k, u_k, D_k), X(t_k, u^*, D^*)) \le h(H(u_k, D_k), H(u^*, D^*)). \tag{13}$$

Now, using relations (9), (11), (12), (13), we have

$$h(X(t_k, u_k, D_k), X(t^*, u^*, D^*)) \le \Delta_k + \delta_k.$$

Since $\Delta_k \to 0,\ \delta_k \to 0,\ k \to \infty$, the last inequality implies the continuity of the multivalued mapping $(t, u, D) \to X(t, u, D)$ on $T \times U \times \Omega(C_n(T_0))$. The theorem has been proven.

Consider a graph of a multivalued mapping $(u, D) \to H(u, D)$, i.e. set

$$\Gamma_H = \{(\omega, x) : \omega = (u, D) \in U \times \Omega(C^n(T_0)), x = x(\cdot) \in H(u, D)\}.$$

Similarly, consider the graph of the set-valued mapping $(t,u,D) \to X(t,u,D)$, i.e. set

$$\Gamma_X = \{(t, \omega, \xi) : t \in T, \omega = (u, D) \in U \times \Omega(C^n(T_0)), \xi \in X(t,u,D)\}.$$

Theorem 2. If the conditions a2) – c2) are fulfilled, the multivalued mappings $(u,D) \to H(u,D)$, $u \in U$, $D \in \Omega(C^n(T_0))$ and $(t,u,D) \to X(t,u,D)$, $t \in T$, $u \in U$, $D \in \Omega(C^n(T_0))$ are closed, i.e. their schedules are closed.

Proof. Let us prove the closedness of the set Γ_H. Closure Γ_X is proved in a similar way.

Let $(\omega_k, x_k) \in \Gamma_H$, $(\omega_k, x_k) \to (\omega^*, x^*)$, i.e.

$$\omega_k = (u_k, D_k),\ x_k = x_k(\cdot) \in H(u_k, D_k),\ u_k = u_k(\cdot) \in U,\ D_k \in \Omega(C^n(T_0)), x_k \to x^*$$

(in the metric $C^n(T_1)$), $u_k \to u^*$ (in the metric $L_2^m(T)$), $D_k \to D^*$ (in the Hausdorff metric) for $k \to \infty$, $u^* = u^*(\cdot) \in U$, $D^* \in \Omega(C^n(T_0))$. Let's show that $x^* \in H(u^*, D^*)$.

It's clear that

$$\rho(x_k, H(u^*, D^*)) \le h(H(u_k, D_k), H(u^*, D^*)) \quad (14)$$

Let $\xi_k^*(\cdot) \in H(u^*, D^*)$ such that

$$\rho(x_k, H(u^*, D^*)) = \left\| x^*(\cdot) - \xi_k^*(\cdot) \right\|_{C^n(T_1)}. \quad (15)$$

By Lemma 3, the set is $H(u^*, D^*)$compact, and therefore such an element $\xi_k^*(\cdot)$ exists. Now, using (14) and (15), we have

$$\rho(x^*, H(u^*, D^*)) \le \left\| x^*(\cdot) - x_k(\cdot) \right\|_{C^n(T_1)} + \rho(x_k, H(u^*, D^*)) \le$$

$$\le \left\| x^*(\cdot) - x_k(\cdot) \right\|_{C_n(T_1)} + \rho^*(H(u_k, D_k), H(u^*, D^*)).$$

Passing here to the limit at $k \to \infty$ and taking into account the continuity of the multivalued mapping $(u,D) \to H(u,D)$, we obtain that $\rho(x^*, H(u^*, D^*)) = 0$. Since $H(u^*, D^*)$is a closed set, this implies the relation $x^* \in H(u^*, D^*)$. The theorem has been proven.

Theorem 3. Let conditions a2) – d2) be satisfied. Then the multivalued mappings $(u,D) \to H(u,D)$ and $(u,D) \to X(t,u,D)$, $u \in U$, $D \in co\Omega(C^n(T_0))$($t \in T$) are convex and closed.

Proof. By Theorem 4, it suffices to show that the following sets are convex:

$$\Gamma_H^0 = \{(\omega^0, x) : \omega^0 = (u, D^0) \in U \times co\Omega(C^n(T_0)), x = x(\cdot) \in H(u, D^0)\},$$

$$\Gamma_X^0(t) = \{(\omega^0, \xi) : \omega^0 = (u, D^0) \in U \times co\Omega(C^n(T_0)), \xi^0 \in X(t,u,D^0)\}, t \in T.$$

Let us prove the convexity Γ_H^0. The convexity of $\Gamma_X^0(t)$ $t \in T$, follows from the convexity of the set Γ_H^0. Take arbitrary points

$$(u_i, D_i^0) \in U \times \Omega^0(C^n(T_0)),\ x_i(\cdot) \in H(u_i, D_i^0),\ \ \alpha_i \ge 0,\ i = \overline{1,2},\ \alpha_1 + \alpha_2 = 1.$$

Let's consider a function $z(t) = \alpha_1 x_1(t) + \alpha_2 x_2(t),\ t \in T_1$. Because

$$(\dot{x}_1(t),\psi)\le c(F(t,x_1(t),x_1(t-h_1(z)),...,x_1(t-h_k(t)),u_1(t)),\psi),$$
$$(\dot{x}_2(t),\psi)\le c(F(t,x_2(t),x_2(t-h_1(z)),...,x_2(t-h_k(t)),u_2(t)),\psi)$$

for almost all $t\in T$ and all $\psi\in R^n$, then using the condition d2), we have:

$$(\dot{z}(t),\psi)=\alpha_1(\dot{x}_1(t),\psi)+\alpha_2(\dot{x}_2(t),\psi)\le\alpha_1 c(F(t,x_1(t),x_1(t-h_1(t)),...,$$
$$x_1(t-h_k(t)),u_1(t)),\psi)+\alpha_2 c(F(t,x_2(t),x_2(t-h_1(t)),...,x_2(t-h_k(t)),u_2(t)),\psi)\le$$
$$\le c(F(t,z(t),z(t-h_1(t)),...,z(t-h_k(t)),\alpha_1u_1(t)+\alpha_2u_2(t)),\psi).$$

By virtue of the properties of support functions [2], this implies that

$$\dot{z}(t)\in F(t,z(t),z(t-h_1(t)),...,z(t-h_k(t)),\alpha_1u_1(t)+\alpha_2u_2(t)) \quad (16)$$

almost everywhere on *T*. Further, $x_{1T_0}(\cdot)\in D_1^0$, $x_{2T_0}(\cdot)\in D_2^0$, since

$$\alpha_1 x_{1T_0}(\cdot)+\alpha_2 x_{2T_0}(\cdot)\in\alpha_1 D_1^0+\alpha_2 D_2^0\in co\Omega(C^n(T_0)). \quad (17)$$

Relations (16) and (17) show that

$$z(\cdot)\in H(\alpha_1u_1+\alpha_2u_2,\alpha_2D_1^0+\alpha_2D_2^0),$$

those. set Γ_H^0 is convex. The theorem has been proven.

THE DISCUSSION OF THE RESULTS

The results obtained in Theorem 1 give conditions for the continuous dependence of the set of solutions and the ensemble of trajectories on the parameters of the Cauchy problem (2). Theorems 2 and 3 give conditions for the closedness and convexity of multivalued mappings $(u,D)\to H(u,D)$ and $(t,u,D)\to X(t,u,D)$.

Regarding the comments received as comments, we present the following:

1. Given the lemma 4 and, arguing similarly to the proof of Theorem 1, we can easily show that multivalued mappings $(u,D)\to H(u,D)$ And $(t,u,D)\to X(t,u,D)$ uniformly continuous $U\times\Omega(C^n(T_0))$ And $T\times U\times\Omega(C^n(T_0))$ respectively.

2. The statements of Theorem 1 are preserved if we replace condition b2) with the following condition: B 2) the multivalued mapping $(t,x,y,u)\to F(t,x,y,u)$ is measurable with respect to $t\in T$ at $\forall\,(x,y,u)\in R^n\times R^{nk}\times V$, continuous with respect to u with $\forall\,(t,x,y)\in T\times R^n\times R^{nk}$, and satisfies the Lipschitz condition with (x,y) respect to almost all $t\in T$, i.e.

$$h(F(t,x,y,u),F(t,\xi,\eta,v))\le l(t)[\|x-\xi\|+\sum_{i=1}^{k}\|y_i-\eta_i\|],\ \forall\,(x,y,u),(\xi,\eta,v)\in R^n\times R^{nk}\times V,$$

where is $l(t)-$the square summable function on T ($l(\cdot)\in L_2(T)$).

3. Let a set-valued mapping $F(t,x,y,u)=A(t)x+\sum_{i=1}^{k}A_i(t)y_i+b(t,u)$ satisfy the conditions of Corollary 2. Then all the assertions of Theorem 3 are preserved.

4. The statement of Theorem 5 regarding the convexity of the graphs of multivalued mappings $(u,D)\to H(u,D)$ and $(u,D)\to X(t,u,D)$ $(t\in T)$ is equivalent to the relations:

$$\alpha_1 H(u_1, D_1^0) + \alpha_2 H(u_2, D_2^0) \subset H(\alpha_1 u_1 + \alpha_2 u_2, \alpha_1 D_1^0 + \alpha_2 D_2^0),$$
$$\alpha_1 X(t, u_1, D_1^0) + \alpha_2 X(t, u_2, D_2^0) \subset X(t, \alpha_1 u_1 + \alpha_2 u_2, \alpha_1 D_1^0 + \alpha_2 D_2^0),$$
$$t \in T,\ \alpha_1 \geq 0, \alpha_2 \geq 0,\ \alpha_1 + \alpha_2 = 1,\ u_i \in U,\ D_i^0 \in \Omega^0(C^n(T_0)),\ i = \overline{1,2}.$$

This, in turn, implies the concavity of the support function $c(X(t,u,D^0),\psi)$ with respect to $(u, D^0) \in U \times \Omega^0(C^n(T_0))$ for all $t \in T,\ \psi \in R^n$.

CONCLUSION

The paper considers a class of controlled differential inclusions with a delay argument. For this model of dynamical systems, the Cauchy problem is studied. Methods of the theory of differential inclusions, multivalued and convex analysis are used. Some properties of the set of solutions of a differential inclusion with delays and an ensemble of trajectories are studied depending on the control parameters and the initial state. Sufficient conditions for the continuity of such multivalued mappings are found. The conditions for their closure and convexity are indicated .

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