IMPROVING THE METHODOLOGICAL SYSTEM OF TEACHING ALGEBRA ELEMENTS IN THE INITIAL MATHEMATICS COURSE

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Abstract. *There are examples and solutions throughout, but of course there is always more than one way to arrive at an answer. The ability to recognize and understand multiple methods is of great value to a teacher because it is quite impressive how creative kids can be with their solutions.*

Keywords: *mathematics, algebra, geometry, teaching methods, element of algebra, formation, education, training.*

Let's begin by discussing the term elementary mathematics. First and foremost, the words elementary and easy are NOT synonyms. Elementary mathematics is the foundation that all of mathematics is built on, and you know it as the mathematics curriculum covered in kindergarten through eighth grade. Because it is taught at such an early age many people brush it off as basic, missing out on the rich and interesting mathematics hidden within.

There are examples and solutions throughout, but of course there is always more than one way to arrive at an answer. The ability to recognize and understand multiple methods is of great value to a teacher because it is quite impressive how creative kids can be with their solutions.

To develop this ability, whenever you see an example whose solution is labeled Possible Solution, then you should take the time to think about and write down alternate solutions[1-2].

Finally, you will see the label Property. Statements under this label are either facts that we have discovered or shortcuts we have found along our path of discovery that we think will be useful to have at our disposal moving forward. In our exploration, we will discover possible properties by looking at examples. However, that does not necessarily mean the statement is always valid. Maybe it won't work with a different example.

Consequently, before applying this property moving forward, we must be sure it is valid. Therefore, for anything labeled as a property you should record an argument justifying the validity of the said property in general. In other words, your argument should explain why the statement is true no matter what example you are using [5].

Algebraic Thinking An understanding of algebraic concepts is extremely important to a student's success in mathematics. Finding ways to ground students, even at an early age, in algebraic thinking will promote their success later on. In this chapter we will be looking at a few examples that will develop a student's algebraic thinking. By altering the questions, you ask and how far you progress, activities of this nature can be used at any grade level. In other words, even if you are teaching first grade you can help students get ready for algebra.

Patterns Consider the figures below.

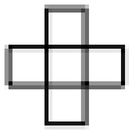


Figure #1

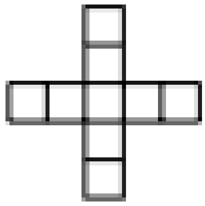


Figure #2

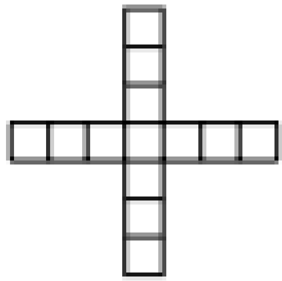


Figure #3

Question 1.1.1. What does figure #4 look like? Example

1.1.2. How many squares are in figure #4? Possible Solution. If the growth follows the pattern of adding one block to each leg, then Figure #4 has 13 squares. Now, here's where some algebraic thinking begins. Did you actually count all of the squares to get 13, or did you have a quicker way to figure out that there were 13 squares?

Example 1.1.3. Without actually counting each square, determine how many squares figure #5 will have. Possible Solution. There are lots of ways to figure this one out, but here's one way. We can think of this figure as a square in the center and four arms. Notice that figure #1 has 1 square in each arm, figure #2 has 2 squares in each arm, etc. Using this structure, we found in the figures, we can now determine the arithmetic needed to find the number of blocks in each figure.

Figure #	Work	Number of Square
1	$4 \times 1 + 1$	5
2	$4 \times 2 + 1$	9
3	$4 \times 3 + 1$	13
4	$4 \times 4 + 1$	17

Looking at the pattern in the table, we can see that figure #5 will have 5 squares in each arm, so we have four arms with 5 in each arm plus the square in the middle. Using this information we see that figure #5 will have $4 \times 5 + 1$ or 21 squares.

Example 1.1.4. Determine how many squares figure #35 will have?

Possible Solution. Figure #35 will have 4 arms, with 35 squares in each arm. Plus, it will also have one square in the middle.

Therefore to determine the number of squares we need to compute $4 \times 35 + 1$, so there are 141 squares in figure #35[7].

As stated above, there is more than one way to count the squares. Suppose a student saw the figures growing. It starts with figure #1, and at each step four squares are added on the end of each arm. Now figure #2 has 1 set of 4 squares added. Figure #3 has 2 sets of 4 squares added. Figure #4 has 3 sets of 4 squares added. Continuing we see figure #5 has 4 sets, figure #6 has 5 sets, etc. Therefore, we can conclude that figure #35 will have 34 sets. Using this information, we see that we could have arrived at 141 squares by computing the base figure plus 34 sets of 4. In other words, $5 + 4 \times 34$.

Question 1.1.5. A student arrived at 141 squares in figure #35 by doing the following arithmetic, $3\times35+ 36$. What structure do you think the student sees in the figures that led to this arithmetic?

Let's now return to our first method of counting, namely four arms and a square in the center. For figure #35 we had the expression 4 × 35 + 1. If we go out to figure #100, what will change in the expression? Will the 4 change? Will the 35 change? Will the 1 change? In answering these questions, we see that the corresponding expression for figure #100 would be 4 × 100 + 1. Consequently, there are 401 squares in figure #100.

We can easily see that as we change figures, the 4 and the 1 remain the same in the expression because the 4 represents the number of arms and the 1 represents the square in the middle. Therefore, there is only one number that varies in the expressions. We can now see how to find an algebraic expression that represents the number of squares in figure #n. There will still be four arms and 1 square in the middle, the only change is the number of squares in an arm. For figure #35 there were 35 squares in an arm, for figure #100 there were 100 squares in an arm. Thus, we can see that in figure #n there will be n squares in an arm. Therefore, the expression 4n + 1 tells us how many squares are in figure #n. [8].

In reviewing what we did above, we essentially looked for a pattern in the many expressions we wrote down. In addition, we were able to correlate each number in the expression to what it represented in the figure. If students are able to do these two things successfully, then the jump from concrete numbers to abstract variables will be an easier transition. One of the struggles students have in algebra is understanding the different roles a variable can play. When we found the formula for the nth figure above, the variable played the role of "any whole number". In other words, n did not stand for a specific number that we were solving for, but rather it represented a number that can vary. The corresponding expression represented the number of squares as the figure number varied.

Let's look at a seemingly different problem but that can be attacked in a similar way. For Examples 1.1.6 through 1.1.8, consider the sequence of numbers below.

3, 10, 17, 24, 31, 38, . . .

Example 1.1.6. What is the next number in the sequence?

Possible Solution. Looking at the pattern we see that to get from one term to the next we add 7. Therefore, the next number will be 45.

Example 1.1.7. What is the 35th number in the set?

Possible Solution. Let's use the idea with the squares above where we look at our work to see a pattern. To get from 3 to 10 we added 7. To get from 10 to 17 we added 7, which means to get from 3 to 17 we must add 7 twice. Similarly to get from 3 to 24 we must add 7 three times. We can summarize these results in the table below.

n	work	*n*th number in the set
1	-	3
2	3 + 7	10
3	3 + 2 × 7	17
4	3 + 3 × 7	24
5	3 + 4 × 7	31
6	3 + 5 × 7	38

Now we can see a pattern. Notice that the 3 doesn't change and the 7 doesn't change. The only thing that changes is how many 7's we are adding. Notice that for the 4th term we added 3 sevens, in the 5th term we added 4 sevens, in the 6th term we added 5 sevens, etc. Therefore, we can see that in the 35th term we will add 34 sevens [6]. Therefore, we can get the 35th term by computing $3 + 34 \times 7$. So, 238 is the 35th number in this set. Example 1.1.8. Will the number 576 appear in the sequence?

Possible Solution. From our above work we know that numbers in this set come from an expression like $3 + \times 7$, where we can fill in the blank with some whole number. In other words we are trying to finding a whole number n, so that $3 + n \times 7$ is equal to 576. Therefore, we need to solve the equation $3+ n \times 7 = 576$.

$$3 + n \times 7 = 576$$
$$n \times 7 = 573$$
$$n \approx 81.86$$

Since n does not turn out to be a whole number, then 576 is not in the set.

This section gave just a very brief introduction to what algebraic thinking might look like. The more students are introduced to pattern recognition and description of patterns in general the more prepared they will be to deal with the notion of variable as a ranging quantity. Notice in the last example, the variable started out as "any number" because the formula represented the value of a term where n represented any term number[9].

However, once we were asked to determine if the specific number, 576, was in the sequence we then shifted to determine a single value for the variable. In other words, we then solved to find a specific value for n. [10]. Algebra is full of variables playing different roles, which often leads to confusion. In particular, students tend to think of variables as what you solve for, so if they are exposed to some of these algebraic thinking activities throughout their elementary career, they will have the ground work laid to see variables playing a different role.

Exercises 1. Consider the pattern of block figures shown below

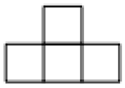
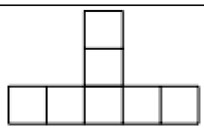
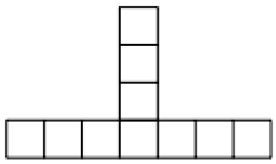
		

Figure #1	Figure #2	Figure #3

(a) Describe a structure you see in the figures, and use that structure to determine the number of blocks in Figure 3.

(b) Using the structure, you described above to produce a corresponding arithmetic expression that will give the number of blocks in Figure 68?

(c) Using the structure, you described above to produce a corresponding algebraic expression that will give the number of blocks in Figure n?

(d) Use the structure you described above to determine if there is a figure in this pattern that has 316 blocks. Your explanation should rely solely on the blocks themselves.

(e) Show the algebraic solution that corresponds to your contextual solution in Problem 1d.

2. Consider the pattern of block figures below.

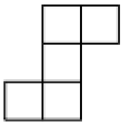
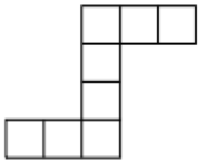
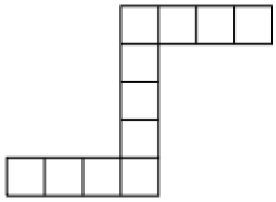
		

Figure #1	Figure #2	Figure #3

(a) Max says that there are 11 blocks in Figure 3, and he used the arithmetic expression $3 + 2 \times 4$ to arrive at his answer. If Max continues the same thinking, what arithmetic expression do you think he will use to find the number of blocks in Figure 5? Please explain how you arrived at your answer.

(b) Macy is looking at the figures and she says that she sees the figures as one long piece in the middle and two short arms off the side. So, for example in figure #3 Macy sees a long piece in the middle with 5 blocks and two arms that each have 3 blocks.

Complete the table below using the structure Macy sees in the figures.

Figure #	work	Number of Squares
1		5
2		8
3		11
4		14

(c) Use Max's structure to produce an algebraic expression that will give the number of blocks in Figure n?

(d) Use Macy's structure to produce an algebraic expression that will give the number of blocks in Figure n?

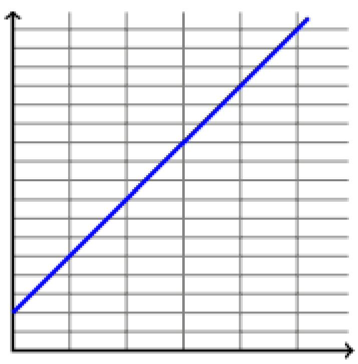
3. Another student, Marta, is working on the same pattern of blocks in Exercise 2, and her formula for Figure n is $3n + 2$.

(a) What structure do you think Marta is seeing in the figures?

(b) Marta's teacher asked the class a question about the blocks. Marta thinks about her teacher's question, and then writes down the inequality $3n + 2 \geq 100$. What question do you think the teacher asked about the blocks?

(c) In solving the inequality $3n + 2 \geq 100$, Marta's second step is $3n \geq 98$. Use the context of the blocks to explain why the second inequality follows from the first.

(d) Marta wrote down her formula, $T = 3n + 2$, and then graphed it. Her graph is shown below.



i. What should the labels on the x- and y-axes be?

ii. What does the y-intercept of the line represent in the context of the squares? iii. What does the slope of the line represent in the context of the squares?

4. Consider the pattern of block figures below. Please justify all responses to the following questions.

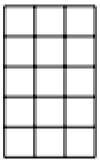
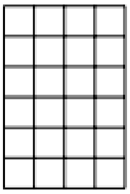
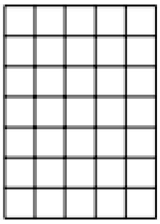
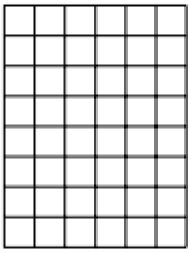
			

Figure 3	Figure 4	Figure 5	Figure 6

(a) How many blocks will be in Figure 20?
(b) How many blocks will be in Figure n?
(c) Is there a figure number for which there will be 63 blocks?
(d) Is there a figure number for which there will be 100 blocks?

5. Consider the pattern of block figures below.

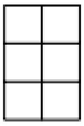
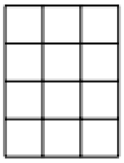
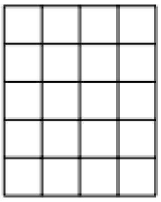
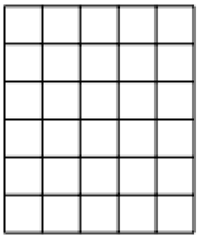
			

Figure 1	Figure 2	Figure 3	Figure 4

Jazlyn says her formula for the number of blocks in Figure n is $(n + 1)(n + 2)$.

Jacarri says his formula for the number of blocks in Figure n is $n^2 + 3n + 2$. Jazlyn used algebra to see that her formula and Jacarri's formula are equivalent, however she does not see how Jacarri could have gotten that formula from the pictures. Where does each part of Jacarri's formula appear in the figures?

6. For each of the algebraic expressions below, create a pattern of rectangular figures for which the expression would represent the number of squares in Figure n.

(a) $n^2 + 5n + 6$
(b) $n^2 + 5n + 4$
(c) $n^2 + 8n + 12$

7. For each of the algebraic expressions below it is not possible* to create a pattern of rectangular figures for which the expression would represent the number of squares in Figure n. Instead, create patterns of figures that are as close as possible to rectangles. Indicate how many squares too much or too little you are from creating rectangles. (*Note: We are assuming only full squares can be used. If we remove that criteria, then it could be possible.)

(a) $n^2 + 2n + 3$
(b) $n^2 + 8n + 5$
(c) $n^2 + 6n + 7$ 8.

In Exercises 6 and 7 we explored quadratic expressions that either could or could not be represented by a rectangular array of blocks. What is the connection between those two exercises and factoring quadratic expressions? This is the question should be answered following the instructions above.

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