VIBRATION OF HYSTERESIS-TYPE ELASTIC DISSIPATIVE CHARACTERISTIC OF VARIABLE CROSS SECTION

[1]Dusmatov O.M., [2]Kasimova F.U.
Samarkand State University
https://doi.org/10.5281/zenodo.10231594

Abstract. *This article is devoted to the study of the vibrations of a hysteresis-type elastic dissipative characteristic cross-section under kinematic excitations. Analytical expression of the transmitter function was determined for the purpose of checking the dynamics of the stern.*

Keywords: *variable cross-section, stergen, elasticity, dissipative, hysteresis, oscillations, transfer function.*

It is important to develop mathematical models, taking into account the elasticity and dissipative properties of them, and to correctly choose the structural parameters in the design of structures of the stern type in many fields of engineering and technology. In certain processes, there are issues of evaluating the vibrational movements of elastic rods, determining their specific frequencies and dynamic characteristics, while changing the cross-section of elastic rods.

[1] - in the work, the problems of determining the solutions of the differential equations of the vibrations of a stern with variable cross-section in kinematic motions using the approximate method based on the asymptotic expansion of the sub-parameter e in terms of degree were considered. The equations for determining the Stergen deformation function and its specific frequencies and the formula for obtaining amplitude resonance lines of vibrations are given.

[2] – in the work, transverse vibrations of a boom with a variable cross-section were studied. In cases where different boundary conditions and cross-sections are obtained according to different variable functions, the specific frequencies of oscillations are determined analytically and numerically, the analysis results and conclusions are presented.

[3] – in the work, the frequency equation of a variable cross-section boom under various boundary conditions is defined and methods of solving it are given. The change in the cross-sectional area is taken as the level of the sinusoid. Using substitutions, the differential equation of motion was converted to Legendre's equation. This allowed us to search for and identify the solution of this system in the form of graded series. In addition, the effect of the change of the cross-sectional surface on the vibration behavior was studied using numerical calculations.

[4] – in the work, forced vibrations of the stern under the influence of periodic force were studied. Transverse vibration equations are obtained according to the Hamilton–Ostrogratsky principle. The specific vibrational forms have been proposed in several different ways and the resonant frequency has been determined. The numerical results of the calculation are given in tabular and graphical forms.

[5] – in the work, the transverse vibrations of the stern are expressed in the form of a matrix and they are solved by the Cauchy sequence method. Eigenfrequencies are determined for various boundary conditions and cases with constant and variable cross section. The almost direct compatibility of the numerical methods with the existing simple software tools made it possible to analyze the character of the sturgeon based on these programs from the solutions of the matrix equations obtained by the Cauchy sequence method. The obtained analytical and numerical solutions are compared.

In this work, we consider transverse oscillations of a hysteresis-type elastic dissipative rod under kinematic excitations. The length of the stern l, width b, thickness h and be mounted on a moving base. In this case, the height of the boom is constant, the width is variable, and the material of the boom is non-single-valued, representing imperfect elasticity. $\vec{\bar{\Phi}}(\xi_0)$ the function is replaced by the following linear expression using the harmonic linearization method [6]

$$H(\xi) = (-q_1 + jq_2)\xi_0, \qquad (1)$$

here q_1, q_2 - linearization coefficients; ξ_0 - deformation. For harmonic processes

$$q_1 = -\frac{1}{\pi a}\int_0^{2\pi} \varepsilon\vec{\bar{\Phi}}(a\cos\varphi)\cos\varphi d\varphi,$$
$$q_2 = -\frac{1}{\pi a}\int_0^{2\pi} \varepsilon\vec{\bar{\Phi}}(a\cos\varphi)\sin\varphi d\varphi, \qquad (2)$$

here ε - small parameter; a, φ - respectively $\xi(t)$ amplitude and phase of the variable.

Here $\xi_0(t)$ variable is obtained as follows:

$$\xi_0(t) = a\cos\varphi, \qquad (3)$$

We write the differential equation of motion for transverse oscillations of the stern in kinematic motions

$$\frac{\partial^2 M}{\partial x^2} + m_0\frac{\partial^2 w}{\partial t^2} = -m_0 W_0; \qquad (4)$$

here M- bending moment; w - bending function of the stern; m_0 - unit mass of the length of the boom; W_0 - acceleration of base movement.

We write down the bending moment of the cross-section of the stern

$$M = 2b(x)\int_0^{h/2}\sigma_n z dz, \quad (5)$$

here $\sigma_n -$ normal voltage.

(5) the connection between the normal stress and the relative deformation for the considered sturgeon can be written according to the relationship [6]

here

$$\sigma_n = E\left[1 + (-L_1 + jL_2)\left[C_0 + f(\xi_0)\right]\right]\xi_0, \qquad (6)$$

E - modulus of elasticity of the stern material; ξ_0 - relative deformation;

$L_1, L_2 -$ are experimentally determined constant coefficients depending on the dissipative properties of the stergen material; $f(\xi_0)$- is the decrement of the sturgeon oscillations and is expressed as:

$$f(\xi_0) = C_1\xi_0 + .C_2\xi_0^2 + ... + C_r\xi_0^r, \qquad (7)$$

$C_0, C_1, ...C_r -$ coefficients of the hysteresis contour, determined from experience, depending on the damping properties of the stern material [7].

For the relative deformation, the following relationship is appropriate:

$$\xi_0 = z\frac{\partial^2 w}{\partial x^2}, \tag{8}$$

Substituting the expressions (6) and (7) into the bending moment expression (5), we get it

$$\begin{aligned} M &= 2b(x)\int_0^{h/2}\sigma_n z dz = 2b(x)E\frac{h^3}{24}\frac{\partial^2 w}{\partial x^2}\left(1+C_0\left(-L_1+jL_2\right)\right)+ \\ &+2b(x)E(-L_1+jL_2)\frac{\partial^2 w}{\partial x^2}\int_0^{h/2} f\left(\xi_0\right)z^2 dz = EI\left[\frac{\partial^2 w}{\partial x^2}\left(1+C_0\left(-L_1+jL_2\right)\right)\right]+ \\ &+EI\left[\frac{24}{h^3}(-L_1+jL_2)\frac{\partial^2 w}{\partial x^2}\int_0^{h/2} f\left(\xi_0\right)z^2 dz\right], \end{aligned} \tag{9}$$

here $I = \frac{b(x)h^3}{12}$;

If we put the obtained expression (9) into the differential equation of the stergen (4), we get

$$\begin{aligned} &EI\frac{\partial^2}{\partial x^2}\left\{\frac{\partial^2 w}{\partial x^2}\left(1+C_0\left(-L_1+jL_2\right)+\frac{24}{h^3}\left(-L_1+jL_2\right)\int_0^{h/2} f\left(\xi_0\right)z^2 dz\right)\right\}+ \\ &+2E\frac{\partial I}{\partial x}\frac{\partial}{\partial x}\left\{\frac{\partial^2 w}{\partial x^2}\left(1+C_0\left(-L_1+jL_2\right)+\frac{24}{h^3}\left(-L_1+jL_2\right)\int_0^{h/2} f\left(\xi_0\right)z^2 dz\right)\right\}+ \\ &+E\frac{\partial^2 I}{\partial x^2}\frac{\partial^2 w}{\partial x^2}\left\{\left(1+C_0\left(-L_1+jL_2\right)+\frac{24}{h^3}\left(-L_1+jL_2\right)\int_0^{h/2} f\left(\xi_0\right)z^2 dz\right)\right\}+ \\ &\qquad +m_0\frac{\partial^2 w}{\partial t^2} = -m_0 W_0; \end{aligned} \tag{10}$$

(9) to find the solution of the equation, we expand the bending function of the stern into a series according to its eigenforms, i.e.

$$w(x,t) = \sum_{k=1}^{\infty} U_k(x)T_k(t), \tag{11}$$

here $T_k(t)$ - function of time; $U_k(x)$ - functions are a special form of stergen oscillations and satisfy the condition of orthogonality, i.e

$$\int_0^l U_i(x)U_m(x)dx = 0, \quad i \neq m. \tag{12}$$

$U_k(x)$ - the following well-known equation is appropriate for the function:

$$EI\frac{d^4 U_k(x)}{dx^4} - m\omega_k^2 U_k(x) = 0, \tag{13}$$

here ω_k - the eigenfrequencies of the stern; $T_k(t)$ - while the function view

$$T_k = T_a \cos a;\ \varphi(a) = \sum_{i=0}^{n} C_i T_{ka}^i \left| z\frac{\partial^2 U_1}{\partial x^2}\right|^i. \tag{14}$$

(11) Substituting the solution and expression (14) into the differential equation (10) and taking into account the differential equation (13), we get:

$$\sum_{k=1}^{\infty}\left\{\left[\ddot{T}_k+\left(1+C_0\left(-L_1+jL_2\right)\right)\omega_k^2T_k\right]U_k+\frac{3EI}{m_0}\left(-L_1+jL_2\right)T_k\sum_{i=1}^{n}C_iT_{ka}^i\frac{h^i}{2^i(i+3)}\frac{\partial^2}{\partial x^2}\left(\frac{\partial^2}{\partial x^2}\left|\frac{\partial^2U_k}{\partial x^2}\right|^i\right)\right\}+$$

$$+\frac{E}{m_0}\frac{\partial^2 I}{\partial x^2}\sum_{k=1}^{\infty}\frac{\partial^2U_k}{\partial x^2}T_k\left[1+C_0\left(-L_1+jL_2\right)+3\left(-L_1+jL_2\right)\sum_{i=1}^{n}C_iT_{ka}^i\frac{h^i}{2^i(i+3)}\left|\frac{\partial^2U_k}{\partial x^2}\right|^i\right]=-W_0\,; \qquad (15)$$

Both sides of the resulting differential equation $U_m(x)$ multiply by and integrate in the interval [0;l]. As a result, using the integral and the property of Dirac's delta function, and after performing some calculations, we have the following differential equation:

$$\ddot{T}_k+\left\{\left(1+C_0\left(-L_1+jL_2\right)\right)\omega_k^2+\frac{3EI}{m_0d_{2i}}\left(-L_1+jL_2\right)\sum_{i=1}^{n}C_iT_{ka}^i\frac{h^i}{2^i(i+3)}\times\right.$$

$$\times\int_0^l U_k\frac{\partial^2}{\partial x^2}\left(\frac{\partial^2}{\partial x^2}\left|\frac{\partial^2U_k}{\partial x^2}\right|^i\right)dx+\frac{E}{m_0}\frac{\partial^2 I}{\partial x^2}\int_0^l U_k\frac{\partial^2U_k}{\partial x^2}\left[1+C_0\left(-L_1+jL_2\right)+\right. \qquad (16)$$

$$\left.\left.+3\left(-L_1+jL_2\right)\sum_{i=1}^{n}C_iT_{ka}^i\frac{h^i}{2^i(i+3)}\left|\frac{\partial^2U_k}{\partial x^2}\right|^i\right]dx\right\}T_k=-d_iW_0\,;$$

that $d_i=\dfrac{d_{1i}}{d_{2i}};\ d_{1i}=\int_0^l U_kdx;\ \ d_{2i}=\int_0^l U_k^2dx.$

First, differential equation (16) in order to find the transfer function of the anti-vibration boom $s=\dfrac{d}{dt}$ we bring to the algebraic equation through the differential operator, i.e

$$\left\{s^2+\left\{\left(1+C_0\left(-L_1+jL_2\right)\right)\omega_k^2+\frac{3EI}{m_0d_{2i}}\left(-L_1+jL_2\right)\sum_{i=1}^{n}C_iT_{ka}^i\frac{h^i}{2^i(i+3)}\times\right.\right.$$

$$\times\int_0^l U_k\frac{\partial^2}{\partial x^2}\left(\frac{\partial^2}{\partial x^2}\left|\frac{\partial^2U_k}{\partial x^2}\right|^i\right)dx+\frac{E}{m_0}\frac{\partial^2 I}{\partial x^2}\int_0^l U_k\frac{\partial^2U_k}{\partial x^2}\left[1+C_0\left(-L_1+jL_2\right)+\right. \qquad (17)$$

$$\left.\left.\left.+3\left(-L_1+jL_2\right)\sum_{i=1}^{n}C_iT_{ka}^i\frac{h^i}{2^i(i+3)}\left|\frac{\partial^2U_k}{\partial x^2}\right|^i\right]dx\right\}\right\}T_k=-d_iW_0\,;$$

In this we define variables

$$T_k=-\frac{d_i}{N_1}W_0, \qquad (18)$$

here

$$N_1=s^2+\left\{\left(1+C_0\left(-L_1+jL_2\right)\right)\omega_k^2+\frac{3EI}{m_0d_{2i}}\left(-L_1+jL_2\right)\sum_{i=1}^{n}C_iT_{ka}^i\frac{h^i}{2^i(i+3)}\times\right.$$

$$\times\int_0^l U_k\frac{\partial^2}{\partial x^2}\left(\frac{\partial^2}{\partial x^2}\left|\frac{\partial^2U_k}{\partial x^2}\right|^i\right)dx+\frac{E}{m_0}\frac{\partial^2 I}{\partial x^2}\int_0^l U_k\frac{\partial^2U_k}{\partial x^2}\left[1+C_0\left(-L_1+jL_2\right)+\right.$$

$$\left.\left.+3\left(-L_1+jL_2\right)\sum_{i=1}^{n}C_iT_{ka}^i\frac{h^i}{2^i(i+3)}\left|\frac{\partial^2U_k}{\partial x^2}\right|^i\right]dx\right\}.$$

We write the absolute acceleration of the anti-vibration boom as follows:

$$W_a = W + W_0, \qquad (19)$$

that $W = \dfrac{\partial^2 w}{\partial t^2}$.

(19) the analytical expression of the transfer function in terms of the ratio of the absolute acceleration of the stern to the base acceleration is as follows:

$$\Phi(s,x) = 1 + \frac{U_k s^2 T_k}{W_0}. \qquad (20)$$

We write the transfer function expression (20) taking into account relation (18).

$$\Phi(s,x) = \frac{N_1 + (-d_i U_k s^2)}{N_1}. \qquad (21)$$

here

$$N_1 = N_{11} + jN_{12}$$

$$N_{11} = -\omega^2 + \omega_k^2 - C_0 L_1 \omega_k^2 - \frac{3EI}{m_0 d_{2i}} L_1 \sum_{i=1}^{n} C_i T_{ka}^i \frac{h^i}{2^i (i+3)} \int_0^l U_k \frac{\partial^2}{\partial x^2} \left(\frac{\partial^2}{\partial x^2} \left| \frac{\partial^2 U_k}{\partial x^2} \right|^i \right) dx +$$

$$+ \frac{E}{m_0} \frac{\partial^2 I}{\partial x^2} \int_0^l U_k \frac{\partial^2 U_k}{\partial x^2} dx - C_0 L_1 \frac{E}{m_0} \frac{\partial^2 I}{\partial x^2} \int_0^l U_k \frac{\partial^2 U_k}{\partial x^2} dx - 3L_1 \frac{E}{m_0} \frac{\partial^2 I}{\partial x^2} \int_0^l U_k \frac{\partial^2 U_k}{\partial x^2} \times$$

$$\times \sum_{i=1}^{n} C_i T_{ka}^i \frac{h^i}{2^i (i+3)} \left| \frac{\partial^2 U_k}{\partial x^2} \right|^i dx;$$

$$N_{12} = C_0 L_2 \omega_k^2 + \frac{3EI}{m_0 d_{2i}} L_2 \sum_{i=1}^{n} C_i T_{ka}^i \frac{h^i}{2^i (i+3)} \int_0^l U_k \frac{\partial^2}{\partial x^2} \left(\frac{\partial^2}{\partial x^2} \left| \frac{\partial^2 U_k}{\partial x^2} \right|^i \right) dx +$$

$$+ C_0 L_2 \frac{E}{m_0} \frac{\partial^2 I}{\partial x^2} \int_0^l U_k \frac{\partial^2 U_k}{\partial x^2} dx + 3L_2 \frac{E}{m_0} \frac{\partial^2 I}{\partial x^2} \int_0^l U_k \frac{\partial^2 U_k}{\partial x^2} \times$$

$$\times \sum_{i=1}^{n} C_i T_{ka}^i \frac{h^i}{2^i (i+3)} \left| \frac{\partial^2 U_k}{\partial x^2} \right|^i dx.$$

The expression (21) is the transfer function of the transverse vibrations of the cross section of the rod with the elastic dissipative characteristic of the hysteresis type in the kinematic excitations.

We write the modulus of this transfer function as follows:

$$|\Phi(j\omega, x)| = \sqrt{\frac{\left(N_{11} + d_i U_k \omega^2\right)^2 + N_{12}^2}{N_{11}^2 + N_{12}^2}}. \qquad (22)$$

The obtained analytical expression of the transfer function allows to study the dynamics of transverse vibrations of the considered hysteresis-type elastic dissipative characteristic, variable cross section.

REFERENCES

1. Писаренко Г.С.,Богинич О.Е. Колебания кинематически возбуждаемых механических систем с учетом диссипации энергии.-К.: Наук. думка, 1981.-220с.
2. Павлов В.П. Поперечные колебания стержня с переменным поперечным сечением и вычисление его собственных частот методом сплайнов// Вестник УГАТУ. 2017, Т.21.№2(76).-с.3-16.
3. Yardimoglu B., Aydin L. Exact longitudinal vibration characteristics of rods with variable cross – sections. Shock and Vibration. Volume 18, 2011, Pp.555-562, DOI 10.3233/SAV-2010-0561
4. Mkrtchyan K.S. Study of forced transverse vibrations of elastic hinge-operated rod taking into account rotative movement. Mech. Solids. Volume 54, 2019, Pp. 112-121. https://doi.org/10.3103/S0025654419010096
5. Chaohe Ch., Yong H., Guangfan L. Calculation of the natural frequencies of transverse vibration of complex beams using the differential-matrix equations. Advanced Materials Research. Volume 243-249, 2011, Pp.284-289, https://doi.org/10.4028/www.scientific.net/AMR.243-249.284
6. Павловский М.А., Рыжков Л.М., Яковенко В.Б., Дусматов О.М. Нелинейные задачи динамики виброзащитных систем. – К.: Техника, 1997. – с. 204.
7. Писаренко Г.С., Яковлев А.П., Матвеев В.В. Вибропоглощающие свойства конструкционных материалов. Справочник.-Киев: Наук.думка, 1971–327 с.