INTERESTING EQUATIONS AND INEQUALITIES

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Abstract. *In this article, a new method is used to solve some equations and inequalities, and it is important for students to learn differential calculus. The use of differential calculus allows to solve complex equations and inequalities in a simple way, and this method can also be used to solve some geometric problems.*
Keywords: *equation, inequality, triangle, rectangle, center, radius, bisector.*

The main focus of this topic is that a new method is used to solve equations and inequalities, and in addition to students learning differential calculus, the use of differential calculus not only allows solving complex equations and inequalities in a simple way, but also some geometrical methods from this method can also be used to solve problems.

1) Let's assume $Q_1, Q_{2,}, \ldots, Q_n$ (1). If (1) are integers corresponding to Eq.

$$(x-a_1)(x-a_2)\ldots(x-a_m)-(-1)^n\cdot(n!)^2=0$$

We find r that has an entire solution

$$r=\frac{Q_1+Q_2+\cdots+Q_{2n}}{2n}$$

Solving: r$\neq a_i$ it is seen.

$$[(r-a_1)(r-a_2)\ldots(r-a_{2n})]\geq|(1)(2)\ldots(n)\cdot(-1)(-2)\ldots(-n)|=(n!)^2$$

For Eq

$$\{r-a_{1,}r-a_{2,,,}r-a_{2n}\}=\{1,2,\ldots,n,-1,-2,\ldots,(-n)\}$$

should be therefore

$$(r-a_1)(r-a_2)\ldots(r-a_{2n})=2nr-(a_1+a_2+\cdots+a_{2n)}=$$

$$=2nr-a_1+..+a_{2n}$$

$$1+2+..+n+(-1)+(-2)+..+(-n)=0=r=\frac{a_1+a_2+..+a_{2n}}{2n}$$

2) If $t_{1,}t_{2,\ldots,}t_n$ for the numbers

$$n^2+1>(t_{1,}t_{2,\ldots,}t_n)\left(\frac{1}{t_1}+\frac{1}{t_2}+,\ldots,+\frac{1}{t_n}\right)$$

If the relationship is appropriate, then $1\leq i<j<k\leq n$ [7] if the inequality is true $\forall\, i,j,k$ for s $t_{i,}t_{j,}t_k$ prove that the numbers are the lengths of the sides of a triangle.

Proof: Let's assume that the numbers $t_{1,}t_{2,}t_3$ satisfying the condition of the problem cannot be the lengths of the sides of a triangle, i.e. $t_3+t_2\leq t_1$

so be it

$$(t_{1,}t_{2,\ldots,}t_n)\left(\frac{1}{t_1}+\frac{1}{t_2}+,\ldots,+\frac{1}{t_n}\right)=n+\left(\frac{t_1}{t_2}+\frac{t_2}{t_1}\right)+\left(\frac{t_1}{t_3}+\frac{t_3}{t_1}\right)+\cdots+\left(\frac{t_n}{t_{n-1}}+\frac{t_{n-1}}{t_n}\right)^{AM-iM}\geq$$

$$n+\frac{t_2+t_3}{t_1}+t_1\left(\frac{1}{t_2}+\frac{1}{t_3}\right)+2+2+\cdots 2 =>$$

$$\text{n}+\frac{t_1+t_3}{t_2\cdot t_3}\cdot\left(\frac{t_2\cdot t_1}{t_1}+t_1\right)+n(n-1)-4=n^2-4+\frac{t_2+t_3}{t_2\cdot t_3}\cdot\left(\frac{t_2 t_3}{t_1}+t_1\right)\geq$$

$$n^2-4+\frac{t_1+t_3}{t_2\cdot t_3}\left(\frac{t_2+t_3}{t_2\cdot t_3}+t_3+t_2\right)\geq n^2-4+1+\frac{(t_2+t_3)^2}{t_2\cdot t_3}\geq$$

$$\geq n^2-3+\frac{4t_2\cdot t_3}{t_2\cdot t_3}=1+n;\ \text{ because } t_3+t_2\leq t_1 \text{ was.}$$

We are conflicted. That is, our hypothesis is wrong. The issue has been completely resolved.

3) ABCD- Let the circle be an inner rectangle. D from the point BC, BA and AB

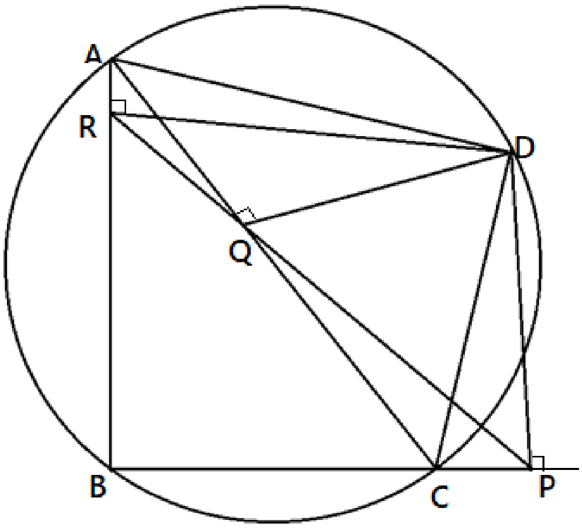
The bases of the perpendicular drawn to the sides are respectively R,Q,R if. PQ=QR to be ABC and ADC bisectors of angles AC

Prove that it is necessary and sufficient that it intersects in a straight line.

Proof: According to Simpson's theorem Q,P,R points lie on a straight line.

CQD+CDP=180° and ARD=AQD=90°=>PCQD RADQ circle radii respectively $R_1=\frac{CD}{2}$ and $R_2=\frac{AD}{2}$ is equal to. [4]

In that case PQ=2$R_1\sin QCP=2\frac{CD}{2}\sin(\pi-ABC)=CD\cdot\sin ACD$



4) a,b,c belongs to R^+ and abc=1 If so, prove it.

$$S=\frac{1}{a^3(b+c)}+\frac{1}{b^3(c+a)}+\frac{1}{c^3(a+b)}\geq\frac{3}{2}$$

Proof: S$\cdot\big(a(b+c)+b(c+a)+c(a+b)\big)\geq\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)^2=(ab+bc+ac)^2$

$$=> S\geq\frac{\big((ab+bc+ac)\big)^2}{2(ab+bc+ac)}=\frac{ab+bc+ac}{2}\geq\frac{3\sqrt[3]{a^2b^2c^2}}{2}=\frac{3}{2}$$

5) If a,b,c belongs to R for $ab+bc+ac=0$ if

P(a-b)+P(b-c)+P(c-a)=2P $(a+b+c)$

Find all polynomials with real coefficients R(x) satisfying the condition.

Proof: a=b=0=> P(0)+P(-c)+P(c)=2P(c)=P(c)= P(0)+P(-c)=P(0)=0=P(c)=P(-c)=P(x) couple So P(x) is a polynomial.

P(x)=$a_n x^{2n} + a_n x^{2n-2} + \cdots + a_2 x^{2n-1} + .. + a_1 x^2$ (a belongs to R, $a_n \neq 0$) in appearance

$$a=x \quad b=2x \quad c=-\frac{2}{3} \quad (x€R) \text{ if } ab+bc+ac = 2x^2 - \frac{4x^2}{3} - \frac{2x^2}{3} = 0 \text{ So}$$

$$b-c=\frac{8}{3}x, \quad c-a=-\frac{5}{3}x, \quad a+b+c = \frac{7}{3}x$$

P(-x)+P($\frac{8}{3}x$) $+ P\left(-\frac{5}{3}x\right) = 2P\left(\frac{7}{3}x\right) = (x^{2n}$ if we equate the coefficients in front of)

$$a_n\left((-1)^{2n} + \left(\frac{8}{3}\right)^{2n} + \left(\frac{5}{3}\right)^{2n}\right) = 2a_n\left(\frac{7}{3}\right)^{2n} = 3^{2n} + 8^{2n} + 5^{2n} = 2\cdot 7^{2n} =$$

$$8^{2n} < 2\cdot 7^{2n} = \left(\frac{8}{7}\right)^{2n} < 2; \qquad (2)$$

But n≥ 3 da $\left(\frac{8}{7}\right)^{2n}\left(1+\frac{1}{7}\right)^{2n} > 1 + C_{2n}^{1}\frac{1}{7} + C_{2n}^{2}\frac{1}{49} = 1 + \frac{2n}{7} + \frac{n(2n-1)}{49} \geq$ $\frac{106}{49} > 2$ So $(*)$ the inequality is valid for n≤ 2 in the form of P(x)= $a_2x^4 + a_1x^2$ as a result of direct inspection P(x)= $a_2x^4 + a_1x^2$, $a_2\, a_1$€R it is not difficult to see the multiple issue condition. So the result P(x)= $a_2x^4 + a_1x^2$ $(\forall a_{2,} a_1$€R$)$.

6) Let's say AB≠AC in acute angle ABC. A circle drawn as a diameter with side BC intersects sides AB and AC at points M and N, respectively. If we mark the middle of the side BC with O, then the bisectors of the angles BAC and MON intersect at the point R, prove that the circles drawn outside the angles BM and CNR intersect on the side BC.

Proof: ΔMOR=ΔNOR because MO=NO,MOR=NOR, RO common side. So MR=NR; BMCN cyclicity of the rectangle BCA=AMN ABC=ANM: according to the condition AB≠

$$AC, ABC \neq ACB => AMN \neq ANM \qquad (2)$$

ΔMRA and ΔNRA the radius of the outer circle.

Conclusion

In conclusion, from what has been analyzed and discussed above in the proving process it can be inferred that chosen formula has been proved with its clear process of checking and calculation which has been intended to do.

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