# STUDY OF CONINUITY OF FUNCTION: FROM VISUALLY-INTUITIVE UNDERSTANDING TO THE PRECISE ONE

[1]R.Ibragimov, [2]B.T.Kalimbetov
[1]Sout Kazakhstan State Pedagogical University, Shymkent, Kazakhstan
[2]Universitety Akhmed Yasawi, Turkestan, Kazakhstan
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***Abstract.*** *In the paper, methodical approaches to the study of continuity of functions are presented, knowledge of which has a significant impact on both the mathematical and methodological training of future specialists. The primary problem for understanding the concept of continuity is the formation of visually-intuitive understanding. The level of assimilation of concepts should be evaluated, basically, at the level of identification, but not by the ability to reproduce definitions of continuity of a function at a point.*
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**Introduction.** Mathematics as an expression of the human mind reflects the active will, the contemplative reason, and the desire for aesthetic perfection. Its basic elements are logic and intuition, analysis and construction, generality and individuality. Though different traditions may emphasize different aspects, it is only the interplay of these antithetic forces and the struggle for their synthesis that constitute the life, usefulness, and supreme value of mathematical science (R.Courant, H.Robbins, 1947).

One of the fundamental mathematical concepts is continuity of a function. Continuity is the basic and rather complex notion on which mathematical analysis is constructed, and therefore all modern mathematics. Students are faced with this concept in the first year of study. But they learn it very formally – they do not understand the meaning of the definition of continuity and logic of the proofs of theorems, they can not give examples of really existing processes that are described by conituous functions. It is important for them to understand the main idea inherent in this concept – the idea of the "small" deviation from each other of the function values, when the argument values, to which these values correspond, are sufficiently "close" (L.R.Gaysina, 2007). The definition of the continuity of a function is based on the concept of a limit. The continuity of a function at a point is one of the most important properties. Students get an intuitive idea on this property by constructing graphs of various functions. Forming the table of values of the argument and the corresponding values of the function, marking out points and connecting these points by a solid line, they obtain graphs of functions. It is known that this procedure is possible only if the function under study is continuous (then its graph is a continuous line) or differentiable (then its graph is a continuous line without corner points).

Visually-intuitive understanding of continuity functions. Since the continuous function is the basic material for differentiation and integration, the concept of continuity accompanies these operations throughout their study. Therefore, when studying relevant topics related to these sections, it is necessary to "lose" these approaches to the definition of the concept of continuity in the corresponding "operational" situation .

In this paper, we consider practical examples explaining the concept of continuity of a function. They are connected with the life experience of students, their knowledge in other sciences. In the process of analyzing the proposed situations, students become acquainted with the concept of continuity at first on an intuitive level; then, come to a strict definition of a new concept. *A problem from the course in physics*. It is considered the process of melting of naphthalene. It is required to determine the change in its temperature as a function of the heating time. Time is measured in minutes, and temperature is measured in degrees Celsius. The temperature of naphthalene is a function of the time determined on the interval $[0;t_0]$ where $t_0$ is the duration of observation. Let's start the experiment from the moment when the temperature of solid naphthalene was 55 °C. When heated for the first 5 minutes, the temperature rises until it reaches 80 °C – the melting point. Then during the entire melting time (the next 4 minutes), it does not change, although the heating process continues. And only after all the naphthalene has melted, its temperature begins to rise again. We finish the observation when the temperature of the formed liquid rises to 90 °C. The graph of the function $T(t)$ is shown in Fig. 1.

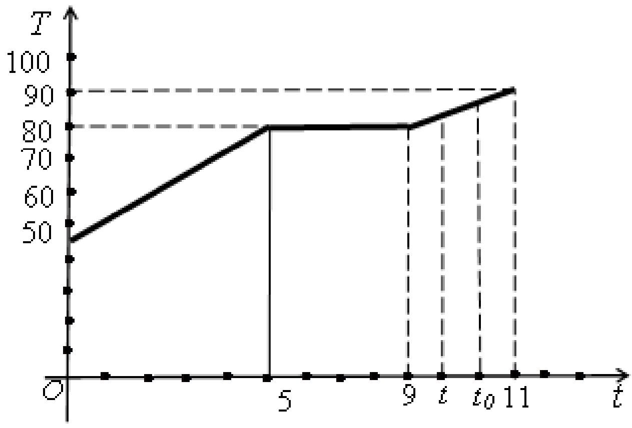


***Figure 1. Graph of the temperature of naphthalene*** $T(t)$

In the following, the function $T(t)$ can be used to illustrate a function that is continuous everywhere on the interval $[0;t_0]$, but not differentiable at the points $t=5$ and $t=9$. The rate of change in the temperature of naphthalene can not be determined uniquely at these times, it is in a transitional state between the solid and liquid.

2. *A problem from the course in strength of materials* (V.P.Kolpakov, A.V.Ponkin, E.E.Rikhter, 2014). The main element of any building construction is a beam. Balconies are supported by cantilever beams, which are embedded in the wall with only one end, while their second end is left free. Before applying the beam to any structure, it must be calculated for strength, since the beam bends under the action of the load. Take the beam, the left end of which is embedded in the wall, and distribute uniformly the load $Q$ on the beam. Let the length of the free beam section be equal to $l$. The ratio $Q/l$ is called the load intensity.

The load intensity becomes larger, the beam bends more strongly. Denote by $y$ the beam deflection at a point located at a distance $x$ from its left (free) end. Each $x$ corresponds to a unique value of the deflection, hence, the beam deflection $y$ is a function of the distance $x$ with the domain $[0;l]$. The greatest deflection will be at the free end of the beam, i.e. at $x=l$. The graph of the function $y(x)$ is depicted in Fig. 2.

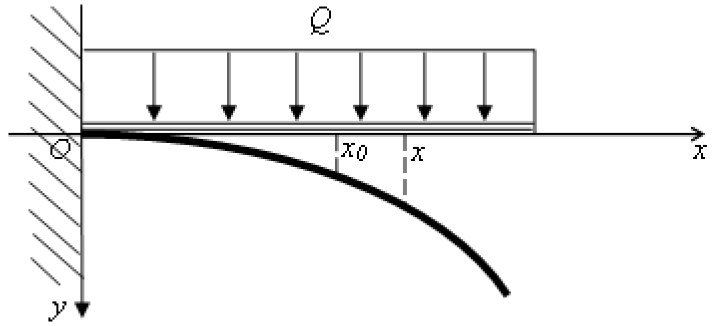


***Figure 2. Graph of the beam deflection*** $y(x)$

3. A problem from the course of mathematical analysis (V.A.Il'in, V.A.Sadovnichij, B.H.Sendov, 2013; B.S.Thomson, J.B.Bruckner, A.M.Bruckner, 2008). Consider the function $y=\{x\}$ where $\{x\}$ is the fractional part of a number $x$. Its graph is shown in Fig. 3.

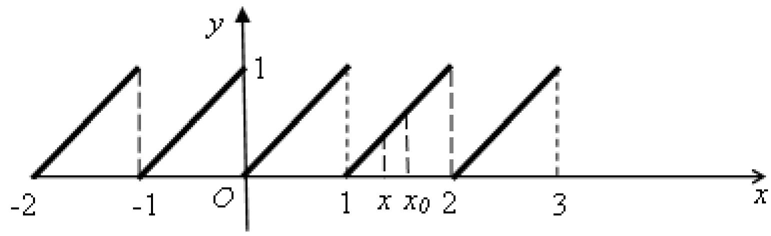


***Figure 3. Graph of*** $y=\{x\}$.

Invite students to compare the graphs on Figures 1-3, find in them common features and differences from each other. Obviously, the graphs of functions $T(t)$ and $y(x)$ are similar to one another in the sense that they are continuous lines, i.e. lines that can be drawn with one hand movement, “without taking the pencil from the paper”. We agree to call such functions *continuous*. The graph of $y=\{x\}$ differs significantly from the first two ones, it can not be drawn with one hand movement. This graph consists of separate segments. Obviously, the function $y=\{x\}$ is continuous in the above sense at each of the intervals $(n;n+1)$ $(n \in \mathrm{Z})$, and there is not continuity at the points $x=n$. In such cases we say that the function suffers a discontinuity at these points, and function is called discontinuous.

Further the teacher should explain that not all functions are continuous, but all functions corresponding to real processes in nature and technique are continuous. The change in atmospheric pressure depending on the altitude above sea level, the change in temperature and humidity of air over a time period, the change in the growth or weight of a person over time, etc. – all these processes proceed continuously. Therefore continuous functions are especially important for practical purposes.

After examples, students have no difficulty to determine whether a function is continuous or discontinuous according its graph. They also realize that plotting a function is often very laborious, technically difficult, and the reliability of the graph is highly questionable without a preliminary analytical study of the function[12].

A mathematician can not confine himself to a purely descriptive definition of a continuous function. Using mathematical concepts without an accurate understanding of their meaning can lead to gross errors. To state general statements about continuous functions, it is necessary first of all to give a clear definition of this mathematical concept that will not intuitive, but strictly and unequivocally establish whether this function is continuous or not[14].

So, having formed an intuitive idea of continuity among students, we set before them the following, qualitatively new task: to introduce suitable terminology, definitions, to formulate the properties of continuous functions and to prove them.

Exact definition of continuity functions. Starting with the study of functional dependencies, we must, of course, first of all, with the help of an expedient classification, introduce at least some order into the diverse world that is to come. The first such classifying and organizing principle is usually (and justifably) separation of all functions into continuous and discontinuous, moreover the mathematical analysis actually deals almost exclusively with continuous functions, involving the simplest of discontinuous only in relatively rare cases. Continuous functions have a number of special properties that, generally speaking, are not inherent in discontinuous functions; thanks to these properties, the study and application of continuous functions are greatly facilitated, so that the study of these properties becomes extremely important for analysis (A.Ya.Khinchin, 1948). First of all, we need to determine exactly what is meant when we say that the function $f(x)$ is continuous at the point $x_0$.

Let's attempt to describe quantitatively the difference between continuous and discontinuous functions (V.A.Il'in, E.G.Pozdnjak, 2005; K.A.Ross, 2013). The functions shown in Fig. 1-2, have the following property: for close values of the argument, for example, $x$ and $x_0$, the corresponding values of the function, i.e. $f(x)$ and $f(x_0)$, are also close to each other. This property is also possessed by the function $y=\{x\}$ at all points except for integers. The situation is different at the points $x=n$, in which there is no continuity. Let's see how the function behaves in a neighborhood of these "bad" points. On each interval $[n;n+1)$, it increases continuously, approaching arbitrarily close to the value 1, but does not reach this value, and at the point $x=n+1$, it returns suddenly, abruptly to the value zero. At points where there is no continuity, the closeness of the two values of the argument does not necessarily imply the proximity of the corresponding values of the function. Obviously, this circumstance can be taken as the basis for determining the continuity of a function at a point (I.A.Maron, 2008; W.F.Trench, 2013). First of all, it is necessary to give an exact meaning to the statement: if $x$ is close to $x_0$, then $f(x)$ is close to $f(x_0)$.

In the following, the notions "increment of the argument" and "increment of the function" and the corresponding symbols are introduced. The meaning of notations $\Delta y \to 0$ and $\Delta x \to 0$ is explained. Using them, we obtain the following definition of continuity at a point (V.A.Il'in, V.A.Sadovnichij, B.I.Sendov, W.F.Trench, 2013).

**Definition 1.** A function $f$ is said to be *continuous at a point* $x_0$, if $\Delta y \to 0$ at $\Delta x \to 0$, i.e.

$$\lim_{\Delta x \to 0} \Delta y = 0. \qquad (1)$$

Now the definition of continuity of a function at a point can be given different formulations, different in form, but identical in content with definition 1. Rewrite expression (1) in the following form:

$$\lim_{x \to x_0}\left(f(x)-f(x_0)\right)=\lim_{x \to x_0} f(x)-\lim_{x \to x_0} f(x_0)=\lim_{x \to x_0} f(x)-f(x_0)=0,$$

what implies

$$\lim_{x \to x_0} f(x) = f(x_0).$$

We obtain the following definition of continuity of a function at a point (V.A.Il'in, E.G.Pozdnjak, 2005; B.S.Thomson, J.B.Bruckner, A.M.Bruckner, 2008).

**Definition 2.** We say a function $f$ is continuous at $x_0$ if $f$ is defined on an open interval $(a,b)$ containing $x_0$ and

$$\lim_{x \to x_0} f(x) = f(x_0). \tag{2}$$

It should be emphasized that to establish the fact of continuity of a function at a point one can use any of the definitions, but in different cases it is advisable to use different definitions of continuity.

To prove continuity of a function at $x_0$ with the help of Definition 1, one must:

give the argument $x_0$ an increment $\Delta x$;

find the corresponding increment of the function

$$\Delta y = f(x_0 + \Delta x) - f(x_0);;$$

study $\Delta y$ at $\Delta x \to 0$; if we obtain $\lim\limits_{\Delta x \to 0} \Delta y = 0,$ then we conclude that $f(x)$ is continuous at $x_0$.

It should be noted that $y = f(x)$ is continuous if its graph is its graph is a smooth, nowhere "interrupted" curve. To determine whether $y = f(x)$ is continuous at $x = x_0$, let the independent variable $x$ be approached continuously from the right and from the left to $x_0$. In this case, the values of the function $y = f(x)$ change unless this function is constant in a neighborhood of $x_0$. If the value of $y = f(x)$ comes arbitrarily close to $y = f(x_0)$ at the chosen point $x = x_0$ ("tends to the limit of $y = f(x)$"), and moreover, regardless of whether $x$ tends to $x_0$ from the left or from the right, then $y = f(x)$ is said to be continuous at $x_0$. A function *f* is said to be continuous on an open interval $(a,b)$ if it is continuous at every point in $(a,b)$.

Although every function represented by a smooth graph is continuous, one can easy to define functions that are not continuous everywhere (R.Courant, H.Robbins, 1947). For example, the function in Fig. 4 defined for all values of $x$ with the help of formulas

$$f(x) = 1 + x \quad \text{at } x > 0,$$
$$f(x) = -1 + x \quad \text{at } x \le 0$$

is discontinuous at $x_0 = 0,$ in which it has the value—$1$. If we approach to $x_0 = 0$ from the right, then $f(x)$ tends to +1. But this value is different from the vaue of the function at this point, namely, 1.

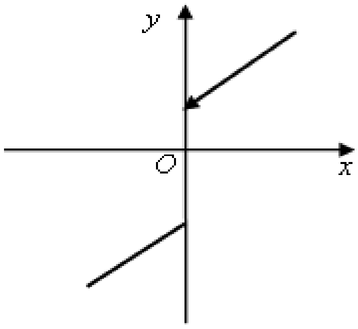


***Figure 4. Graph of*** $f(x)=\begin{cases}1+x, & x>0,\\ -1+x, & x\le 0.\end{cases}$

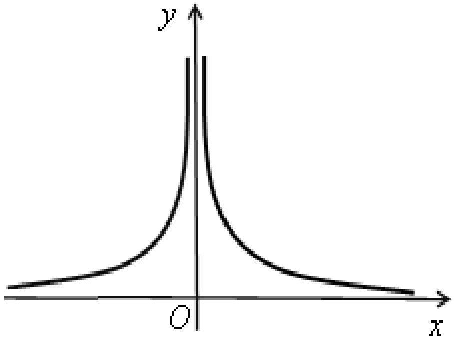


***Figure 5. Graph of*** $f(x)=\dfrac{1}{x^2}$.

One circumstance that the function tends to $-1$ when $x$ tens to zero is still insufficient to establish continuity of $f(x)$.

A function $f(x)$ defined by

$$f(x)=\begin{cases}0, & x\neq 0,\\ 1, & x=0\end{cases}$$

has a discontinuity of another kind at $x_0=0$. For this case, the limits from the left and on the right exist and are equal to each other, but this total limit value is different from $f(0)$. The function $y=f(x)=\dfrac{1}{x^2}$, the graph of which is shon in Fig.5, has a still different type of discontinuity at $x=0$. If $x$ tends to $0$ from either side, then $y$ will tend always to infinity. The graph of the function is "interrupted" at this point, and small changes in the independent variable $x$ at a neighborhood of $x=0$ can correspond to very large changes in the dependent variable $y$. Strictly speaking, the value of the function is not defined for $x=0$; since infinity is not considered a number, that's why one can not say that $f(x)$ is equal to infinity at $x=0$. So, we say only that $f(x)$ "tends to infinity" as $x$ approaches zero.

The above examples show several different typical cases when a function ceases to be continuous at a point $x=x_0$.

1) It may be possible that the function will be continuous at $x=x_0$ after defining or redefining its value at $x=x_0$. For example, the function $y=\dfrac{x}{x}$ is constantly equal to 1 at $x\neq 0$; it is not defined at $x=0$ since $\dfrac{0}{0}$ is a meaningless symbol. But if in this example, we agree that the value $y=1$ also corresponds to the value $x=0$, then the function "extended" in this way becomes continuous at all points without exception. The same result will be achieved if we change the value of the function at $x=0$ in the second of the examples above and set $f(0)=0$ instead of $f(0)=1$

Discontinuities of this kind are said to be removable.

2) The function approaches different limits depending on $x$ tends to $x_0$ from the left or from the right, as in Fig. 4.

3) There is no limit from any one nor the other side.

4) The function tends to infinity as $x$ approaches $x_0$ (Fig. 5).

Discontinuities of the last three types are said to be essential or unremovable, they can not be removed by properly defining the function at the point $x = x_0$ alone.

Now we give the precise definition of continuity of a function. First, we give the definition for the limit of a function (I.A.Maron, 2008; V.A.Il'in, E.G.Poznjak, 2005; K.A.Ross, 2013).

**Definition 3.** We say that the function $f(x)$ has the limit $a$ as $x$ tends to the value $x_0$ if, for every positive number $\varepsilon$, no matter how small, there may be found a positive number $\delta$ (depending on $\varepsilon$) such that

$$|f(x)-a|<\varepsilon$$

for all $x \neq x_0$ satisfying the inequality

$$|x-x_0|<\delta.$$

When this is the case, it is customary to write $f(x)\to a$ as $x \to x_0$ or $\lim_{x\to x_0} f(x) = a$.

If, in the Cauchy limit determination, we substitute instead of $a$ its value $y = f(x_0)$, then the continuity condition takes the following form:

**Definition 4.** The function $y = f(x)$ is said to be continuous at $x = x_0$ if, for any sufficientle small positive number $\varepsilon$, there is a positive number $\delta$ (depending on $\varepsilon$) such that the inequality

$$|f(x)-f(x_0)|<\varepsilon$$

takes place for every $x$ satisfying the condition

$$|x-x_0|<\delta$$

(the restriction $x \neq x_0$, introducing in the definition of the limit, is unnecessary here because the inequality $|f(x)-f(x_0)|<\varepsilon$ is automatically satisfied at $x = x_0$).

This is one of the precise definitions of the continuity of a function at a point $x = x_0$.

As an example, we try to establish continuity of the function $y = x^3$ at the point $x_0 = 0$ (R.Courant, H.Robbins, 1947). We have

$$y_0 = f(x_0) = x^3\big|_{x_0=0} = 0.$$

Choose sufficiently small positive number $\varepsilon$, for example, $\varepsilon = 1000^{-1}$. We must show that by confining values of $x$ to values sufficiently near $x_0 = 0$, we obtain the corresponding values of $f(x)$, which will not differ from 0 more than $1000^{-1}$, i.e. will lie between $-1000^{-1}$ and $+1000^{-1}$. It can be seen that the values of $f(x)$ do not leave this margin if we restrict $x$ to values delivering from zero by less than $\delta = \sqrt[3]{1000^{-1}} = 10^{-1}$. In fact, if $|x| < \frac{1}{10}$, then $|f(x)| = |x^3| < \frac{1}{1000}$. In exactly the same way one can take instead of $\varepsilon = 10^{-3}$ any smaller value $\varepsilon = 10^{-4}, 10^{-5}$, etc.; the numbers $\delta = \sqrt[3]{\varepsilon}$ will be satisfied our requirement, since the inequality $|x| = \sqrt[3]{\varepsilon}$ implies the inequality $|f(x)| = |x^3| < \varepsilon.$

Based on the $\varepsilon-\delta$-definition of continuity, similarly it can be shown that all polynomials, rational and trigonometric functions are continuous except, maybe, for isolated values of $x$ where the functions become infinite.

Connecting the definition continuity of a function $y = f(x)$ with its graph, one can give it the following geometric form. Choose any positive number $\varepsilon$ and draw straight lines parallel to the $x$-axis at a height $f(x_0)-\varepsilon$ and $f(x_0)+\varepsilon$ above it. Then it must be possible to find a positive number $\delta$ such that the whole portion of rge graph which lies within the vertical band of width $2\delta$ about $x_0$ is also contained within the horizontal band of width $2\varepsilon$ about $f(x_0)$. Fig. 6 shows a function which is continuous at $x_0$, while Fig. 7 shows a function which has the dicontinuity at this point. In the latter case, no matter how narrow the vertical band about $x_0$ is, it will always include a portion of the graph that lies outside the vertical band of width $2\varepsilon$.

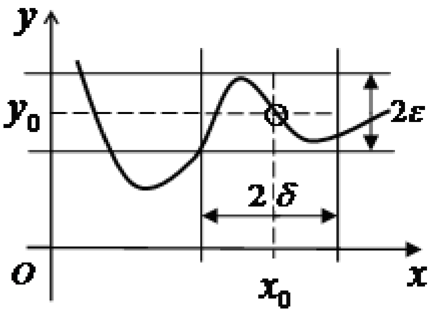


*Figure 6.*

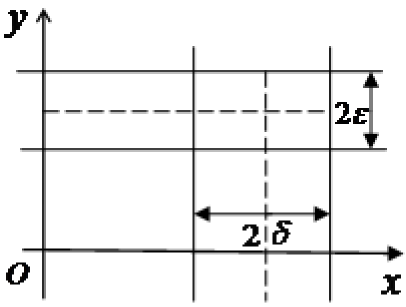


*Figure 7.*

If the given function $y = f(x)$ is continuous at the point $x = x_0$, then one can choose any positive number $\varepsilon$, sufficiently small, but definite. Then one can find a positive number $\delta$ such that the inequality $|x - x_0| < \delta$ implies the inequality $|f(x) - f(x_0)| < \varepsilon$.

Example. Determine whether the function

$$f(x) = \frac{2x+1}{2x^2 - x - 1}$$

is continuous at the point a) $x = 2$; b) $x = 1$.

Solution. According to definition, three conditions must be met in order to ensure continuity at the point $x = a$.

a) To determine whether the given function is continuous at the point $x = 2$, check the coninuity conditions at $x = 2$.

**Condition 1.** The function $f(x)$ should be define at the point $x = 2$. Determine $f(2)$:

$$f(2) = \frac{2\cdot 2+1}{2\cdot(2)^2 - 2 - 1} = 1.$$

Thus, $f(2) = 1$ is a real number.

**Condition 2.** For $f(x)$, there exists $\lim\limits_{x\to a} f(x)$. Does it exist $\lim\limits_{x\to 2} f(x)$?

$$\lim_{x\to 2} f(x) = \lim_{x\to 2}\frac{2x+1}{2x^2 - x - 1} = \frac{\lim\limits_{x\to 2}(2x+1)}{\lim\limits_{x\to 2}(2x^2 - x - 1)} = \frac{2\cdot 2+1}{2(2)^2 - 2 - 1} = 1.$$

Using the properties of limits, we prove that $\lim\limits_{x\to 2} f(x)$ exists.

**Condition 3.** $\lim_{x\to a} f(x) = f(a)$. Is there a limit $\lim_{x\to 2} f(x) = f(2)$? We have found that $\lim_{x\to 2} f(x) = 1$ and $f(2) = 1$. Thus, as $x$ approaches 2, corresponding values of $f(x)$ approach the value of at 2: $\lim_{x\to 2} f(x) = f(2)$.

Since all three conditions of continuity of a function at the point are valid, we conclude that the function $f(x)$ is continuous at the point 2.

b) To determine continuity of the function $f(x) = \dfrac{2x+1}{2x^2 - x - 1}$ at the point $x = 1$, we check the conditions of continuity for $x = 1$.

**Condition 1.** Define the function $f(x)$ at the point $x = 1$. Since the denominator becomes zero for $x = 1$ and $x = -1/2$, the function $f(x)$ is not defined at $x = 1$.

Since one of the three conditions is not satisfied, we conclude that the function $f(x) = \dfrac{2x+1}{2x^2 - x - 1}$ is not continuous at $x = 1$. Equivalently, we can say that it has a discontinuity at the point $x = 1$.

So, $f(2) = 1$ is a real number.

Thus, for continuity of a function at $x_0$, it is required, firstly, the existence of $\lim_{x\to a} f(x)$, and, secondly, the coincidence of this limit with the value that the function takes when $x = x_0$. It goes without saying that the first does not imply the second, as the example of the function

$$f(x) = \begin{cases} x^2 & \text{for } x \neq 0, \\ 1 & \text{for } x = 0 \end{cases} \qquad (3)$$

shows.

With reference to this definition, it should be first of all noted that so understood continuity is a local property of a function, i.e. the property that a function can possess at one point and not possess in another one: so, the function (3) is discontinuous (i.e. is not continuous) at $x = 0$ and it is continuous at any another value of $x$; this is a very important circumstance that must never be overlooked.

Further, we say that a function is continuous on the given segment $[a,b]$ if it is continuous at every point of the interval and, in addition at $x = a$ the function must be continuous only from the right, i.e. $\lim_{x\to a+0} f(x) = f(a)$, and at $x = b$ – only from the left, this continuity is defined by a similar relation. It should be noted that mathematicians have long ago used very convenient notations

$$\lim_{x\to a+0} f(x) = f(a+0), \qquad \lim_{x\to a-0} f(x) = f(a-0),$$

with the help of which, definition of the continuity of a function $f(x)$ at a point $x = a$ can be written by means of a very simple relation

$$f(a+0) = f(a-0) = f(a),$$

this notation can not lead to any confusions if we only remember that $f(a+0)$ and $f(a-0)$ represent not the values of $f(x)$ at any points, but the limits of such values for certain changes in the value of $x$.

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