STUDYING THE STABILITY OF THE ZERO SOLUTION OF THE HILL EQUATION IN PARAMETRIC MOTION ACCORDING TO THE QUASI-RECTANGULAR SINE LAW

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Abstract. *In this article, the state of stability in parametric excitation according to the quasi-rectangular sine law of motion expressed by Hill's equation is studied. The stability of changing systems and their solutions over a certain period of time was analyzed.*

Keywords: *quasi-rectangular sine law, parametric excitation, Hill's equation, stability, fundamental matrix.*

INTRODUCTION

It is known that human life consists of movement and various processes. Where there is movement, work is done, energy is spent, and energy is generated. This, in turn, creates a process that satisfies human needs. It is important that the movement of anything or system is continuous and stable. This issue is one of the pressing issues in our developing society. As the human mind develops, the processes activated by it become more complex. These systems of actions and processes are represented in science by various functional-differential equations and systems of equations. We can check the stability of such complex systems using these equations. Several equations have been proposed by scientists to study the stability of the movement of complex systems in life. When studying the stability of motion, it is important to express it with an equation. Hill's equations are very effective for this. In the following article, the stability of the movement represented by Hill's equation in the parametric excitation according to the quasi-rectangular sine law is studied.

LITERATURE REWIEW

The equations and concepts used in this article are studied in one chapter of Д.Р.Меркин's book named "Introduction to the theory of stability of motion". The author showed that when studying the stability of non-autonomous systems, that is, parametric driven systems, it is enough to study the stability of the solutions of Hill's equations [1]. In А.Х. Гелиг's book named "Absolute stability of linear systems with non-unique equilibrium states in critical states" the conditions of absolute stability imposed on the fundamental solutions of the equations of motion were considered [2].

RESEARCH METHODOLOGY AND DISCUSSION

Consider a simple system, the following excited motion equation described by Hill's equation

$$\ddot{x} + [\delta + \varepsilon\psi(t)]x = 0 \qquad (1)$$

$\psi(t)$ changes with the excitation function according to the quasi-rectangular sine law (Fig. 1). The period T of the excitation function is equal to the sum of time T_1 when the function $\psi(t)$ equals +1 and time T_2 when $\psi(t) = -1$. At $T_1 = T_2$, we get a typical quasi-rectangular sine.[1]

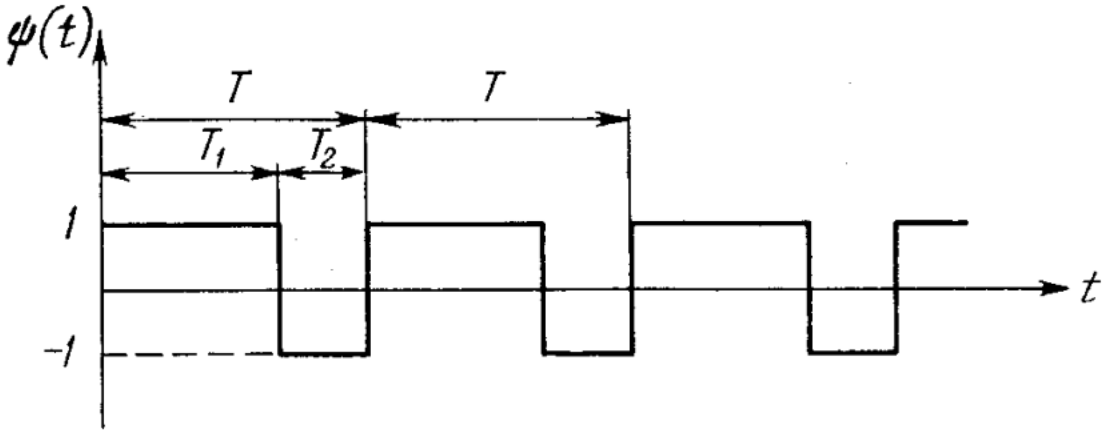

Figure 1

In particular, with the help of equation (1), the systems whose stiffness changes from time to time are studied using the relay device. For us, this problem means not only that its solution can be used to analyze the stability of the behavior of certain systems, but also that it is necessary to construct a fundamental matrix $X(T)$ of solutions satisfying the condition $X(0) = E$ for one period $[0, T]$,

$$\det(A-\rho E)=\begin{Vmatrix} a_{11}-\rho & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22}-\rho & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn}-\rho \end{Vmatrix}=0.$$

shows the construction of the matrix $A = X(T)$ of the characteristic equation and the determination of stability conditions for the solutions $x = 0, \dot{x} = 0$.[2]

In the equation (1), the number ε is equal to the pulsation depth, and the number δ is equal to the square of the frequency k of natural vibrations at $\delta > 0$ and $\varepsilon = 0$, i.e. $\delta = k^2$. We match the beginning of time t with the beginning of any period T. Then, for the first part of the period $0 \leq t \leq T_1$, equation (1) looks like this:

$$\ddot{x} + (k^2 + \varepsilon)x = 0 \qquad (0 \leq t \leq T_1), \qquad (2)$$

and for the second part of the period $0 \leq t \leq T_2$

$$\ddot{x} + (k^2 - \varepsilon)x = 0 \qquad (T_2 \leq t \leq T). \qquad (3)$$

Let's look at the first equation (2). Assuming that $x_1 = x,\ x_2 = \dot{x}$ as before, we reduce equation (2) to a system of two first-order equations

$$\dot{x}_1 = x_2,\ \dot{x}_2 = -k_1^2 x_1 \quad (0 \leq t \leq T), \qquad (4)$$

here

$$k_1^2 = k^2 + \varepsilon. \qquad (5)$$

(4) system is simply solved. Two linearly uncoupled solutions of this system satisfying the condition $X(0) = E$ are as follows:

$$x_{11} = cosk_1t,\quad x_{12} = \frac{1}{k_1}sink_1t,\quad x_{21} = -k_1sink_1t,\quad x_{22} = cosk_1t \qquad (6)$$

Thus, the fundamental matrix of the solution in the first part of the period is as follows

$$X(T) = \begin{Vmatrix} cosk_1t & \frac{1}{k_1}sink_1t \\ -k_1sink_1t & cosk_1t \end{Vmatrix} \qquad (0 \leq t \leq T_1). \qquad (7)$$

It can be seen that $X(0) = E$.

Let's go to the second part of the period ($T_2 \le t \le T$). Equation (3) after replacing $k_2^2 = k^2 - \varepsilon$ in system (4) with $k_1^2 = k^2 + \varepsilon$ has the following form:[3]

$$\dot{x}_1 = x_2,\ \dot{x}_2 = -k_2^2 x_1 \quad (T_1 \le t \le T). \tag{8}$$

RESULTS

This is in the general solution of the system

$$x_1 = C_1 cosk_2(t - T_1) + C_2 sink_2(t - T_1),$$
$$x_2 = -k_2 C_1 sink_2(t - T_1) + k_2 C_2 cosk_2(t - T_1) \tag{9}$$

to determine the first eigenvalues, we choose integral constants C_1 and C_2. For this, the solution of (9) must coincide with the solutions of x_{11}, x_{21} in system (6) at $t = T_1$. We have

$$cosk_1 T_1 = C_1, \quad -k_1 sink_1 T_1 = k_2 C_2.$$

We put the values of C_1 and C_2 in these equations into (9) and find the first specific solution of equation (8) in the second part of the period $T_1 \le t \le T$:

$$x_{11} = cosk_1 T_1 cosk_2(t - T_1) - \frac{k_1}{k_2} sink_1 T_1 sink_2(t - T_1),$$
$$x_{21} = -k_2 cosk_1 T_1 sink_2(t - T_1) - k_1 sink_1 T_1 cosk_2(t - T_1),$$
$$x_{12} = \frac{1}{k_1} sink_1 T_1 cosk_2(t - T_1) + \frac{1}{k_2} cosk_1 T_1 sink_2(t - T_1),$$
$$x_{22} = -\frac{k_2}{k_1} sink_1 T_1 sink_2(t - T_1) + cosk_1 T_1 cosk_2(t - T_1). \tag{10}$$

These expressions determine the elements of the fundamental matrix $X(T)$ in the second part of the period $T_1 \le t \le T$. If we put $t = T$ in (10), we get matrix elements $A = X(T)$. Considering $a_{kj} = x_{kj}(T)$, we construct the characteristic equation $\det(A - \rho E)$:

$$\begin{vmatrix} x_{11}(T) - \rho & x_{12}(T) \\ x_{21}(T) & x_{22}(T) - \rho \end{vmatrix} = 0.$$

We put the value of $x_{kj}(T)$ in (10) into this equation and, taking into account $k_1^2 = k^2 + \varepsilon,\ k_2^2 = k^2 - \varepsilon,\ T - T_1 = T_2$, directly -by correct calculations we find the following:

$$\rho^2 + a\rho + 1 = 0, \tag{11}$$

here

$$a = 2\left[\frac{1}{\sqrt{1-\mu^2}} sink_1 T_1 sink_2 T_2 - cosk_1 T_1 cosk_2 T_2\right], \tag{12}$$
$$\mu = \varepsilon / k^2 = \varepsilon / \delta. \tag{13}$$

In this example, all the coefficients of the characteristic equation were obtained by direct calculations. It follows from the general theory of the Hill equation that the free term of equation (11) is equal to one.[1] In order for the motion to be stable, it is necessary and sufficient that the inequality $|a| < 2$ is satisfied. In our example, the stability condition (simple but not asymptotic) is as follows:

$$\left|\frac{sink_1 T_1 sink_2 T_2}{\sqrt{1-\mu^2}} - cosk_1 T_1 cosk_2 T_2\right| < 1. \tag{14}$$

If all numbers $\delta, \varepsilon,\ T_1$ and T_2 are given, it is not difficult to check this condition. Without stopping at the detailed analysis of the inequality (14), we establish the conditions for the emergence of parametric resonance only at $\mu = \varepsilon/\delta \ll 1$. (14) without taking into account all the participating quantities μ higher than one and taking into account that the parametric resonance for the Hill equation already occurs in the stability region, we will have

$$|\cos(k_1T_1 + k_2T_2)| = 1.$$

From here

$$k_1T_1 + k_2T_2 = \pi n \quad (n = 1,2,3,\dots). \tag{15}$$

And now we consider the values of k_1 and k_2

$$k_1 = \sqrt{k^2+\varepsilon} = k\sqrt{1+\mu}, \;\; k_2 = \sqrt{k^2-\varepsilon} = k\sqrt{1-\mu}.$$

For sufficiently small $\mu = \varepsilon/k^2 = \varepsilon/\delta$, we have the following:

$$k_1 = k\left(1 + {}^1\!/_2\,\mu\right), \quad k_2 = k\left(1 - {}^1\!/_2\,\mu\right).$$

We substitute these values for k_1 and k_2 in (15) and find the following ($T_2 + T_1 = T$):

$$kT + {}^1\!/_2\,\mu k(T_2 - T_1) = \pi n,$$

or to the accuracy of the main limits

$$\omega = 2\frac{k}{n} \quad (n = 1,2,3,\dots) \tag{16}$$

where $\omega = 2\pi/T$ is the frequency of pulsation, $k = \sqrt{\delta}$ is the frequency of specific oscillations of the system in the absence of parametric excitations.[4]

CONCLUSION

It can be seen from the expression (16) that at a sufficiently small pulsation depth, the parametric resonance ε occurs at countless values of its frequency ω. (16) expression does not depend on T_1 and T_2 parts of the period in parametric excitation according to the quasi-rectangular sine law for critical values of the pulsation frequency, and it is critical in parametric excitation according to the simple sine (cosine) law overlaps the corresponding values of the frequency. Indeed, if the Mathieu equation is written in the following form

$$\ddot{x} + (k^2 + \varepsilon cos\omega t)x = 0,$$

where k is the frequency of natural oscillations of the system without parametric excitations, in this case, using the formula $\omega t = \tau$, passing to dimensionless time, we get the canonical form of this equation, where $\delta = k^2/\omega^2$. ε is defined by the equations $\delta = n^2/4$ or $k^2/\omega^2 = n^2/4$ with a small critical point, where $n = 1,2,3, \dots$. $\omega = 2k/n$, i.e. formula (16).

To conclude this example, we should note that the stability condition (14) holds when one or all of the numbers $\delta, \delta + \varepsilon = k_1^2$ and $\delta - \varepsilon = k_2^2$ are negative. For this, it is enough to go from trigonometric functions of abstract arguments to hyperbolic functions of real quantities.

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