MATHEMATICAL MODELING OF DEFORMATION OF A PARABOLOID OF REVOLUTION IN A MAGNETIC FIELD

[1]Indiaminov Ravshan Shukurovich, [2]Kholzhigitov Sobir Mamaraupovich

[1]Doctor of Physical and Mathematical Sciences, Professor of the Samarkand branch of the Tashkent University of Information Technologies

[2]Lecturer at the Jizzakh State Pedagogical University

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Abstract. *The development of science and new technologies poses more subtle, specific, and detailed requirements for solving technical and technological problems in various fields, such as mechanical engineering, instrument making, electrical engineering, etc. Scientific and technological progress constantly demands the formulation and solution to new, increasingly complex model problems. The process of deformation of a paraboloid of revolution under the influence of variable electromagnetic forces and mechanical loads is mathematically modeled in the article. Numerical results were obtained and electromagnetic effects were analyzed.*

Keywords: *shell, strain, stress, electromagnetic field, magnetoelasticity.*

I. Introduction. The dynamics of electrically conductive media in electric and magnetic fields is currently an extensive and well-developed science, formed in the intersection of mechanics and electrodynamics. Initially, the main direction developed was related to the study of the motion of electrically conductive solids in electric and magnetic fields. The solution to such problems was stimulated, first, by the needs in electrical engineering and electromechanics.

The needs of technology have led to problems that consider the motion of complex mechanical systems with electrically conductive and magnetized areas in electric and magnetic fields. In particular, problems of metallurgy, nuclear energy, and astronautics required consideration of problems in the motion of a conducting rigid body with cavities containing a conducting liquid in a magnetic field.

An important place in the mechanics of conjugate fields is occupied by the study of the motion of a continuous medium, taking into account electromagnetic effects. When constructing such models of the mechanics of a deformable rigid body, the influence of the electromagnetic field on the thermomechanical behavior of the body is realized through ponderomotive forces and their moments, as well as through sources of additional energy that arise when the body interacts with an external electromagnetic field. In this case, Maxwell's macroscopic equations of electrodynamics are formulated, describing the field in the external medium and in the body, taking into account field characteristics such as conduction currents, polarization, and magnetization.

Today, there are several approaches to obtaining macroscopic equations of electrodynamics of bodies capable of polarization and magnetization and determining the characteristics of the electromagnetic field in the body and the energy in it.

The most common approaches in the literature sources are the statistical model, the Lorentz model, the two-dipole model, and the Maxwell-Minkowski model.

Macroscopic fields and Maxwell's equations in the statistical model are determined by statistical averaging of electromagnetic fields and electrodynamic equations at the micro level, caused by the movement of point charge carriers (electrons, nuclei) within stable structures (atoms, molecules, ions); the relations for polarization and magnetization at the macro level are obtained as statistical averages of the magnetic and dipole moments in the body.

The motion of an elastic conductive medium in a magnetic field is described by a coupled system of equations of electrodynamics of a slowly moving medium and equations of the dynamic theory of elasticity considering ponderomotive forces [1,2,3].

II. Formulation of the problem. Models of magnetic elasticity. We will consider a truncated current-carrying paraboloid of revolution under the influence of non-stationary electromagnetic and mechanical fields. Ignoring the influence of polarization and magnetization processes, we assume that an alternating electric current is supplied to the end of the shell from an external source. It is assumed that the external electric current in an unperturbed state is uniformly distributed over the body (the current density does not depend on the coordinates).

We also assume that the electromagnetic hypotheses are satisfied regarding the electric field strength $\vec{E}$ and magnetic field strength $\vec{H}$ [1]. These assumptions are some electrodynamic analogs of the hypothesis of non-deformable normals and, together with the latter, constitute the hypotheses of magnetoelasticity of thin bodies. Accepting these hypotheses allows us to reduce the problem of the deformation of a three-dimensional body to the problem of the deformation of an arbitrarily chosen coordinate surface.

Let us consider shells of variable thickness in the meridional direction, the middle surface of which is closed in the circumferential direction. We assume that the shell is under the influence of axisymmetric non-stationary mechanical and magnetic loads. We ignore the processes of polarization and magnetization.

The middle surface of the shell in an unstrained state refers to a curvilinear orthogonal coordinate frame $\alpha = s, \beta = \theta$, where $s -$ is the length of the generatrix arc (meridian), measured from a certain fixed point, θ is the central angle in a parallel circle.

Coordinate lines $s = const$ and $\theta = const$ are the lines of the principal curvatures of the middle surface of the shell.

Counting coordinate γ along the normal to this surface, we relate the entire shell to the orthogonal spatial coordinate frame s, θ, γ.

Using the equations of magnetoelasticity of shells of revolution [2], after appropriate transformations, we obtain a resolving system of equations of one-dimensional current-carrying shells of revolution of variable rigidity in a magnetic field in the s coordinate.

$$\frac{\partial u}{\partial s} = \frac{1-\nu^2}{Eh} N_s - \frac{\nu \cos\varphi}{r} u - \left(\frac{1}{R_s} + \frac{\nu \sin\varphi}{r}\right) - \frac{1}{2}\vartheta_s^2 \ ,$$

$$\frac{\partial w}{\partial s} = -\vartheta_s + \frac{u}{R_s},$$

$$\frac{\partial \vartheta_s}{\partial s} = \frac{12(1-\nu^2)}{Eh^3} M_s - \frac{\nu \cos\varphi}{r} \vartheta_s \, ,$$

$$\frac{\partial N_s}{\partial s} = \frac{\cos\varphi}{r}[\nu - 1] N_s + Eh\left(\frac{\cos\varphi}{r} u + \frac{\sin\varphi}{r} w\right) - \frac{1}{R_s} Q_s - P_s -$$

$$-hJ_{\theta cm} B_\gamma - \sigma h \left[E_\theta B_\gamma + 0{,}5 \frac{\partial w}{\partial t} B_\gamma (B_s^+ + B_s^-) - \frac{\partial u}{\partial t} B_\gamma^2\right] + \rho h \frac{\partial^2 u}{\partial t^2},$$

$$\frac{\partial Q_s}{\partial s} = -\frac{\cos\varphi}{r} Q_s + \left(\frac{1}{R_s} + \frac{\nu \sin\varphi}{r}\right) N_s + Eh \frac{\sin\varphi}{r}\left(\frac{\cos\varphi}{r} u + \frac{\sin\varphi}{r} w\right) - P_\gamma +$$

$$+0{,}5hJ_{\theta cm}(B_s^+ + B_s^-) + \sigma h\left[0{,}5E_\theta(B_s^+ + B_s^-) + 0{,}25\frac{\partial w}{\partial t}(B_s^+ + B_s^-)^2 + \right.$$
$$\left. + \frac{1}{12}\frac{\partial w}{\partial t}(B_s^+ - B_s^-)^2 - 0{,}5\frac{\partial u}{\partial t}B_\gamma(B_s^+ + B_s^-)\right] + \rho h\frac{\partial^2 w}{\partial t^2}, \qquad (3.1)$$
$$\frac{\partial M_s}{\partial s} = \frac{\cos\varphi}{r}\left[(\nu - 1)M_s + \frac{Eh^3}{12}\frac{\cos\varphi}{r}\vartheta_s\right] + Q_s + N_s v_s -$$
$$-\frac{\sin\varphi}{r}\left(\nu M_s + \frac{Eh^3}{12}\frac{\cos\varphi}{r}\vartheta_s\right)\vartheta_s,$$
$$\frac{\partial E_\theta}{\partial s} = -\frac{\partial B_\gamma}{\partial t} - \frac{\cos\varphi}{r}E_\theta,$$
$$\frac{\partial B_\gamma}{\partial s} = -\sigma\mu\left[E_\theta + 0{,}5\frac{\partial w}{\partial t}(B_s^+ + B_s^-) - \frac{\partial u}{\partial t}B_\gamma\right] - \frac{(B_s^+ - B_s^-)}{h}.$$

Here the sought-for functions are $u, w, \vartheta_s, N_s, Q_s, M_s, E_\theta, B_\gamma$. The resulting system of eighth-order nonlinear differential equations describes mathematical models of magnetoelasticity of a truncated flexible paraboloid of revolution located in a non-stationary magnetic field.

III. Methods for solving the magnetoelasticity problem. The methods for solving the nonlinear problem of magnetoelasticity of a current-carrying paraboloid of revolution is based on the sequential use of the Newmark scheme, the quasi-linearization method, and the discrete orthogonalization method [2,3,4,5].

When studying the stressed state of flexible current-carrying shells of revolution of arbitrary cross-section, and conjugate shells of various shapes under the influence of a non-stationary magnetic field, we select the following functions as the sought-for ones:

$$u_x, u_z, \vartheta_s, N_x, N_z, M_s, E_\theta, B_\gamma, \qquad (3.2)$$

where $u_x, u_z -$ are radial and axial displacements, $\vartheta_s -$ is the normal revolution angle, $N_x, N_z -$ are radial and axial forces, $M_s -$ is the bending moment, $E_\theta -$ is the electric field strength, $B_\gamma -$ is the magnetic field induction.

These quantities are related to displacements u, w and forces N_s, Q_s as:

$$N_x = N_s\cos\varphi + Q_s\sin\varphi, \quad u_x = u\cos\varphi + w\sin\varphi,$$
$$N_z = N_s\sin\varphi - Q_s\cos\varphi, \quad u_z = u\sin\varphi - w\cos\varphi. \qquad (3.3)$$

The use of these functions makes it possible to construct an algorithm for solving problems for shells of revolution of an arbitrary cross-section when the conditions for the simplest conjugation of shells of revolution are met. In this case, the elasticity relations [2] are written as:

$$\varepsilon_{ss} = \frac{1-\nu^2}{Eh}(N_x\cos\varphi + N_z\sin\varphi) - \frac{\nu}{r}u_x, \qquad \varepsilon_{\theta\theta} = \frac{u_x}{r},$$
$$\chi_{ss} = \frac{12(1-\nu^2)}{Eh^3}M_s - \frac{\nu\cos\varphi}{r}\vartheta_s, \qquad \chi_{\theta\theta} = \frac{\cos\varphi}{r}\vartheta_s, \qquad (3.4)$$
$$N_\theta = \nu(N_x\cos\varphi + N_z\sin\varphi) + \frac{Eh}{r}u_x, \qquad M_\theta = \nu M_s + \frac{Eh^3}{12}\frac{\cos\varphi}{r}\vartheta_s.$$

Considering ((3.3), (3.4)), we obtain a resolving system of nonlinear differential equations for the magnetoelasticity of shells of revolution of an arbitrary cross-section:

$$\frac{\partial u_x}{\partial s} = \frac{1-\nu^2}{Eh}(\cos\varphi\, N_x + \sin\varphi\, N_z)\cos\varphi + \frac{\nu\cos\varphi}{r}u_x + \frac{1}{R_s}u_z - \sin\varphi\,\vartheta_s - \frac{\cos\varphi}{2}\vartheta_s^2$$
$$\frac{\partial u_z}{\partial s} = \frac{1-\nu^2}{Eh}(\cos\varphi\, N_x + \sin\varphi\, N_z)\sin\varphi + \frac{\nu\sin\varphi}{r}u_x - \frac{1}{R_s}u_x + \cos\varphi\,\vartheta_s - \frac{\sin\varphi}{2}\vartheta_s^2$$

$$\frac{\partial \vartheta_s}{\partial s} = \frac{12(1-\nu^2)}{Eh^3} M_s - \frac{\nu \cos\varphi}{r} \vartheta_s ,$$
$$\frac{\partial N_x}{\partial s} = \frac{\cos\varphi}{r}(\nu - 1)N_x + \left(\frac{1}{R_s} + \frac{\nu \sin\varphi}{r}\right) N_z + \frac{Eh}{r^2} u_x - (P_r + \rho F_r^{\wedge}) +$$
$$+\rho h \left(\frac{\partial^2 u_x}{\partial t^2}\cos\varphi + \frac{\partial^2 u_z}{\partial t^2}\sin\varphi\right),$$
$$\frac{\partial N_z}{\partial s} = -\frac{\cos\varphi}{r} N_z - \frac{1}{R_s} N_x - (P_z + \rho F_z^{\wedge}) + \rho h \left(\frac{\partial^2 u_x}{\partial t^2}\sin\varphi - \frac{\partial^2 u_z}{\partial t^2}\cos\varphi\right),$$
$$\frac{\partial M_s}{\partial s} = \frac{\cos\varphi}{r}(\nu - 1)M_s + \frac{Eh^3}{12}\frac{\cos^2\varphi}{r^2}\vartheta_s + \cos\varphi\, N_x - \sin\varphi\, N_z +$$
$$+(\cos\varphi\, N_x + \sin\varphi\, N_z)\vartheta_s - \frac{\nu \sin\varphi}{r} M_s \vartheta_s - \frac{Eh^3}{12}\frac{\cos\varphi \sin\varphi}{r}\vartheta_s^2 , \quad (3.5)$$
$$\frac{\partial E_\theta}{\partial s} = -\frac{\partial B_\gamma}{\partial t} - \frac{\cos\varphi}{r} E_\theta ,$$
$$\frac{\partial B_\gamma}{\partial s} = -\sigma\mu \left[E_\theta + 0{,}5\left(\frac{\partial u_x}{\partial t}\sin\varphi - \frac{\partial u_z}{\partial t}\cos\varphi\right)(B_s^+ + B_s^-) - \mu J_{\theta cm} -\right.$$
$$\left. -\left(\frac{\partial u_x}{\partial t}\cos\varphi + \frac{\partial u_z}{\partial t}\sin\varphi\right) B_\gamma\right] + \frac{B_s^+ - B_s^-}{h},$$

where

$$P_r = P_s \cos\varphi + P_\gamma \sin\varphi, \quad P_z = P_s \sin\varphi - P_\gamma \cos\varphi ,$$
$$F_r^{\wedge} = F_s^{\wedge}\cos\varphi + F_\gamma^{\wedge}\sin\varphi, \quad F_z^{\wedge} = F_s^{\wedge}\sin\varphi - F_\gamma^{\wedge}\cos\varphi.$$

The Lorentz force components $F_s^{\wedge}, F_\gamma^{\wedge}$ take the following form:

$$\rho F_r^{\wedge} = hJ_{\theta cm}B_\gamma + \sigma h\left[E_\theta B_\gamma + 0{,}5\left(\frac{\partial u_x}{\partial t}\sin\varphi - \frac{\partial u_z}{\partial t}\cos\varphi\right)(B_s^+ + B_s^-)B_\gamma -\right.$$
$$\left. -\left(\frac{\partial u_x}{\partial t}\cos\varphi + \frac{\partial u_z}{\partial t}\sin\varphi\right)B_\gamma^2\right]$$
$$\rho F_z = -0{,}5hJ_{\theta cm}(B_s^+ + B_s^-) - \sigma h\{0{,}5E_\theta(B_s^+ + B_s^-) + \left(\frac{\partial u_x}{\partial t}\sin\varphi - \frac{\partial u_z}{\partial t}\cos\varphi\right)$$
$$\times\left[0{,}25(B_s^+ + +B_s^-)^2 + \frac{1}{12}(B_s^+ - B_s^-)^2\right] -$$
$$-0{,}5\left(\frac{\partial u_x}{\partial t}\cos\varphi + \frac{\partial u_z}{\partial t}\sin\varphi\right)B_\gamma(B_s^+ + B_s^-)\}. \quad (3.6)$$

Here $B_s^{\pm}$ refers to the value of magnetic induction on the surfaces of the shell:

$$B_s\left(s, \pm\frac{h}{2}, t\right) = B_s^{\pm}(s,t). \quad (3.7)$$

Thus, the system of eighth-order nonlinear differential equations (3.5) describes the stress-strain state of a flexible current-carrying paraboloid of revolution of an arbitrary cross-section located in a non-stationary magnetic field.

The notations generally accepted in the theory of shells and the theory of electromagnetoelasticity are used in relations.

Let us study the behavior of a conductive shell of variable thickness in a magnetic field. Figure 1 shows the distribution of maximum values of shell stress.

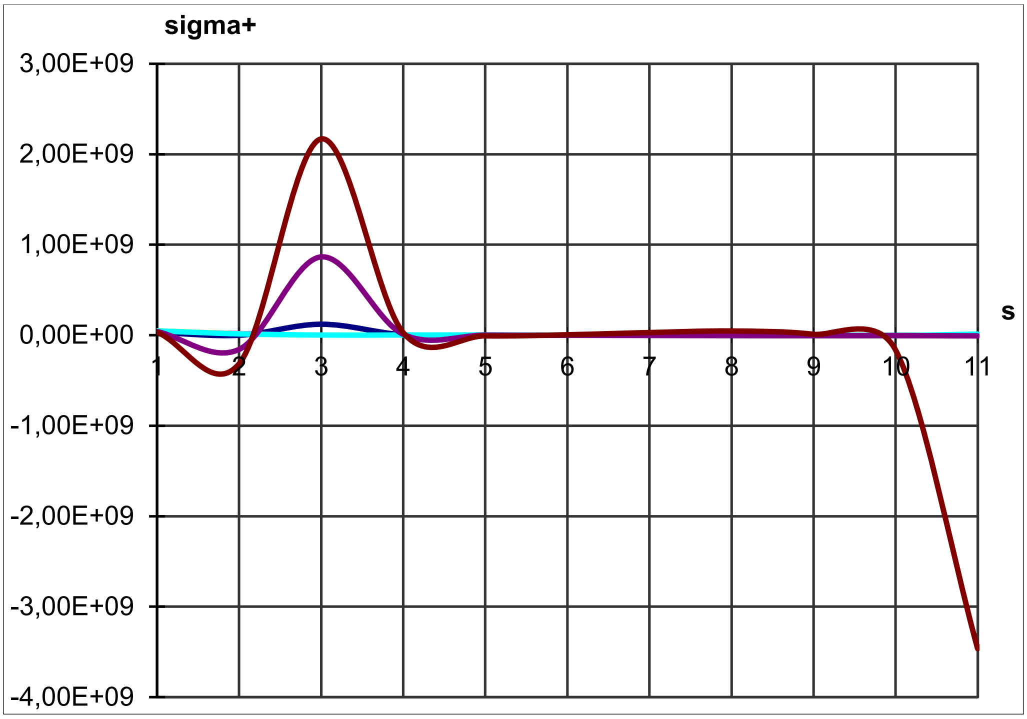

Fig.1. Stress distribution.

The presented results make it possible to evaluate the influence of external electric current and magnetic induction on the shell and their combined effect.

IV. Conclusion. The study presents an analysis of the stressed state of a truncated flexible paraboloid of revolution under the influence of a time-varying mechanical force and a time-varying external electric current. The results obtained show the influence of anisotropy of conductive properties, external electric current and external magnetic field on the stressed state of a paraboloid of revolution; the consideration of geometric nonlinearity allows us to significantly clarify the pattern of strain. Note that the proposed approach allows solving new problems of magnetoelasticity of flexible current-carrying shells of revolution, which is illustrated by the example of a paraboloid of revolution.

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