SECOND-ORDER HOMOGENEOUS WITH CONSTANT COEFFICIENTS MATHEMATICS IN TEACHING DIFFERENTIAL EQUATIONS REPLACE PACKAGES

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Abstract. *In this article, it is homogeneous of the second order with constant coefficients it is intended to solve differential equations using analytical methods of solving using the Maple mathematical package, to show this process in specific practical problems, to create an algorithm and program for solving the problem.*

Keywords: *ordinary differential equations, second-order homogeneous differential equation with constant coefficients, mathematical package, maple, dsolve, method, Maple environment.*

INTRODUCTION

One of the areas of computer application in modern education is the study of mechanical processes and mathematical models of objects with the help of calculation methods and computer software tools. Computational mathematics methods and modern capabilities of computers together help to reveal the hitherto unknown properties of mechanical processes and objects and thus improve technological processes is serving.

Today, as science and technology develops, the role of mathematics is increasing. In addition, mathematics is used in physics, mechanics and astronomy, as well as in solving economic problems, in the analysis of biological processes, and in many other areas. The mathematical model of processes in these areas is called differential equations.

Recall that a discrete random variable is given by a list of all possible values and their probabilities. This mode of transmission is not general; it cannot be used, for example, for continuous random variables.

In fact, let's look at the random variable X whose possible values (a,b) fill the interval. shows. For this purpose, the integral function of the distribution is introduced.

Let's say that x is a real number. We define the probability of an event consisting of X taking a smaller value than x, that is, the probability of the event $X<x$ by F(x). Of course, as x changes, F(x) also changes , that is, it is a function of x.

The integral function of the distribution is the function F(x) that determines the probability that the random variable X will take a value smaller than x for each value, i.e.

$$F(x)=P(X<x)$$

This equality can be interpreted from a geometric point of view; The function f(x) is the probability of a random variable taking a value represented by the point lying to the left of the point x on the number line.

Now, we can give a unique definition of a continuous random variable: if the integral function F(x) of the random variable distribution is continuously differentiable, then the random variable is called continuous.

Properties of the integral function

Property 1. The values of the integral function belong to the section $[0,1]$;

$0 \le F(x) \le 1.$

Proof 1. This property comes from defining an integral function as a probability: A probability is always a number that is not negative and is not greater than.

Property 2. F (x) is a nondecreasing function, that is, if X1>X1, then

$F(x_2) \ge F(x\ )$

METHODOLOGY

This scientific article is related to computational mathematics and the use of computers in scientific research, and is relevant from a scientific and practical point of view [4]. The article deals with the analytical and approximate solution of second-order homogeneous differential equations with constant coefficients using the Maple program. Below is a sequential algorithm for setting the problem and solving it. Calculation methods necessary for solving second-order homogeneous differential equations with constant coefficients are described.

DISCUSSION AND RESULTS

In practice, a chain of "elementary" calculations and substitutions, which can be performed with the help of an optional mathematical package, allows solving even complex problems (for example, solving simple differential equations, boundary value problems). The Maple software package allows you to find solutions to many problems in special branches of mathematics. The technology of working in the Maple environment can be found in special literature [5-6]. The Maple math package can be used in practical classes on "Differential Equations" and "Higher Mathematics", seminar classes, simple differential equation and system of equations, numerical solution of boundary value problems.

In this case, it is a homogeneous differential of the second order with constant coefficients we look at the method of solving equations [1-3].

Let us be given the following linear differential equation.

$$y'' + py' + qy = 0 \quad (1)$$

where p q is a constant number.

equation (1) is called a second-order homogeneous differential equation with constant coefficients. The characteristic equation for this equation is as follows.

$$k^2 + pk + q = 0 \quad (2)$$

The solution of differential equations with constant coefficients depends on the roots of the characteristic equation. The characteristic equation of the second-order homogeneous differential equation with constant coefficients is a square of the form (2) will be an equation.

We consider the quadratic equation in the following three cases according to its properties.

1. The discriminant of the characteristic quadratic equation is positive, i.e. D>0. Then the characteristic equation (2) has two different real roots k_1 and k_2. In this case, the solution of the second-order homogeneous differential equation with constant coefficient (1) is as follows.

$y\ (x) = c_1 e^{k1\ x} + c_2 e^{k2x}$

2. The discriminant of the characteristic quadratic equation is equal to zero, i.e. D=0. Then the characteristic equation (2) has a double root k. In this case, the solution of the homogeneous differential equation of the second constant k order with constant coefficient (1) is as follows.

$y(x)=(c_1+c_2x)e^{kx}$

3. The discriminant of the characteristic quadratic equation is negative, i.e. D<0. Then the characteristic equation (2) will not have a real root. Equation (2) has a complex root, i.e. $k_{1.2}=\alpha+i\beta$. In this case, the solution of the second-order homogeneous differential equation with constant coefficient (1) is as follows will be.

$$y(x)=e^{\alpha x}(c_1\cos\beta x+c_2\sin\beta x)$$

The three considered cases can be easily presented in the form of a table.

General solution of a second-order homogeneous differential equation with constant coefficients		
Roots of the characteristic equation	Discriminant of the characteristic equation	General solution
have two different real roots k_1 and k_2	*D>0*	$y(x)=c_1e^{k_1x}+c_2e^{k_2x}$
k has a double root	D=0	$y(x)=(c_1+c_2x)e^{kx}$
has a complex root, i.e $k_{1.2}=\alpha+i\beta$	D<0	$y(x)=e^{\alpha x}(c_1\cos\beta x+c_2\sin\beta x)$

We will look at examples of general solution of second-order homogeneous differential equations using the Maple package and graphing the solution of the Cauchy problem.

Example 1. $y''+4y'+3y=0,\ \ y(0)=1,\ \ y'(0)=1$ solve the problem.

Solution:

$$> ode_1 := y''(x)+4\cdot y'(x)+3\cdot y(x)=0:$$

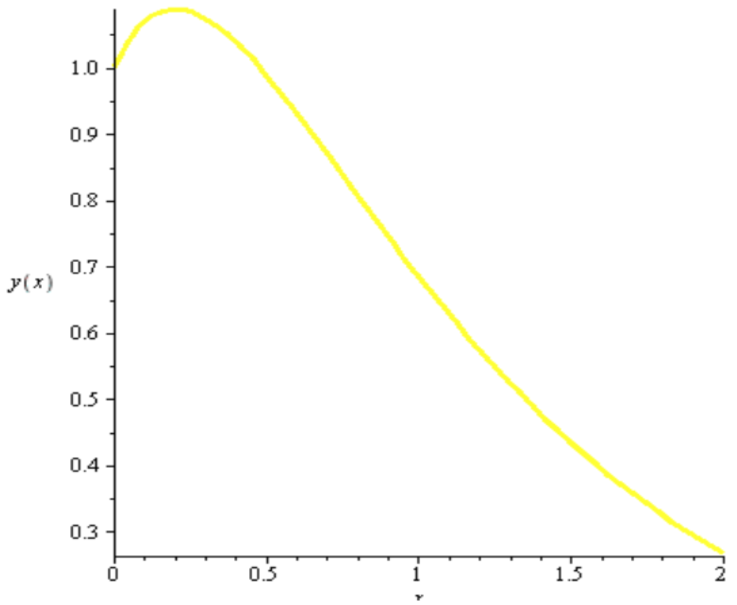
$$> sol_1 := dsolve(ode_1, useInt)$$

$$sol_1 := y(x)=\_C1\,e^{-x}+\_C2\,e^{-3x}$$

$$> DE1 := \left(\frac{d^2}{dx^2}y(x)\right)+4\left(\frac{d}{dx}y(x)\right)+3\cdot y(x)=0$$

$$> DEplot(DE1, y(x), x=0..2, [[y(0)=1, D(y)(0)=1]])$$

$$DE1 := \frac{d^2}{dx^2}y(x)+4\left(\frac{d}{dx}y(x)\right)+3\,y(x)=0$$



Example 2. $y''+4y'+5y=0,\ \ y(0)=0,\ \ y'(0)=-1$ solve the problem.
Solution:

$$> ode_3 := y''(x) + 4\cdot y'(x) + 5\cdot y(x) = 0 :$$

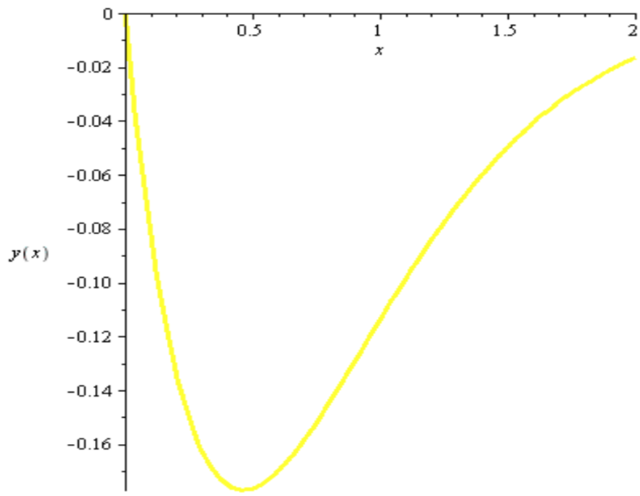
$$> sol_3 := dsolve(ode_3, useInt)$$

$$sol_3 := y(x) = \_C1\, e^{-2x} \sin(x) + \_C2\, e^{-2x} \cos(x)$$

$$> DE3 := \left(\frac{d^2}{dx^2} y(x)\right) + 4\cdot\left(\frac{d}{dx} y(x)\right) + 5\cdot y(x) = 0$$

$$> DEplot(DE3, y(x), x = 0..2, [[y(0) = 0, D(y)(0) = -1]])$$

$$DE3 := \frac{d^2}{dx^2} y(x) + 4\left(\frac{d}{dx} y(x)\right) + 5\, y(x) = 0$$



CONCLUSION

If it is necessary to solve such problems of second-order homogeneous differential equations with constant coefficients in a simple mathematical way, and to create its graph, it requires a lot of time and skills from students, researchers and teachers. As you can see from the above problem, it can be easily solved in the Maple environment and its graph can be created at the same time.

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