COMPARISON OF DIFFERENT SCHEMES FOR SOLVING EQUATIONS OF HYPERBOLIC TYPE

Toshboyev Murodjon Xudoyqul o'g'li
Termiz State University 2nd stage graduate student of Applied Mathematics
https://doi.org/10.5281/zenodo.7336365

Abstract. *Different mathematical models often lead to differential equations of hyperbolic type. Such equations rarely have perfect analytical solutions. The most common are digital methods. They are widely used in solving quasi-linear gas dynamics equations (numerical methods of gas dynamics). This article deals with the comparison of different schemes for solving equations of hyperbolic type.*

Keywords. *Equation of hyperbolic type, differential circuits, quasi-linear equation, gas dynamics, analytical solutions.*

СРАВНЕНИЕ РАЗЛИЧНЫХ СХЕМ РЕШЕНИЯ УРАВНЕНИЙ ГИПЕРБОЛИЧЕСКОГО ТИПА

Аннотация. *Различные математические модели часто приводят к дифференциальным уравнениям гиперболического типа. Такие уравнения редко имеют совершенные аналитические решения. Наиболее распространены цифровые методы. Они широко используются при решении квазилинейных уравнений газовой динамики (численные методы газовой динамики). В данной статье проводится сравнение различных схем решения уравнений гиперболического типа.*

Ключевые слова. *Уравнение гиперболического типа, дифференциальные цепи, квазилинейное уравнение, газовая динамика, аналитические решения.*

INTRODUCTION

Equations and systems of hyperbolic type form an important part of mathematical models used to solve various practical problems. Among many possible examples, we consider Euler's equations for unsteady and supersonic steady flows of compressible gas, unsteady Maxwell's equations and various combinations of them with Euler's equations, in particular the equations of magnetic gas dynamics and a number of plasma physics other mathematical models, non-stationary equations of the mechanics of deformable solids for a number of rheological models and geometries (both in multidimensional continuum models and in a number of theories of bar and shell structures), unstable one- and two-shallow dimensional equations of incompressible fluid flow in the gravity field in water, unsteady equations of incompressible fluid flow in elastically deformable pipes (in particular, in large blood vessels and in the trachea-bronchial tract describing the pulse flow of blood in air), energy transmission in electric power and radio engineering systems moisture equations, heavy traffic in megacities, intense inflation. organizational flows in telecommunications and computer networks, etc. The field of mathematical models is wider, in which the hyperbolic part of the operator is an important component of them, which mainly determines the properties of the desired solutions and the numerical methods used (for example, Navier-Stokes equations, etc.).

Since the appearance of the first numerical methods for equations of hyperbolic type, their number has reached an incalculable value and is constantly updated with new developments. On the one hand, this indicates the importance of this class of problems in various applications, but such abundance also shows that there is no universal method that meets all the requirements of users. Reviews (there are even reviews of reviews) and monographs on this issue contain references to many hundreds of original articles, which does not allow even a cursory analysis here.

MATERIALS AND METHODS

The first numerical methods of hyperbolic type equations and systems were based on the use of the characteristic properties of these equations both to approximate the equations themselves and to construct a network of differences (the first proposed direct methods of characterization). Late 19th century by Massau - (Massau, 1899). In fact, these were the first difference schemes on unstructured networks to adapt to the currently very popular and rapidly developed solution. Characteristic properties of hyperbolic equations were also actively used to prove the convergence of the numerical solution to an exact one - (Courant, Friedrich, Lewis, 1928) - the well-known Courant-Friedrich-Lewi stability condition and Courant number), including in the non-linear case (Courant, Isaacson and Rees, 1952). Similar numerical methods (for two-dimensional problems of gas dynamics - (Zhukov, 1960 (Chushkin, 1960) and others), for three-dimensional problems - (Coburn, Dolph, 1949), (Butler, 1960), (Moretty, 1963), (Katskova, Chushkin, 1965), (Zapryanov, Minostsev, 1965), (Magomedov, 1966), (Borisov, Mikhailov, 1982), etc.) only to construct continuous and often very smooth solutions was suitable and the requirements were clear. Selection of singularities (points, lines or surfaces of discontinuities in solutions) by setting appropriate boundary conditions for discontinuities significantly complicates the entire calculation algorithm, especially in the multidimensional case and when new discontinuities appear, in their detection and it is required to be included in the calculation scheme. A detailed overview of this direction can be found, for example, in a review (Chushkin, 1968).

RESULTS

A characteristic feature of hyperbolic type equations is the limited propagation speed of vibrations in the field of integration (wave processes) and the presence of characteristic manifolds - characteristic lines and surfaces (restricting the spheres of influence of dependence and solutions), their dimensions, the original of equations in partial derivatives system (the number of independent variables) can be reduced to one. Another characteristic of hyperbolic type equations and systems is the emergence of continuous solutions, in the nonlinear case, even under smooth (including analytical) boundary conditions, which is a well-known gradient catastrophe. The source of discontinuities for linear equations and systems can be continuous functions in the initial or boundary conditions, as well as inconsistency of the initial and boundary conditions. In the case of nonlinearity, these properties severely limit the possibilities of constructing analytical solutions even in the study of classical boundary value problems, therefore, the most common approach to solving such problems is currently discrete schemes.

DISCUSSION

The simplest equations of the hyperbolic type are the transport equation and the wave equation, therefore, the review of various well-formed formulas of boundary value problems and numerical algorithms for their solution is carried out primarily for these examples. Information on the theory of hyperbolic equations and systems is used (characteristic properties of equations, divergent notations of equations, continuous systems, etc.). In this article, we consider the generalization of relevant problem sets for systems of hyperbolic nonlinear (quasilinear and divergent) equations and computational algorithms for solving them. Numerical methods, including explicit and implicit conservative versions of such difference schemes on compact lattice templates, are also considered, which were used to numerically solve the posed problems.

CONCLUSION

To find a unique solution, the equation is filled with initial and boundary conditions, since the equation is second-order in time, there are two initial conditions: for the function itself and for its derivative. The Kirchhoff formula is used for the analytical solution of equations in an infinite domain, which is expressed as the d'Alembert formula in one-dimensional case and Poisson-Parceval formula in two-dimensional case. Fourier separation of variables method and its modifications can be used to solve inhomogeneous equations for an analytical solution in a finite region. For the numerical solution, the finite element method, the finite difference method, their combination (in time they are solved by finite differences, in space - by finite elements), as well as other numerical methods suitable for the task are used.

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