ON THE MOTION OF THE MAXWELL PENDULUM

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Abstract. *This paper investigates the motion of the Maxwell pendulum in a uniform gravitational field. In addition, using the Maxwell pendulum, the theoretically and experimentally found values were compared and analyzed. The threads on which the axis and the disk of the pendulum have been suspended are assumed to be weightless and inextensible, and the characteristic linear size of the disk is assumed to be small compared to the lengths of threads.*

Theoretical values of the nonlinear properties are derived using the conservation of energy and with respect to small deviations of the threads from the vertical.

Keywords: *Pendulum, the conservation of energy, equation of motion, predictions.*

О ДВИЖЕНИИ МАЯТНИКА МАКСВЕЛЛА

Аннотация. *В данной работе исследуется движение маятника Максвелла в однородном гравитационном поле. Кроме того, с помощью маятника Максвелла были сопоставлены и проанализированы теоретически и экспериментально найденные значения. Нити, на которых подвешены ось и диск маятника, предполагаются невесомыми и нерастяжимыми, а характерный линейный размер диска малым по сравнению с длинами нитей.*

Теоретические значения нелинейных свойств получены с использованием закона сохранения энергии и с учетом малых отклонений нитей от вертикали.

Ключевые слова: *маятник, закон сохранения энергии, уравнение движения, предсказания.*

INTRODUCTION

The Maxwell pendulum is a device consisting (Fig. 1) of a massive disk rigidly fixed on an axis perpendicular to the disk plane and passing through its center of gravity. Furthermore, the apparatus consists of a disk of radius R having an axis of radius r (with r << R) (Fig.2). The axis is suspended on two identical long and thin threads. The threads are considered to be weightless and inextensible; in the equilibrium position of the disk, the axis occupies a horizontal position. Maxwell pendulum is used to illustrate the properties of the plane motion of a rigid body in a uniform gravitational field, in particular, to demonstrate the transition of the kinetic energy of a rotating body into its potential energy and vice versa.

Fig.1.The Maxwell's Pendulum

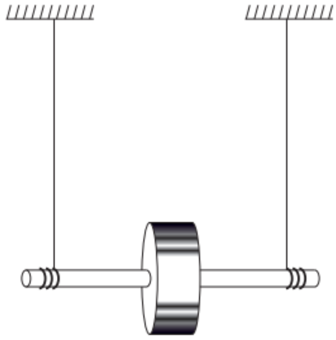
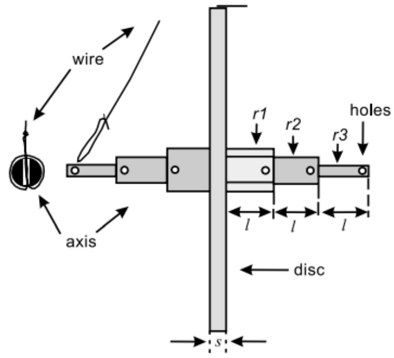


Fig.2



MATERIALS AND METHODS

The mechanical energy of a system is the result of the sum of kinetic and potential energies. When there are only conservative forces, the principle stating that "during transformation, partial energies are transformed, whereas mechanical energy is preserved" is valid. Winding the wires around the axis will load the flywheel to reach a certain height from the reference plane. When released, the flywheel starts to go down, gathering speed. When it arrives at the lowest point allowed by unwinding wires, the pendulum will rewind in the opposite direction and starts going upwards again. In ideal conditions, it would come back to the same starting height; however, motion is damped by friction with wires and with the medium (air), and after a certain number of oscillations, the pendulum will stop at the lowest point allowed by wires. The principle of energy conservation is used to determine the pendulum's period, that is, the time the flywheel spends going down and up: the kinetic energy of both translation and rotation will compensate for the variations of potential energy. All the energy is potential at the max height, whereas all the energy available at the lowest point is kinetic. That could continue ad infinitum in an ideal system, but the wheel will stop at a certain point because of friction. The system must be equipped with two sensors (a force sensor and a distance sensor) that are interfaced to the datalogger and PC to study system kinematics and dynamics. The pendulum hooks onto the force sensor via a pair of parallel wires tied to the pendulum ends on the central axis. The position sensor is arranged at the system's base, enabling the assessment of the speed with which the wheel arrives at the end of its travel using the sonar technique. The system used in this equipment is characterized by a high frequency of data acquisition and by versatile data processing software enabling to study of the up and down movement of the pendulum in a practical way. When conducting an experiment, the thread is neat, and a coil to the coil is wound on the axis, due to which the disk rises to a certain height. If the disk is released, then under the influence of gravity, it will begin to fall down, rotating around the axis. In this case, the threads will be stretched, and the axis will be horizontal. This movement continues until the threads are entirely unwound. After this moment, the disk rises up while the threads remain taut and are wound on the axis. If threads are inextensible and the resistance of the environment is negligible, then the disk will rise to its original height, and the process is repeated. The frequency of this process gives reason to call the device in question a pendulum. In the described movement, the angle the threads make with a fixed vertical plane (passing through the points of suspension) equals zero. The article's primary purpose is to study the dynamics of the pendulum's motion and compare it with experimental results.

1. The relation between the speed and position

Let be the pendulum's mass (equal to the sum of the masses of the disk and the axis on which it is fixed). We assume that the axis is a cylinder of radius r, and the center of mass of the pendulum coincides with the center of mass of the disk. We denote the moment of inertia of the pendulum about the axis perpendicular to the plane of the disk and passing through the center of mass.

RESULTS

We based the model on energy conservation. Let the starting point be a height of h- the wounded state of the pendulum. In this, $U(x)$ −referred to the potential energy of a system ($U(x)$=0, when $x = 0$). The total energy of the system $E = mgh$ (when $x = h, v = 0$) turns into translational kinetic energy $E_t = \frac{1}{2}mv^2$ and rotational kinetic energy $E_r = \frac{1}{2}I\omega^2$ during the fall. Assuming minimal energy loss, we can

$$E = E_p + E_t + E_r \quad (1), \text{ or}$$

$$E = mgh + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \; (2)$$

This energy balance explains why the linear acceleration is less than g. To test the model, we should write the acceleration as a function of directly measurable quantities. To do this, we exploit the fact that the translational and rotational motions are not independent.

Using an upward vertical axis with the origin in the equilibrium position, the rotation angle α(t) is related to the displacement $x(t)$ by: $x(t) = r\alpha(t)$ (where r is the axis radius), while the angular velocity is related to the linear velocity by:

$$v(t) = r\omega(t) \quad (3)$$

Using these equations, we can write all the quantities as a position and velocity function. Thus, the equation for the conservation of energy becomes:

$$mg(h - x) = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \quad (4)$$

, where $\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$ is equal to $\frac{1}{2}m(1 + \frac{I}{mr^2})$. We can also name $(1 + \frac{I}{mr^2})$ as a constant k.

Therefore, the energy balance is

$$mg(h - x) = \frac{1}{2}(mk)v^2 \; (5)$$

Solving with respect to velocity, we get

$$v(t) = \sqrt{\frac{2g}{k}(h - x)} \quad (6)$$

From this we can get that the motion of the disk is very much the same as the free-falling object.

The acceleration of the disk is

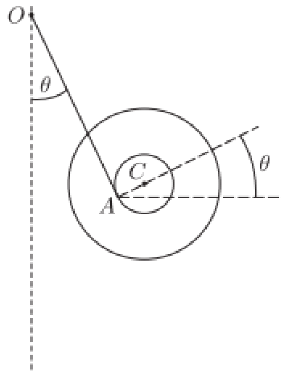
$$a = \frac{g}{k} = \frac{g}{1 + I/mr^2} \quad (7)$$

Note that for r<<R, the moment of inertia of the disc is approximately $I \approx mR^2/2$

2. Equation of motion

We assume that the axis is a cylinder of radius r, and the center of mass C of the pendulum coincides with the center of mass of the disk. Let l be the length of each of the threads in their unwound state. D is the distance from the suspension point O of any of the threads to point A of its descent from the axis of the pendulum (Fig.3). Two generalized coordinates set the position of the pendulum: the angle θ between the direction of the threads and the vertical plane passing through the points of suspension of the threads, and the value.

Fig.3.Deriving the equation of motion



Because the thread is inextensible, we have the following non-retaining bond

$$\xi \geq 0 \quad (2.1)$$

The vectors v_A and v_B of the velocities in points A and C are related by the following equation

$$v_C = v_A + \omega \times AC \quad (2.2)$$

, where ω is the vector of the instantaneous angular velocity of the disk, and this vector has a horizontal direction. Since the threads are inextensible, the vector v_A is collinear to the vector AC, and for the modulus of the vectors ω and v_A, we have the following

$$\omega = \left|\dot{\theta} - \frac{\dot{d}}{r}\right|, \quad v_A = d|\dot{\theta}| \quad (2.3)$$

The dots refer to the differentiation with respect to time.

From Koenig's theorem, taking equations (2.2) and (2.3) into account, we obtain an expression for the kinetic energy of the pendulum,

$$E_k = \frac{1}{2}m(l-\xi)^2\dot{\theta}^2 + \frac{1}{2}(I_c + mr^2)\left(\dot{\theta} + \frac{\dot{\xi}}{r}\right)^2. \quad (2.4)$$

The Potential energy of a pendulum

$$E_p = mg[\xi cos\theta + rsin\theta + l(1-cos\theta], \quad (2.5)$$

g is the acceleration of gravity

For $\xi > 0$, the motion of the pendulum corresponds to the Langrage function $L = E_k - E_p$.

For $\xi = 0$, the motion of pendulum has the character of an absolute elastic impact. Therefore, we have

$$\dot{\xi}^+ = -\dot{\xi}^-. \quad (2.6)$$

Using (2.6), we can find that $\tau = 2\sqrt{2\alpha kl/g}$, where α and k are the constant numbers referring to the followings

$$\alpha = \frac{h}{l} \ (0 < \alpha < 1), \quad k = \left(1 + \frac{I_C}{mr^2}\right) \ (k > 1). \quad (2.7)$$

The following equation corresponds to the unperturbed periodic motion of the pendulum. At $0 \leq \tau < 1$,

$$\theta = 0, \quad \xi = -\frac{g}{2k}t^2 + \sqrt{2\frac{\alpha gl}{k}}t \quad (2.8)$$

DISCUSSION

The initial moment of time t=0 is taken as the moment when $\xi = 0$, and as we found in the previous section, the center of the disk starts its movement with $\sqrt{2\frac{\alpha g l}{k}}$ velocity. At $t = \tau/2$, the pendulum reaches the maximum height of h. *

*[The same results were derived in this way in the paper “On the stability of motion of the Maxwell pendulum’’ by A.P. Markeev (eq. 2.8)]

3. Checking predictions

The moment of inertia of a homogenous cylinder, as we mentioned, above is $I = mr^2/2$.

Our disk is made of 2 cylinders. However, R is much bigger than r, which makes it possible to approximately calculate the moment of inertia with the following equation

$$I = mR^2 = \pi\rho R^4/2$$

, where R=49mm. This makes the value of k to be $\approx (R/r)^2/2$.

CONCLUSION

In figure 4, the predicted values were $k \approx 270$ and $g/k = 0.036\ m/s^2$ .The experimental value $g/k = 0.046\ m/s^2$. In performing the experiment, we neglected some important features, such as the assumption of zero thickness of the string and the effective value of the radius. Therefore those results agree very well with the experimental results.

From the experiments, we can see that the motion of the pendulum is similar to that of a free-falling object, but with different acceleration.

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