CHIZIQLI ALGEBRAIK TENGLAMALAR SISTEMALARIGA OID MASALALAR

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https://doi.org/10.5281/zenodo.6636435

Annotatsiya. *Mazkur maqolada chiziqli algebraik tenglamalar sistemalariga oid masalalar keltirilgan bo'lib, undagi masalalar 3 xil: Gauss, Kramer va Matritsa usullari bilan yechib ko'rsatilgan. Quyida keltirib o'tilgan 1-masalada qurilish ishlaridagi beton quyuvchilar va chilangarlar sonini yuqorida aytib o'tilgan usullar yordamida hisoblab ko'rsatilgan. 2-masalada esa dizayner ishchilarning buyurtmani necha kunda bajarishligi masalasiga javob topishimiz mumkin.*

Kalit so'zlar: *chiziqli algebraik tenglamalar, chiziqli algebraik tenglamalar sistemasi, beton quyuvchilar, chilangarlar va dizayner.*

ЗАДАЧИ СИСТЕМ ЛИНЕЙНЫХ АЛГЕБРАИЧЕСКИХ УРАВНЕНИЙ I

Аннотация. *В данной работе представлены задачи, связанные с системами линейных алгебраических уравнений, которые решаются 3 типами: методом Гаусса, Крамера и матриц. В задаче 1, которая приводится ниже, количество бетонщиков и сантехников на строительных работах рассчитывается с использованием методов, описанных выше. В вопросе 2 мы можем найти ответ на вопрос, за сколько дней рабочие-конструкторы выполнят заказ.*

Ключевые слова: *линейные алгебраические уравнения, система линейных алгебраических уравнений, бетонщики, сантехники и проектировщики.*

PROBLEMS OF SYSTEMS OF LINEAR ALGEBRAIC EQUATIONS I

Abstract. *This paper presents problems related to systems of linear algebraic equations, which are solved by 3 types: Gaussian, Kramer and Matrix methods. In Problem 1, which is given below, the number of concrete pourers and plumbers in the construction work is calculated using the methods described above. In issue 2, we can find the answer to the question of how many days the designer workers will complete the order.*

Keywords: *linear algebraic equations, system of linear algebraic equations, concrete pourers, plumbers and designer.*

KIRISH

1-masala. Ma'lumki, Covid-19 nomi bilan “mashhur” bo‘lgan tojdor virus butun dunyo pandemiyasini keltirib chiqardi. O‘zbekistonda ham 2020 yilning 15 martida tojdor virus aniqlangan birinchi holat qayd etilishi bilanoq unga oid bir qator qat'iy choralar belgilandi. Jumladan, 2020 yil 16 martdan barcha davlat, nodavlat va xususiy ta'lim muassasalarida, jumladan bog‘cha, maktab, kollej, litsey va OTMlarda muddatdan oldin ta'til e'lon qilindi va o‘qishlar to‘xtatildi; 2020 yil 23 martdan boshlab barcha korxona va muassasalarga asosiy ish faoliyatiga ta'sir etmagan holda ishchi va xodimlarni qonunchilikda belgilangan tartibda mehnat ta'tiliga chiqarildi, qolgan ishchilarning faoliyatini maksimal darajada masofadan turib (onlayn yoki distansion tarzda) tashkil etildi.

TADQIQOT MATERIALLARI VA METODOLOGIYASI

Shunday sharoitda 2020 yil 1 may kuni Sardoba suv omborida to‘g‘on yorilishi natijasida halokat ro‘y berdi. Jabr ko‘rganlar uchun ko‘p qavatli uylar quriladigan bo‘ldi. Dastlab qurilayotgan uylarda ishlayotgan beton quyuvchi va chilangar (svarkachi)lar jami soni 450 nafar edi. Karantin sharoitlari yumshagani hamda aholini tezroq uy-joy bilan ta’minlash maqsadida beton quyuvchilar soni 2 barobar, chilangarlar soni esa 5 barobar oshirildi va ularning umumiy soni 1170 nafar bo‘ldi. Qurilish ishlariga necha nafar beton quyuvchilar va necha nafar chilangarlar jalb etilganligini aniqlang.

Yechish. Dastlabki:

beton quyuvchilar soni x;

chilangarlar soni y

bo‘lsin. Keyinchalik esa:

beton quyuvchilar soni $2x$;

chilangarlar soni $5y$

bo‘lgan. Masala shartlariga ko‘ra:

dastlab $x + y = 450,$

keyin $2x + 5y = 1170$

munosabatlarga egamiz.

Masalani yechish ushbu

$$\begin{cases} x + y = 450, \\ 2x + 5y = 1170 \end{cases}$$

chiziqli tenglamalar sistemasini yechishga olib keladi.

Bu tenglamalar sistemasini yechishning bir necha usulini keltiramiz.

1-usul (Gauss usuli). Bu tenglamalar sistemasining asosiy matritsasi

$$A = \begin{pmatrix} 1 & 1 \\ 2 & 5 \end{pmatrix},$$

kengaytirilgan matritsasi

$$A_{keng} = \begin{pmatrix} 1 & 1 & 450 \\ 2 & 5 & 1170 \end{pmatrix}.$$

A matritsaning rangi 2 ga teng, chunki undagi satrlar ($(1 \quad 1)$ va $(2 \quad 5)$) hamda ustunlar $\left(\begin{pmatrix} 1 \\ 2 \end{pmatrix} va \begin{pmatrix} 1 \\ 5 \end{pmatrix}\right)$ chiziqli erkli. Ya’ni $rank\ A = rank\ A_{keng} = 2$. Shuning uchun Kroneker-Kapelli teoremasiga ko‘ra bu sistema yagona yechimga ega. Shu yechimni topish uchun kengaytirilgan matritsa ustida elementar almashtirishlar bajaramiz:

$$\begin{pmatrix}1 & 1 & 450\\ 2 & 5 & 1170\end{pmatrix}\Big| \begin{matrix}\times 2\\ -\end{matrix} \sim \begin{pmatrix}1 & 1 & 450\\ 0 & 3 & 270\end{pmatrix}.$$

Demak, berilgan sistema ushbu

$$\begin{cases} x + y = 450,\\ 3y = 270\end{cases}$$

sistemaga teng kuchli (Gauss usulining asosi shunda). Oxirgi sistemaning ikkinchi tenglamasidan $y = 90$ bo‘lib, y ning bu qiymati sistemadagi birinchi tenglamaga qo‘yilsa, $x = 360$ bo‘ladi.

Javob: Qurilish ishlarida
Dastlab 360 nafar beton quyuvchi, 90 nafar chilangar,
keyin $2 \times x = 720$ nafar beton quyuvchi, $5 \times y = 450$ nafar chilangar ishlagan.

TADQIQOT NATIJALARI

2-usul (Kramer usuli). Hosil qilingan chiziqli tenglamalar sistemasida tenglamalar soni (2 ta) bilan o‘zgaruvchilar soni (2 ta) teng bo‘lgani uchun determinatlar usulida foydalanish mumkin.

$$\Delta = \begin{vmatrix}1 & 1\\ 2 & 5\end{vmatrix} = 5 - 2 = 3,$$

$$\Delta_1 = \Delta_x = \begin{vmatrix}450 & 1\\ 1170 & 5\end{vmatrix} = 2250 - 1170 = 1080,$$

$$\Delta_2 = \Delta_y = \begin{vmatrix}1 & 450\\ 2 & 1170\end{vmatrix} = 1170 - 900 = 270.$$

Bundan Kramer formulalariga ko‘ra

$$x = \frac{\Delta_x}{\Delta} = \frac{1080}{3} = 360,$$

$$y = \frac{\Delta_y}{\Delta} = \frac{270}{3} = 90.$$

Javob: Qurilish ishlarida
Dastlab 360 nafar beton quyuvchi, 90 nafar chilangar,
keyin 720 nafar beton quyuvchi, 450 nafar chilangar ishlagan.

3-usul (Matritsalar usuli). Masala shartlaridan hosil qilingan tenglamalar sistemasi uchun ushbularni yozib olamiz:

$A = \begin{pmatrix}1 & 1\\ 2 & 5\end{pmatrix}$ – tenglamalar sistemasining asosiy matritsasi,

$X = \begin{pmatrix} x \\ y \end{pmatrix}$ – tenglamalar sistemasining o‘zgaruvchilari (ustun-)matritsasi,

$B = \begin{pmatrix} 450 \\ 1170 \end{pmatrix}$ – tenglamalar sistemasining ozod hadlari (ustun-) matritsasi.

U holda qaralayotgan tenglamalar sistemasini

$$\begin{pmatrix} 1 & 1 \\ 2 & 5 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 450 \\ 1170 \end{pmatrix}$$

matritsa ko‘rinishida. yoki qisqalik uchun

$$AX = B$$

ko‘rinishda yozish mumkin. Agar A matritsaning A^{-1} teskarisi mavjud bo‘lsa, $AX = B$ tenglamaning echimi

$$X = A^{-1} \cdot B$$

kabi topiladi (matritsalar usulining mag‘zi mana shunda).

A^{-1} mavjud bo‘lish sharti $\det A \neq 0$ hisoblanadi:

$$\det A = \begin{vmatrix} 1 & 1 \\ 2 & 5 \end{vmatrix} = 5 - 2 = 3 \neq 0.$$

Teskari matritsaning o‘zi esa

$$A^{-1} = \frac{1}{\det A} \cdot \begin{pmatrix} A_{11} & A_{21} \\ A_{12} & A_{22} \end{pmatrix}$$

formuladan topiladi. Bu erda A_{11} ifoda a_{11} elementning algebraik to‘ldirmasi, A_{12} ifoda a_{12} elementning algebraik to‘ldirmasi, A_{21} ifoda a_{21} elementning algebraik to‘ldirmasi, A_{22} ifoda a_{22} elementning algebraik to‘ldirmasi. Qaralayotgan misolda,

$$A_{11} = a_{22} = 5, \quad A_{12} = -a_{21} = -2, \quad A_{21} = -a_{12} = -1, \quad A_{22} = a_{11} = 1$$

bo‘lgani uchun

$$A^{-1} = \frac{1}{3} \cdot \begin{pmatrix} 5 & -1 \\ -2 & 1 \end{pmatrix} = \begin{pmatrix} \frac{5}{3} & -\frac{1}{3} \\ -\frac{2}{3} & \frac{1}{3} \end{pmatrix}.$$

Demak,

$$X = \begin{pmatrix} \frac{5}{3} & -\frac{1}{3} \\ -\frac{2}{3} & \frac{1}{3} \end{pmatrix} \cdot \begin{pmatrix} 450 \\ 1170 \end{pmatrix} = \begin{pmatrix} \frac{5}{3} \cdot 450 - \frac{1}{3} \cdot 1170 \\ -\frac{2}{3} \cdot 450 + \frac{1}{3} \cdot 1170 \end{pmatrix} = \begin{pmatrix} 750 - 390 \\ -300 + 390 \end{pmatrix} = \begin{pmatrix} 360 \\ 90 \end{pmatrix},$$

ya'ni $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 360 \\ 90 \end{pmatrix}$ bo'lib, bu erdan $x = 360$ va $y = 90$ kelib chiqadi.

Javob: Qurilish ishlarida
Dastlab 360 nafar beton quyuvchi, 90 nafar chilangar,
keyinchalik 720 nafar beton quyuvchi, 450 nafar chilangar ishlagan.

MUHOKAMA

2-masala. Dizayner bitta xonaning ichki dizayni uchun 2 kun, bitta uyning tashqi dizayni uchun 3 kun, bitta hovli landshaft dizayni uchun esa 5 kun sarflaydi. Agar xonalarning ichki dizayni bo'yicha buyurtmalar uyning tashqi dizayni bo'yicha buyurtmalardan 3 taga ko'p, hovli landshaft dizayni bo'yicha buyurtmalardan 4 taga ortiq bo'lib, jami buyurtmalarni bajarishga 21 kun sarflangan bo'lsa, dizayner nechta xonaning ichki dizayni, nechta uyning tashqi dizayni va nechta hovlining landshaft dizayni bo'yicha buyurtma bajargan?

Yechish.

ichki dizayn qilinadigan xonalar soni x,

tashqi dizayn qilinadigan uylar soni y,

landshaft dizayni qilinadigan hovlilar soni z

bo'lsin. Masala shartlariga ko'ra

$x = y + 3,$

$x = z + 4,$

$2x + 3y + 5z = 21$

bo'ladi. Demak, masala ushbu tenglamalar sistemasini echishga olib kelinadi:

$$\begin{cases} x - y = 3, \\ x - z = 4, \\ 2x + 3y + 5z = 21. \end{cases}$$

Hosil qilingan bu sistemaning asosiy matritsasi

$$A = \begin{pmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 2 & 3 & 5 \end{pmatrix},$$

kengaytirilgan matritsasi esa

$$A_{keng} = \begin{pmatrix} 1 & -1 & 0 & 3 \\ 1 & 0 & -1 & 4 \\ 2 & 3 & 5 & 21 \end{pmatrix}$$

bo'ladi. Elementar almashtirishlardan so'ng

$$A = \begin{pmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 2 & 3 & 5 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 5 & 5 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 2 \end{pmatrix},$$

A matritsaning rangi 3 ga tengligini ko'ramiz: $rang\ A = 3$. U holda kengaytirilgan matritsaning rangi ham 3 ga teng bo'ladi: $rang\ A_{keng} = 3$. Kroneker-Kapelli teoremasidan bu sistemaning yechimga egaligi kelib chiqadi.

1-usul (Gauss usuli).

$$A_{keng}=\begin{pmatrix}1&-1&0&3\\1&0&-1&4\\2&3&5&21\end{pmatrix}\sim\begin{pmatrix}1&-1&0&3\\0&1&-1&1\\0&5&5&15\end{pmatrix}\sim\begin{pmatrix}1&-1&0&3\\0&1&-1&1\\0&1&1&3\end{pmatrix}\sim$$
$$\sim\begin{pmatrix}1&-1&0&3\\0&1&-1&1\\0&0&2&2\end{pmatrix}\sim\begin{pmatrix}1&-1&0&3\\0&1&-1&1\\0&0&1&1\end{pmatrix}.$$

Bundan

$$\begin{cases}x-y=3,\\x-z=4,\\2x+3y+5z=21\end{cases}\Leftrightarrow\begin{cases}x-y=3,\\y-z=1,\\z=1\end{cases}\Rightarrow\begin{cases}x=5\\y=2\\z=1\end{cases}$$

Javob: Buyurtmalar quyidagicha bo‘lgan: 5 ta xonaning ichki dizayni, 2 ta uyning tashqi dizayni, 1 ta hovlining landshaft dizayni qilingan.

2-usul (Kramer usuli).

$$\Delta=\begin{vmatrix}1&-1&0\\1&0&-1\\2&3&5\end{vmatrix}=0+2+0-0+5+3=10,$$

$$\Delta_x=\begin{vmatrix}3&-1&0\\4&0&-1\\21&3&5\end{vmatrix}=0+21+0-0+20+9=50,$$

$$\Delta_y=\begin{vmatrix}1&3&0\\1&4&-1\\2&21&5\end{vmatrix}=20-6+0-0-15+21=20,$$

$$\Delta_z=\begin{vmatrix}1&-1&3\\1&0&4\\2&3&21\end{vmatrix}=0-8+9-0-12+21=10.$$

$$x=\frac{\Delta_x}{\Delta}=1,\quad y=\frac{\Delta_y}{\Delta}=5,\quad z=\frac{\Delta_z}{\Delta}=1.$$

Javob: $x=1, y=5, z=1$, ya’ni buyurtmalar quyidagicha bo‘lgan: 5 ta xonaning ichki dizayni, 2 ta uyning tashqi dizayni, 1 ta hovlining landshaft dizayni qilingan.

XULOSA

3-usul (Matritsalar usuli). Masala shartlaridan hosil qilingan tenglamalar sistemasi uchun ushbularni yozib olamiz:

$A=\begin{pmatrix}1&-1&0\\1&0&-1\\2&3&5\end{pmatrix}$ – tenglamalar sistemasining asosiy matritsasi,

$X=\begin{pmatrix}x\\y\\z\end{pmatrix}$ – tenglamalar sistemasining o‘zgaruvchilari (ustun-)matritsasi,

$B=\begin{pmatrix}3\\4\\21\end{pmatrix}$ – tenglamalar sistemasining ozod hadlari (ustun-) matritsasi.

U holda qaralayotgan tenglamalar sistemasini

$$\begin{pmatrix}1 & -1 & 0\\ 1 & 0 & -1\\ 2 & 3 & 5\end{pmatrix}\cdot\begin{pmatrix}x\\ y\\ z\end{pmatrix}=\begin{pmatrix}3\\ 4\\ 1\end{pmatrix}$$

matritsa ko‘rinishida. yoki qisqalik uchun

$AX = B$

ko‘rinishda yozish mumkin. Agar A matritsaning A^{-1} teskarisi mavjud bo‘lsa, $AX = B$ tenglamaning echimi

$X = A^{-1} \cdot B$

kabi topiladi (matritsalar usulining mag‘zi mana shunda).

A^{-1} mavjud bo‘lish sharti $\det A \neq 0$ hisoblanadi:

$$\det A = \begin{vmatrix}1 & -1 & 0\\ 1 & 0 & -1\\ 2 & 3 & 5\end{vmatrix} = 0 + 2 + 0 - 0 + 5 + 3 = 10 \neq 0.$$

Teskari matritsaning o‘zi esa

$$A^{-1} = \frac{1}{\det A}\cdot\begin{pmatrix}A_{11} & A_{21} & A_{31}\\ A_{12} & A_{22} & A_{32}\\ A_{13} & A_{23} & A_{33}\end{pmatrix}$$

formuladan topiladi. Bu erda A_{ij} ifoda a_{ij} elementning algebraik to‘ldirmasi, $i,\ j = 1,\ 2,\ 3$. Mazkur algebraik to‘ldirmalarni hisoblaymiz:

$$A_{11} = (-1)^{1+1}\begin{vmatrix}a_{22} & a_{23}\\ a_{32} & a_{33}\end{vmatrix} = \begin{vmatrix}0 & -1\\ 3 & 5\end{vmatrix} = 3,$$

$$A_{12} = (-1)^{1+2}\begin{vmatrix}a_{21} & a_{23}\\ a_{31} & a_{33}\end{vmatrix} = -\begin{vmatrix}1 & -1\\ 2 & 5\end{vmatrix} = -7,$$

$$A_{13} = (-1)^{1+3}\begin{vmatrix}a_{21} & a_{22}\\ a_{31} & a_{32}\end{vmatrix} = \begin{vmatrix}1 & 0\\ 2 & 3\end{vmatrix} = 3,$$

$$A_{21} = (-1)^{2+1}\begin{vmatrix}a_{12} & a_{13}\\ a_{32} & a_{33}\end{vmatrix} = -\begin{vmatrix}-1 & 0\\ 3 & 5\end{vmatrix} = 5,$$

$$A_{22} = (-1)^{2+2}\begin{vmatrix}a_{11} & a_{13}\\ a_{31} & a_{33}\end{vmatrix} = \begin{vmatrix}1 & 0\\ 2 & 5\end{vmatrix} = 5,$$

$$A_{23} = (-1)^{2+3}\begin{vmatrix}a_{11} & a_{12}\\ a_{31} & a_{32}\end{vmatrix} = -\begin{vmatrix}1 & -1\\ 2 & 3\end{vmatrix} = -5,$$

$$A_{31} = (-1)^{3+1}\begin{vmatrix}a_{12} & a_{13}\\ a_{22} & a_{23}\end{vmatrix} = \begin{vmatrix}-1 & 0\\ 0 & -1\end{vmatrix} = 1,$$

$$A_{32} = (-1)^{3+2}\begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix} = -\begin{vmatrix} 1 & 0 \\ 1 & -1 \end{vmatrix} = 1,$$

$$A_{33} = (-1)^{3+3}\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = \begin{vmatrix} 1 & -1 \\ 1 & 0 \end{vmatrix} = 1.$$

Topilgan bu qiymatlarni formulaga qo‘yib, ushbu matritsani hosil qilamiz

$$A^{-1} = \frac{1}{10}\cdot\begin{pmatrix} 3 & 5 & 1 \\ -7 & 5 & 1 \\ 3 & -5 & 1 \end{pmatrix} = \begin{pmatrix} \frac{3}{10} & \frac{1}{2} & \frac{1}{10} \\ -\frac{7}{10} & \frac{1}{2} & \frac{1}{10} \\ \frac{3}{10} & -\frac{1}{2} & \frac{1}{10} \end{pmatrix}.$$

Hisoblab topilgan bu A^{-1} matritsa A matritsaga teskari bo‘ladi (Haqiqatan ham,

$$A\cdot A^{-1} = \begin{pmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 2 & 3 & 5 \end{pmatrix}\cdot\begin{pmatrix} \frac{3}{10} & \frac{1}{2} & \frac{1}{10} \\ -\frac{7}{10} & \frac{1}{2} & \frac{1}{10} \\ \frac{3}{10} & -\frac{1}{2} & \frac{1}{10} \end{pmatrix} =$$

$$= \begin{pmatrix} 1\cdot\frac{3}{10}+(-1)\cdot\left(-\frac{7}{10}\right)+0\cdot\frac{3}{10} & 1\cdot\frac{1}{2}+(-1)\cdot\frac{1}{2}+0\cdot\left(-\frac{1}{2}\right) & 1\cdot\frac{1}{10}+(-1)\cdot\frac{1}{10}+0\cdot\frac{1}{10} \\ 1\cdot\frac{3}{10}+0\cdot\left(-\frac{7}{10}\right)+(-1)\cdot\frac{3}{10} & 1\cdot\frac{1}{2}+0\cdot\frac{1}{2}+(-1)\cdot\left(-\frac{1}{2}\right) & 1\cdot\frac{1}{10}+0\cdot\frac{1}{10}+(-1)\cdot\frac{1}{10} \\ 2\cdot\frac{3}{10}+3\cdot\left(-\frac{7}{10}\right)+5\cdot\frac{3}{10} & 2\cdot\frac{1}{2}+3\cdot\frac{1}{2}+5\cdot\left(-\frac{1}{2}\right) & 2\cdot\frac{1}{10}+3\cdot\frac{1}{10}+5\cdot\frac{1}{10} \end{pmatrix}$$

$$=$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = E,$$

ya’ni $A\cdot A^{-1} = E$. Xuddi shuningdek, $A^{-1}\cdot A = E$ ekanligini ham ko‘rsatish mumkin).

Demak,

$$X = A^{-1}\cdot B = \begin{pmatrix} \frac{3}{10} & \frac{1}{2} & \frac{1}{10} \\ -\frac{7}{10} & \frac{1}{2} & \frac{1}{10} \\ \frac{3}{10} & -\frac{1}{2} & \frac{1}{10} \end{pmatrix}\cdot\begin{pmatrix} 3 \\ 4 \\ 21 \end{pmatrix} = \begin{pmatrix} \frac{3}{10}\cdot 3+\frac{1}{2}\cdot 4+\frac{1}{10}\cdot 21 \\ -\frac{7}{10}\cdot 3+\frac{1}{2}\cdot 4+\frac{1}{10}\cdot 21 \\ \frac{3}{10}\cdot 3-\frac{1}{2}\cdot 4+\frac{1}{10}\cdot 21 \end{pmatrix} = \begin{pmatrix} 5 \\ 2 \\ 1 \end{pmatrix},$$

ya'ni $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5 \\ 2 \\ 1 \end{pmatrix}$ bo'lib, bu yerdan $x = 5$, $y = 2$ va $z = 1$ kelib chiqadi.

Javob: $x = 1, y = 5, z = 1$, ya'ni buyurtmalar quyidagicha bo'lgan: 5 ta xonaning ichki dizayni, 2 ta uyning tashqi dizayni, 1 ta hovlining landshaft dizayni qilingan.

Foydalanilgan adabiyotlar

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