ANIQ INTEGRALNING BA'ZI BIR TATBIQLARI.

Qurbonov Sardor Shavkatovich
Qashqadaryo viloyati Qamashi tumani 51-maktab matematika o'qituvchisi
Normatova Ozoda Asror qizi
Toshkent davlat pedagogika universiteti talabasi

https://doi.org/10.5281/zenodo.6634868

Annotasiya. *Ushbu maqolada aniq integralni ta'rif yordamida hisoblashga oid misollar yechilgan. Shuningdek, aniq integral yordamida ba'zi bir limitlarga doir misollarni hisoblash o'rganib chiqilgan.*

Kalit so'zlar: *interval, limit, uzluksiz funksiya, aniq integral.*

НЕКОТОРЫЕ ПРИЛОЖЕНИЯ ОПРЕДЕЛЕННОГО ИНТЕГРАЛА.

Аннотация. *В этой статье приведены некоторые примеры того, как вычислить определенный интеграл по определению. А также исследуются некоторые примеры пределов с использованием определенных интегралов.*

Ключевые слова: *интервал, предел, непрерывная функция, определенный интеграл.*

SOME APPLICATIONS OF THE DEFINITE INTEGRAL.

Abstract. This article provides some examples of how to calculate a definite integral by definition. And some examples of limits using definite integrals are also explored.

Keywords: interval, limit, continuous function, definite integral.

KIRISH

Limitlarga oid misollarni yechish matematik analizni o'qitishning muhim yo'nalishini tashkil etadi va bu sohada talabalarni o'rganishni qo'llab-quvvatlovchi o'quv yondashuvlari bo'yicha tadqiqotlarga ehtiyoj bor.

Ushbu maqolada ba'zi bir limitlarga doir misollarni aniq integral yordamida yechishni o'rganib chiqildi.

TADQIQOT MATERIALLARI VA METODOLOGIYASI

Quyidagi masala, chegaralangan oraliqda uzluksiz funksiyasining aniq integralining ta'rifini o'z ichiga oladi. $[a,b]$ oralig'ida $y=f(x)$ uzluksiz funksiya berilgan bo'lsin. Ushbu bo'laklar

$$a=x_0, x_1, x_2, ..., x_{n-2}, x_{n-1}, x_n=b$$

$[a,b]$ oralig'ining ixtiyoriy (tasodifiy tanlangan) bo'laklari bo'lib, u intervalni n ichki intervallarga (bo'laklarga) ajratadi. Ichki intervallardan

$$c_1, c_2, ..., c_{n-2}, c_{n-1}, c_n$$

nuqtalar ixtiyoriy tanlangan bo'lib,

$c_1 \in [x_0, x_1]$, $c_2 \in [x_1, x_2]$,..., $c_{n-2} \in [x_{n-3}, x_{n-2}]$, $c_{n-1} \in [x_{n-2}, x_{n-1}]$, $c_n \in [x_{n-1}, x_n]$ bo'ladi.

Har bir ichki intervalning uzunligi $\Delta x_w = x_w - x_{w-1}$ $(w=1,2,3,...,n)$ va eng katta ichki intervalning uzunligi $\lambda = \max\limits_{1\le w\le n} \Delta x_w$ orqali belgilaymiz.

$y=f(x)$ funksiyaning $[a,b]$ oralig'idagi aniq integrali odatda quyidagicha aniqlanadi.

$$\int_a^b f(x)dx = \lim_{\lambda\to 0}\sum_{w=1}^{n} f(c_w)\Delta x_w .$$

Shunday qilib, har bir ichki intervalning uzunligi bor

$$\Delta x_w = \frac{b-a}{n},\ (w=1,2,3,...,n) \qquad (\delta)$$

bo’lib, hisoblashni qulaylashtirish uchun $c_w\ (w=1,2,3,...,n)$ ni n ta ichki intervallarning o'ng tomonidagi so'nggi nuqtalarni tanlagan nuqtalaridan foydalaniladi. O‘ng tomondagi so‘nggi nuqta formulasi

$$c_w = a + \left(\frac{b-a}{n}\right)\cdot w, \quad (w=1,2,3,...,n) \qquad (\delta\delta).$$

$y = f(x)$ funksiyaning $[a,b]$ oralig'idagi aniq integrali quyidagicha

$$\int_a^b f(x)dx = \lim_{\lambda\to 0}\sum_{w=1}^{n} f(c_w)\Delta x_w$$

aniqlanadi.

Bizga quyidagi taniqli yig'indi qoidalari kerak bo'ladi.

1. $\sum_{w=1}^{n} c = c+c+c+...+c = n\cdot c,\ bu\ \ yerda\ c\ o'zgarmas$;
2. $\sum_{w=1}^{n} w = 1+2+3+...+n = \frac{1+n}{2}\cdot n$;
3. $\sum_{w=1}^{n} w^2 = 1^2+2^2+3^2+...+n^2 = \frac{n\cdot(1+n)\cdot(2n+1)}{6}$;
4. $\sum_{w=1}^{n} w^3 = 1^3+2^3+3^3+...+n^3 = \frac{n^2\cdot(1+n)^2}{4}$;
5. $\sum_{w=1}^{n} c\cdot f(w) = c\cdot\sum_{w=1}^{n} f(w),\ bu\ \ yerda\ c\ o'zgarmas$;
6. $\sum_{w=1}^{n}\left[f(w)\pm g(w)\right] = \sum_{w=1}^{n} f(w) \pm \sum_{w=1}^{n} g(w)$.

TADQIQOT NATIJALARI

Quyidagi muammolarning aksariyati o'rtacha va ba'zilari biroz murakkab. Agar siz ushbu muammolarni hal qilish yo'llarini izlashdan oldin sinab ko'rmoqchi bo'lsangiz, yuqoridagi formulalarni xuddi shunday ishlatib, keng tarqalgan xatolardan qochishingiz mumkin. Quyidagi muammoni hal qilishda yuqoridagi (δ) va $(\delta\delta)$ tengliklarda ko'rsatilganidek, teng o'lchamdagi ichki intervallar va ichki nuqtalari sifatida o'ng nuqtalardan foydalaniladi.

***Misol* 1:** Aniq integralni ta'rifidan foydalanib toping. $\int_{-4}^{0}(x-2)dx$.

***Yechish*.** $[-4,0]$ oralig'ni n ta teng qismlarga bo'lamiz

$$\Delta x_w = \frac{0-(-4)}{n} = \frac{4}{n}, \ (w=1,2,3,...,n).$$

Ichki intervallarning o'ng tomonidagi so'nggi nuqtalarini c_w sifatida tanlaymiz

$$c_w = -4 + \left(\frac{0-(-4)}{n}\right)\cdot w = -4 + \frac{4\cdot w}{n}, \quad (w=1,2,3,...,n).$$

Funksiya $f(x) = x-2$ bo'lib, aniq integral quyidagicha hisoblanadi:

$$\int_{-4}^{0}(x-2)\,dx = \lim_{n\to 0}\sum_{w=1}^{n} f(c_w)\Delta x_w = \lim_{n\to 0}\sum_{w=1}^{n} f\left(-4+\frac{4\cdot w}{n}\right)\cdot\left(\frac{4}{n}\right) = \lim_{n\to 0}\sum_{w=1}^{n}\left(\left(-4+\frac{4\cdot w}{n}\right)-2\right)\cdot\left(\frac{4}{n}\right) =$$

$$= \lim_{n\to 0}\sum_{w=1}^{n}\left(-6+\frac{4\cdot w}{n}\right)\cdot\left(\frac{4}{n}\right) = \lim_{n\to 0}\sum_{w=1}^{n}\left(\frac{-24}{n}+\frac{16\cdot w}{n^2}\right) = \lim_{n\to 0}\left\{\sum_{w=1}^{n}\frac{-24}{n}+\sum_{w=1}^{n}\frac{16\cdot w}{n^2}\right\} =$$

$$= \lim_{n\to 0}\left\{n\cdot\left(\frac{-24}{n}\right)+\frac{16}{n^2}\cdot\sum_{w=1}^{n} w\right\} = \lim_{n\to 0}\left\{-24+\frac{16}{n^2}\cdot\frac{n\cdot(n+1)}{2}\right\} = \lim_{n\to 0}\left\{-24+\frac{8\cdot(n+1)}{n}\right\} =$$

$$= \lim_{n\to 0}\left\{-24+8\cdot\left(\frac{n}{n}+\frac{1}{n}\right)\right\} = \lim_{n\to 0}\left\{-24+8\cdot\left(1+\frac{1}{n}\right)\right\} = -24+8\cdot(1+0) = -16.$$

***Misol* 2:** Aniq integralni ta'rifidan foydalanib toping. $\int_{0}^{4} x^3\,dx$.

Yechish. $[0,4]$ oralig'ni n ta teng qismlarga bo'lamiz

$$\Delta x_w = \frac{4-0}{n} = \frac{4}{n}, \ (w=1,2,3,...,n).$$

Ichki intervallarning o'ng tomonidagi so'nggi nuqtalarini c_w sifatida tanlaymiz

$$c_w = 0 + \left(\frac{4-0}{n}\right)\cdot w = \frac{4\cdot w}{n}, \quad (w=1,2,3,...,n).$$

Funksiya $f(x) = x^3$ bo'lib, aniq integral quyidagicha hisoblanadi:

$$\int_{0}^{4} x^3 dx = \lim_{n\to 0}\sum_{w=1}^{n} f(c_w)\Delta x_w = \lim_{n\to 0}\sum_{w=1}^{n} f\left(\frac{4\cdot w}{n}\right)\cdot\left(\frac{4}{n}\right) = \lim_{n\to 0}\sum_{w=1}^{n}\left(-\frac{4\cdot w}{n}\right)^3\cdot\left(\frac{4}{n}\right) =$$

$$= \lim_{n\to 0}\sum_{w=1}^{n}\left(\frac{64\cdot w^3}{n^3}\right)\cdot\left(\frac{4}{n}\right) = \lim_{n\to 0}\sum_{w=1}^{n}\left(\frac{256\cdot w^3}{n^4}\right) = \lim_{n\to 0}\left\{\frac{256}{n^4}\sum_{w=1}^{n} w^3\right\} = \lim_{n\to 0}\left\{\frac{256}{n^4}\cdot\frac{n^2\cdot(n+1)^2}{4}\right\} =$$

$$= \lim_{n\to 0}\left\{64\cdot\frac{n^2}{n^2}\cdot\frac{n+1}{n}\cdot\frac{n+1}{n}\right\} = \lim_{n\to 0}\left\{64\cdot\left(1+\frac{1}{n}\right)\cdot\left(1+\frac{1}{n}\right)\right\} = 64\cdot(1+0)\cdot(1+0) = 64.$$

***Misol* 3:** Quyidagi limitni aniq integral orqali hisoblang: $\lim_{n\to\infty}\sum_{w=1}^{n}\frac{2\cdot w^2+n\cdot w}{n^3}$.

Yechish. $\lim_{n\to\infty}\sum_{w=1}^{n}\frac{2\cdot w^2+n\cdot w}{n^3} = \lim_{n\to\infty}\sum_{w=1}^{n}\left[2\cdot\left(\frac{w}{n}\right)^2+\frac{w}{n}\right]\cdot\frac{1}{n}$ bundan c_w nuqtani tanlaymiz

$$c_w = \frac{1}{n}, \ (w = 1, 2, 3, ..., n).$$

Ushbu c_w nuqtalar $[0,1]$ dagi teng o'lchamdagi bo'lakchalarning o'ng tomonidagi so'nggi nuqtalarini ifodalaydi va

$$\Delta x_w = \frac{1}{n}, \ (w = 1, 2, 3, ..., n).$$

Shunday qilib,

$$\lim_{n\to\infty} \sum_{w=1}^{n} \left[2 \cdot \left(\frac{w}{n} \right)^2 + \frac{w}{n} \right] \cdot \frac{1}{n} = \lim_{n\to\infty} \sum_{w=1}^{n} \left[2 \cdot c_w^2 + c_w \right] \cdot \Delta x_w$$

$\left(f(x) = 2 \cdot x^2 + x \ \ bo'ladi. \right)$

$$= \lim_{n\to\infty} \sum_{w=1}^{n} f(c_w) \cdot \Delta x_w = \int_0^1 f(x) dx = \int_0^1 \left(2 \cdot x^2 + x \right) dx = \left(\frac{2}{3} \cdot x^3 + \frac{x^2}{2} \right) \Bigg|_0^1 = \frac{7}{6}.$$

MUHOKAMA

***Misol* 4:** Quyidagi limitni aniq integral orqali hisoblang: $\lim_{n\to\infty} \sum_{w=1}^{n} \left(\frac{1 - w + 2n}{1 - w + n} \right) \cdot \frac{1}{n}$.

Yechish. Yig'indidan c_w nuqtani tanlaymiz.

$$\lim_{n\to\infty} \sum_{w=1}^{n} \left(\frac{w - 1 + 2n}{w - 1 + n} \right) \cdot \frac{1}{n} = \lim_{n\to\infty} \sum_{w=1}^{n} \left(\frac{w - 1 + 2n}{w - 1 + n} \right) \cdot \frac{\frac{1}{n}}{\frac{1}{n}} \cdot \frac{1}{n} = \lim_{n\to\infty} \sum_{w=1}^{n} \left(\frac{\frac{w}{n} - \frac{1}{n} + 2}{\frac{w}{n} - \frac{1}{n} + 1} \right) \cdot \frac{1}{n} =$$

$$= \lim_{n\to\infty} \sum_{w=1}^{n} \left(\frac{\frac{w-1}{n} + 2}{\frac{w-1}{n} + 1} \right) \cdot \frac{1}{n}.$$

Bundan $c_w = \frac{w-1}{n}, \ \ (w = 1, 2, 3, ..., n)$ bo'ladi.

Ushbu $c_w = \frac{w-1}{n} \ \ (w = 1, 2, 3, ..., n)$ nuqtalar $[0, \ 1]$ intervalining n teng o'lchamdagi ichki intervallarning chap tomonidagi nuqtalarini ifodalaydi va $\Delta x_w = \frac{w-1}{n}, \ \ (w = 1, 2, 3, ..., n)$ bo'ladi.

Shunday qilib, $\left(f(x) = \frac{x+2}{x+1} \ \ bo'ladi. \right)$

$$\lim_{n\to\infty}\sum_{w=1}^{n}\left(\frac{\frac{w-1}{n}+2}{\frac{w-1}{n}+1}\right)\cdot\frac{1}{n}=\lim_{n\to\infty}\sum_{w=1}^{n}\left(\frac{c_w+2}{c_w+1}\right)\cdot\Delta x_w=\lim_{n\to\infty}\sum_{w=1}^{n}f\left(c_w\right)\cdot\Delta x_w=\int_0^1 f\left(x\right)dx=\int_0^1\frac{x+2}{x+1}dx=$$

$$=x+\ln\left(x+1\right)\Big|_0^1=1+\ln 2.$$

XULOSA

Tadqiqot shuni ko'rsatdiki, ko'p jihatdan ishlangan misollarni o'rganishga tayanadigan o'qitish muammoni hal qilishga urg'u beradigan ko'rsatmalarga qaraganda tajribasiz o'quvchilar uchun samaraliroqdir. Biroq, ba'zi ishlangan misollarni o'rganish bilan bog'liq yo'l-yo'riq ko'proq tajribali o'quvchilarning ish faoliyatini kamaytirishi mumkin.

Bu yerda ba’zi bir limitlarni topish uchun aniq integral qo'llanilgan. Bu usulni bilish muhim, ammo bu texnikani qachon qo'llashni bilish ham muhimdir.

Foydalanilgan adabiyotlar

1. Azlarov T., Mansurov H. ,, Matematik analiz” I-qism T., ,, O‘qituvchi” 1994 y.
2. Sadullayev A., Mansurov H., Xudoyberganov G., ,,Matematik analiz kursidan misol va masalalar to‘plami ” I-qism T., ,, O‘qituvchi” 2008 y.
3. TOSHMETOV O‘, TURGUNBAYEV R. MATEMATIK ANALIZDAN MISOL VA MASALALAR TO’PLAMI. 1-Q. TDPU. 2006 Y.-140 B.
4. Б.П.Демидович ,,Сборник задач и упражнений по математическому анализу“– Москва: Наука, 1997.
5. Gaziyev A., Israilov I., Yaxshibayev M. “Matematik analizdan misol va masalalar” T.: “Yangi asr avlodi” 2006 y.